TRIBOLOGICAL EFFECTS OF HYALURONIC ACID CONCENTRATION ON ARTICULAR CARTILAGE: PIN-ON-PLATE FRICTION TESTS IN PORCINE AND OSTEOARTHRITIC HUMAN TISSUE

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Hyaluronic acid (HA) is the main biopolymer used in intra-articular injections for osteoarthritis (OA). HA is usually applied at 1–3 mg/mL, though the optimal level remains unclear. The purpose of this study was to determine the effect of HA concentration on the friction in osteoarthrosis.

Samples from the osteoarthritic head of a human femur and porcine controls were tested in a pin-on-plate setup. The results showed a statistically significant effect of HA concentration on the friction in a group of porcine cartilage. In a group of osteoarthritic cartilage, such a relationship did not occur. This comparison highlights that degeneration limits HA's effect.

Keywords: osteoarthrosis; hyaluronic acid; joint friction; tribology; biomechanics.



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1. Introduction

Osteoarthritis (OA) is caused by aging joints and is increasingly influenced by lifestyle choices, lack of physical activity, and a diet based on highly processed foods (Allen & Golightly, 2015). It is one of the main causes of functional disability. It can occur in all joints, but the effects of OA of the hand, knee, hip and spine are usually considered the most dangerous. The symptoms of this disease include joint pain, stiffness, and difficulty in movement. This is the result of changes in the structure of the articular cartilage, which loses its physiological shape, structure, and properties. As a result of these changes, pain occurs during movement, resulting in limited mobility for patients (Burr & Gallant, 2012; Aitken et al., 2020). In order to improve mobility in the affected joint, supplementation with hyaluronic acid (HA) preparations is often used to relieve pain and improve its lubricating conditions (Gaumet et al., 2018; Turajane et al., 2007). HA is a component of synovial fluid (SF) responsible for the proper functioning of joints. In a healthy joint, HA has lubricating and shock absorbing functions, reducing friction between surfaces. HA provides the joint lubricant with high viscosity and elasticity, which protects it against mechanical overload. Various authors have analyzed the effect of HA concentration and



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molar mass on the coefficient of friction (CoF) of articular cartilage. De Roy et al. (2024) and Snetkov et al. (2020) showed that cartilage friction is mainly determined by its microscopic structure, while viscoelastic properties are additionally related to macroscopic structure. Viscoelastic and frictional properties showed a weak correlation. Caligaris et al. (2009) studied the effect of OA degeneration on the friction coefficient value. They assessed friction on seven specimens with a degeneration stage ≤ 2 and nine specimens $> 2 \leq 3$ on the ICRS scale. They found no statistically significant differences between the friction coefficient value and the degree of OA. Neu et al. (2010) investigated that CoF of femoral cartilage samples correlates positively with the severity of OA. These results are not consistent, in addition these studies were conducted on HA solutions produced under laboratory conditions, not on HA preparations used for SF supplementation by injection into the joint.

The aim of the study: In clinical practice, the most commonly used preparations contain between 1 mg/mL and 3 mg/mL of hyaluronic acid. However, from a medical point of view, there are no clear recommendations for injecting a preparation containing a specific concentration of HA (Snetkov et al., 2020; Jin & Dowson, 2013). The study conducted here was designed to answer how the concentration of hyaluronic acid in an HA preparation for injection into the joint affects the reduction of cartilage friction. The lubrication efficacy of preparations with different hyaluronic acid contents was evaluated based on the value of the friction coefficient between the articular cartilage and surgical stainless steel.

2. Material and methods

2.1. Material

The study used 18 osteoarthritic cartilage samples taken from 9 heads of osteoarthritic human femur. Two cylindrical specimens of 10 mm in diameter and 15 mm in height were taken from each head. The femoral heads were obtained from patients undergoing hip replacement.

A single freeze-thaw protocol was applied for sample preservation. The heads were immediately frozen at $-22\,^{\circ}$ C after collection. Before examination, they were thawed for 8 hours at $23\,^{\circ}$ C, followed by sampling and examination. The storage protocol was selected based on the findings of Szarko *et al.* (2010), who demonstrated that freezing articular cartilage at $-20\,^{\circ}$ C or $-80\,^{\circ}$ C, followed by controlled thawing at room temperature, maintains the tissue's mechanical properties without causing significant changes.

Figure 1 shows the process of extracting samples for testing. The authors had permission from the local ethics committee to collect and use the material for the study. As a lubricant, a commercially available intra-articular injection product containing 2.2 % high-molecular-weight hyaluronic acid was used. To obtain lower concentrations, the product was diluted with deionized water.

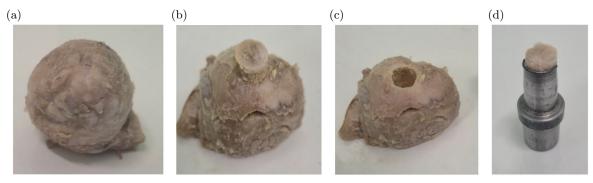


Fig. 1. Steps of sampling for testing: (a) osteoarthritic femoral head; (b), (c) specimen extraction; (d) collected sample.

The control group consisted of 18 samples of pig cartilage taken from pig femoral heads. The bones were obtained from a local slaughterhouse from pigs of the Polish White breed. The heads cut from the bones were frozen immediately after slaughter. The samples were prepared for testing and stored in the same way as human bones.

2.2. Friction coefficient measurements

The tests were conducted using the pin-on-plate method. In this method, an articular cartilage sample was mounted on a stationary pin, while a flat stainless steel counter-sample moved in a rectilinear motion on a moving table. A lubricant containing HA was placed between the samples. Compared with the pin-on-disk method, the principal advantage of the pin-on-plate method is that it ensures a constant relative linear speed between the sample and the counter-sample. In the pin-on-disk method, a stationary pin is in contact with a rotating disk. The disadvantage of this method is that the linear velocity of relative motion between the samples depends on the distance from the axis of rotation of the disk. The linear velocity of the subarea of the sample located on the axis of the rotating disk is zero, whereas subareas farthest from the axis of rotation have the maximum linear velocity. Therefore, the test using the pin-on-plate method more closely reflects the real conditions of movement in the joint. The device used for the study was described by Gordon et al. (2014).

ISO 7206 and ISO 14242 series of standards are frequently used standards for evaluating the wear characteristics of hip implants. They specify methods of measurement, values of loads used for testing, directions of application, environmental conditions of testing and others. Based on an analysis of the parameter values recommended in these standards and those used by other researchers (Furmann et al., 2020; Caligaris et al., 2009), a dedicated test program was developed. The speed of movement between the two samples was 0.05 m/s, which corresponds to slow walking (Furmann et al., 2020). Each test was divided into 5 cycles. Each cycle contained 2 steps: movement and rest. The movement time was 2 seconds followed by a 2-second break. Therefore, one test contained 5 cycles of movement and rest. This was to reflect the way the femoral head is loaded during walking, when it is loaded with body weight in the stance phase and unloaded in the swing phase. The reciprocal pressing force of the samples was 10 N (Furmann et al., 2020).

To lubricate the surfaces, preparations used for intra-articular injections containing HA at concentrations of: 1.0%, 1.5%, 1.8%, 2.0%, and 2.2% HA were used. The preparations did not contain other substances that can affect the coefficient of friction.

3. Results

Tables 1 and 2 show the mean value of the coefficient of friction, median, standard deviation, minimum and maximum values measured for the sample groups tested (pork cartilage and osteoarthritic cartilage). Additionally, Fig. 2 presents these data as bar charts, which allows a direct visual comparison between the two groups.

Parameter	HA concentration in lubricant					
	1.0 %	1.5 %	1.8 %	2.0%	2.2%	
Mean value of the friction coefficient, μ	0.019	0.016	0.015	0.013	0.012	
Standard deviation, SD	0.005	0.004	0.003	0.004	0.003	
Minimum value	0.012	0.011	0.011	0.005	0.006	
Maximum value	0.029	0.023	0.022	0.020	0.019	
Median	0.019	0.016	0.015	0.013	0.0115	

Table 1. Friction coefficient values for pork cartilage – stainless steel pairs.

Parameter	HA concentration in lubricant					
	1.0 %	1.5%	1.8%	2.0%	2.2%	
Mean value of the friction coefficient, μ	0.033	0.031	0.029	0.028	0.026	
Standard deviation, SD	0.007	0.007	0.004	0.004	0.007	
Minimum value	0.021	0.021	0.024	0.021	0.019	
Maximum value	0.044	0.042	0.037	0.035	0.040	
Median	0.033	0.032	0.031	0.029	0.025	

Table 2. Friction coefficient values for osteoarthritic cartilage – stainless steel pairs.

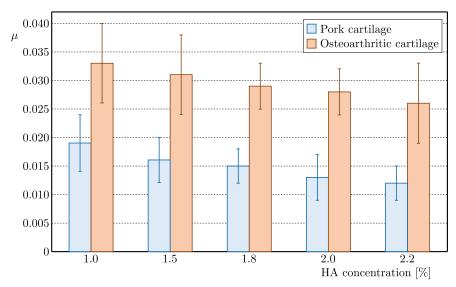


Fig. 2. Effect of hyaluronic acid concentration on the friction coefficient (μ) in porcine and osteoarthritic human cartilage (means with SD error bars).

A statistical analysis of the results was carried out to assess the differences in friction coefficients for lubricants with different HA contents. As a first step, the Kolgomorov–Smirnov and Levene's test was performed at a significance level of $\alpha=0.05$ to check the type of distribution of results. In each sample group, the results had a normal distribution and equal variances. Further analyses of the significance of differences in the coefficient of friction for lubricants with different HA contents were performed using Anova's one-way analysis at a significance level of $\alpha=0.05$. Tukey's test was used to determine which groups had statistically significant differences in the mean values of the friction coefficient. The results of the statistical tests are shown in Tables 3 and 4. All analyses were performed using Statistica 13 software (StatSoft, PL).

Table 3. Tukey's test results for the pork cartilage – stainless steel pair.

HA concentration	1.0%	1.5%	1.8 %	2.0%	2.2 %
1.0 %	_	NS	S	S	S
1.5%	_	_	NS	NS	S
1.8 %	_	_	_	NS	S
2.0 %	_	_	_	_	NS
2.2 %	_	_	_	_	_

An S value means that the difference in mean values between the two groups is statistically significant. An NS value means that statistically, there is no difference between the mean values in two particular groups.

HA concentration	1.0 %	1.5%	1.8 %	2.0%	2.2%
1.0 %	_	NS	NS	NS	NS
1.5%	_	_	NS	NS	NS
1.8 %	_	_	_	NS	NS
2.0 %	_	_	_	_	NS
2.2%	_	_	_	_	_

Table 4. Tukey's test results for the osteoarthritic cartilage – stainless steel pair.

4. Discussion

The friction coefficient values obtained from the study for the pork cartilage-steel friction pair were in the range of 0.019–0.012, while for human osteoarthritic cartilage, they were in the range of 0.033–0.026. For pork cartilage, these values are in line with literature data, i.e., 0.005–0.02 (Furmann et al., 2020; Jin & Dowson, 2013). For osteoarthritic cartilage, these values are higher than for healthy cartilage but comparable with results obtained by other authors (Caligaris et al., 2009).

In both test groups, the average value of the coefficient of friction decreased as the concentration of HA in the lubricant increased. However, statistical analyses showed that the effect of HA concentration in the lubricant on the coefficient of friction was significant only for the pork cartilage. No such relationship was found in the OA group. Porcine samples were the reference group, as cartilage in samples from this group had no pathological changes. The use of human articular cartilage without pathological features in the reference group was not possible due to the lack of approval from the local ethics committee. Nevertheless, porcine cartilage is widely accepted as a suitable model for human articular cartilage (Fackler et al., 2023). Furthermore, the mechanical properties of swine cartilage, including stiffness under defined loading conditions, have been reported to approximate those of human tissue, further supporting its application in biomechanical evaluations (Ronken et al., 2012).

The statistically significant differences, or lack thereof, observed in the study can be explained at the molecular level. At this scale, the variation in concentration-dependent response is primarily determined by the condition of the cartilage surface. In healthy tissue, densely hydrated hyaluronic acid coils adsorb onto the phospholipid-rich superficial zone, forming electrostatic interactions with both phosphatidylcholine head groups and the underlying collagen network. This promotes the formation of a continuous hydration film that substantially reduces friction. In contrast, degenerative changes associated with OA disrupt this lipid-protein interface and expose denatured collagen fibrils, thereby limiting the availability of effective HA-binding sites, which likely accounts for the absence of statistically significant differences observed in osteoarthritic tissue. This interpretation is supported by findings from NMR-based compression experiments, which showed that enzymatic degradation of the collagen fibrillar network leads to mechanical softening and an almost complete loss of swelling capacity due to impaired fluid pressurization and a disrupted pore structure (Greene et al., 2012). These structural alterations reduce the tissue's ability to interact effectively with HA and to maintain a functional lubrication environment under load.

It is important to emphasize that certain methodological challenges are inherent to studies involving biological tissues, such as cartilage. In the case of cartilage, as in the case of the study of other tissues (Kohut *et al.*, 2021; Aleksandrowicz, 2020), the evaluation of biomechanical characteristics by methods used to test structural materials is not straightforward, and the accuracy of measurement may be unsatisfactory. This is due to the specific characteristics of the material, the difficulty of determining the actual way in which the tissue is loaded in the body, and choosing the correct method of conducting the test.

Cartilage lubrication in the joint occurs in two ways: by compression of the interstitial fluid (Ateshian et al., 1998; Krishnan et al., 2004) and boundary lubrication by the SF (Schmidt & Sah, 2007a; Schmidt et al., 2007b). Caligaris et al. (2009) showed that lubrication by compression of interstitial fluid is usually much more effective than boundary lubrication by SF. During OA, the structure of collagen fibers in the upper layers of the particle is damaged, and consequently, its porosity and permeability are higher. Therefore, during cartilage deformation, the increase in fluid pressure in the cartilage matrix with OA is not as great as in healthy cartilage.

In our study, the cartilage samples were 10 mm in diameter, while the steel counter-sample was flat and smooth. Due to the spherical structure of the articular surfaces, it is impossible to obtain relatively flat specimens with larger dimensions on which to perform a more accurate test. The dimensions of the specimen made it difficult to obtain the correct fluid pressure in the cartilage, due to the extrusion of fluid from the specimen and the lack of fluid flow throughout the cartilage. This could also have affected the accuracy of the measurement.

The next factor to analyze was the speed at which the test was conducted. Tests were conducted at the speed of reciprocal surface motion corresponding to slow walking, i.e., $0.05\,\mathrm{m/s}$. At other speeds, due to the non-Newtonian nature of the fluid, the friction coefficient values may be different.

Another factor is changes in morphology in the subchondral layer and the trabecular bone that supports the cartilage. As a result of OA, the shape, structure, as well as mineral content of these tissues may change (Cichański *et al.*, 2010; Topoliński *et al.*, 2012a; 2012b). As a result, the elasticity of the cartilage may also change.

Balazs (2004) showed that the intramedullary injection of HA improves the viscoelasticity and fluidity of SF, alleviates the effects of OA, prevents symptoms of the disease, and allows postponement of surgery. However, it is difficult to determine whether the concentration of injectable HA affects the duration of effective impact. It is highly dependent on the individual characteristics of the patient and many factors, such as the degree of joint damage, the patient's weight, and level of physical activity.

5. Conclusions

Frictional performance of articular cartilage reflects the interplay between tissue condition and the properties of the lubricating medium. To provide a clear, application-oriented summary, we evaluated how stepwise changes in HA concentration affect the coefficient of friction using a standardized pin-on-plate protocol within a range relevant to viscosupplementation practice.

In pin-on-plate friction testing, increasing hyaluronic-acid concentration from $1.0\,\%$ to $2.2\,\%$ was associated with a progressive reduction of the friction coefficient in porcine articular cartilage, with several pairwise comparisons reaching statistical significance. In osteoarthritic human cartilage, friction remained consistently higher across the same concentration range and between-concentration differences did not reach significance under the present protocol.

These findings, obtained within a concentration range commonly used in viscosupplementation, highlight the practical importance of reporting and controlling HA content in tribological assessments. For non-degenerate tissue, higher HA levels can yield a tangible reduction in friction; for osteoarthritic tissue, adjusting HA concentration alone may be insufficient, suggesting the value of exploring more physiologically representative lubricants or combined approaches. Future work should expand the number of specimens per concentration and examine broader loading and speed conditions.

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References

- 1. Aitken, D., Jones, G., & Winzenberg, T.M. (2020). Clinical overview of osteoarthritis (OA) and the challenges faced for future management. In S.I.S. Rattan (Ed.), *Encyclopedia of Biomedical Gerontology* (pp. 420–430). Academic Press. https://doi.org/10.1016/B978-0-12-801238-3.11419-9
- 2. Aleksandrowicz, P. (2020). Modeling head-on collisions: The problem of identifying collision parameters. Applied Sciences, 10(18), Article 6212. https://doi.org/10.3390/app10186212
- 3. Allen, K.D., & Golightly, Y.M. (2015). State of the evidence. *Current Opinion in Rheumatology*, 27(3), 276–283. https://doi.org/10.1097/BOR.000000000000161
- 4. Ateshian, G.A., Wang, H., & Lai, W.M. (1998). The role of interstitial fluid pressurization and surface porosities on the boundary friction of articular cartilage. *Journal of Tribology*, 120(2), 241–248. https://doi.org/10.1115/1.2834416
- 5. Balazs, E.A. (2004). Viscosupplementation for treatment of osteoarthritis: from initial discovery to current status and results. *Surgical Technology International*, 12, 278–289.
- 6. Burr, D.B., & Gallant, M.A. (2012). Bone remodelling in osteoarthritis. *Nature Reviews Rheumatology*, 8, 665–673. https://doi.org/10.1038/nrrheum.2012.130
- Caligaris, M., Canal, C.E., Ahmad, C.S., Gardner, T.R., & Ateshian, G.A. (2009). Investigation of the frictional response of osteoarthritic human tibiofemoral joints and the potential beneficial tribological effect of healthy synovial fluid. Osteoarthritis and Cartilage, 17(10), 1327–1332. https://doi.org/ 10.1016/j.joca.2009.03.020
- 8. Cichański, A., Nowicki, K., Mazurkiewicz, A., & Topoliński, T. (2010). Investigation of statistical relationships between quantities describing bone architecture, its fractal dimensions and mechanical properties. *Acta of Bioengineering and Biomechanics*, 12(4), 69–77.
- 9. Comper, W.D., & Laurent, T.C. (1978). Physiological function of connective tissue polysaccharides. *Physiological Reviews*, 58(1), 255–315. https://doi.org/10.1152/physrev.1978.58.1.255
- De Roy, L., Teixeira, G.Q., Schwer, J., Sukopp, M., Faschingbauer, M., Ignatius, A., & Seitz, A.M. (2024). Structure-function of cartilage in osteoarthritis: An ex-vivo correlation analysis between its structural, viscoelastic and frictional properties. Acta Biomaterialia, 190, 293–302. https://doi.org/10.1016/j.actbio.2024.10.027
- Fackler, N.P., Donahue, R.P., Bielajew, B.J., Amirhekmat, A., Hu, J.C., Athanasiou, K.A., & Wang, D. (2023). Characterization of the age-related differences in porcine acetabulum and femoral head articular cartilage. *Cartilage*, 16(3), 366–375. https://doi.org/10.1177/19476035231214724
- 12. Furmann, D., Nečas, D., Rebenda, D., Čipek, P., Vrbka, M., Křupka, I., & Hartl, M. (2020). The effect of synovial fluid composition, speed and load on frictional behaviour of articular cartilage. *Materials*, 13(6), Article 1334. https://doi.org/10.3390/ma13061334
- Gaumet, M., Badoud, I., & Ammann, P. (2018). Effect of hyaluronic acid-based viscosupplementation on cartilage material properties. Osteoarthritis and Cartilage, 26 (Supplement 1), S136. https://doi.org/10.1016/j.joca.2018.02.294
- 14. Gordon, M., Słomion, M., & Mazurkiewicz, A. (2014). Design device for friction coefficient examination of articular cartilage. In 31st Danubia Adria Symposium on Advances in Experimental Mechanics. September 24–27, 2014 Kempten (Germany). Proceedings (pp. 90–91). VDI Verein Deutscher Ingenieure. https://owncloud.tuwien.ac.at/index.php/s/2IakbkmqJceQx44?dir=/&editing=false&openfile=true
- 15. Greene, G.W., Zappone, B., Banquy, X., Lee, D.W., Söderman, O., Topgaard, D., & Israelachvili, J.N. (2012). Hyaluronic acid–collagen network interactions during the dynamic compression and recovery of cartilage. *Soft Matter*, 8(38), 9906–9914. https://doi.org/10.1039/c2sm26330k
- 16. Jin, Z., & Dowson, D. (2013). Bio-friction. Friction, I(2), 100-113. https://doi.org/10.1007/s40544-013-0004-4
- 17. Kohut, P., Holak, K., Ekiert, M., Młyniec, A., Tomaszewski, K.A., & Uhl, T. (2021). The application of digital image correlation to investigate the heterogeneity of Achilles tendon deformation and

determine its material parameters. Journal of Theoretical and Applied Mechanics, 59(1), 43–52. https://doi.org/10.15632/jtam-pl/127903

- 18. Krishnan, R., Kopacz, M., & Ateshian, G.A. (2004). Experimental verification of the role of interstitial fluid pressurization in cartilage lubrication. *Journal of Orthopaedic Research*, 22(3), 565–570. https://doi.org/10.1016/j.orthres.2003.07.002
- 19. Neu, C.P., Reddi, A.H., Komvopoulos, K., Schmid, T.M., & Di Cesare, P.E. (2010). Increased friction coefficient and superficial zone protein expression in patients with advanced osteoarthritis. *Arthritis & Rheumatology*, 62(9), 2680–2687. https://doi.org/10.1002/art.27577
- Ronken, S., Arnold, M.P., Ardura García, H., Jeger, A., Daniels, A.U., & Wirz, D. (2012). A comparison of healthy human and swine articular cartilage dynamic indentation mechanics. *Biomechanics and Modeling in Mechanobiology*, 11(5), 631–639. https://doi.org/10.1007/s10237-011-0338-7
- 21. Schmidt, T.A., & Sah, R.L. (2007a). Effect of synovial fluid on boundary lubrication of articular cartilage. Osteoarthritis and Cartilage, 15(1), 35–47. https://doi.org/10.1016/j.joca.2006.06.005
- 22. Schmidt, T.A., Gastelum, N.S., Nguyen, Q.T., Schumacher, B.L., & Sah, R.L. (2007b). Boundary lubrication of articular cartilage: Role of synovial fluid constituents. *Arthritis & Rheumatology*, 56(3), 882–891. https://doi.org/10.1002/art.22446
- 23. Snetkov, P., Zakharova, K., Morozkina, S., Olekhnovich, R., & Uspenskaya, M. (2020). Hyaluronic acid: The influence of molecular weight on structural, physical, physico-chemical, and degradable properties of biopolymer. *Polymers*, 12(8), Article 1800. https://doi.org/10.3390/polym12081800
- 24. Szarko, M., Muldrew, K., & Bertram, J.E.A. (2010). Freeze-thaw treatment effects on the dynamic mechanical properties of articular cartilage. *BMC Musculoskeletal Disorders*, 11, Article 231. https://doi.org/10.1186/1471-2474-11-231
- 25. Topoliński, T., Cichański, A., Mazurkiewicz, A., & Nowicki, K. (2012a). The relationship between trabecular bone structure modeling methods and the elastic modulus as calculated by FEM. *The Scientific World Journal*, 2012(1), Article 827196. https://doi.org/10.1100/2012/827196
- 26. Topoliński, T., Cichański, A., Mazurkiewicz, A., & Nowicki, K. (2012b). Applying a stepwise load for calculation of the S-N curve for trabecular bone based on the linear hypothesis for fatigue damage accumulation. *Materials Science Forum*, 726, 39–42. https://doi.org/10.4028/www.scientific.net/MSF.726.39
- 27. Turajane, T., Tanavaree, A., Labpiboonpong, V., & Maungsiri, S. (2007). Outcomes of intra-articular injection of sodium hyaluronate for the treatment of osteoarthritis of the knee. *Journal of the Medical Association of Thailand*, 90(9), 1845–1852. http://www.jmatonline.com/PDF/90-PB-1845-1852.pdf

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