JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
https://doi.org/10.15632/jtam-pl/202666
ONLINE FIRST

PROBABILISTIC ESTIMATION OF THE DYNAMIC GAIT PARAMETERS†

Tomasz WALCZAK[1], Michał GUMINIAK[2]*, Marcin KAMIŃSKI[3]

[1] *Institute of Applied Mechanics, Poznan University of Technology, Poznan, Poland*
[2] *Institute of Structural Analysis, Poznan University of Technology, Poznan, Poland*
[3] *Faculty of Civil Engineering, Architecture & Environmental Engineering, Lodz University of Technology, Lodz, Poland*

*corresponding author, *michal.guminiak@put.poznan.pl*

This study presents an estimation of the dynamic parameters of gait with a random approach. The data necessary for random analysis was obtained through laboratory tests. The study was conducted on a group of healthy people aged 20–25, without diagnosed musculoskeletal diseases. It consisted in walking along a several-metre-long path at free speed and recording the ground reaction forces (GRF) for both limbs using dynamometric platforms. On this basis, the basic dynamic parameters of gait, such as maxima and local minima of the stance phase were determined, and then they were subjected to stochastic analysis.

Keywords: dynamic gait parameters; semi-analytical probabilistic approach; stochastic perturbation technique; Monte Carlo simulation.

1. Introduction

Gait, as the basic form of locomotion, is the most frequently studied human activity. Walking properly is one of the distinguishing features of good health and condition of the examined person. Conversely, disturbed gait with abnormal parameters indicates potential problems with the musculoskeletal system, past injuries, joint dysfunctions, etc. Therefore, it is important to choose the right parameters that quantify the gait characteristics of the examined person. Among the parameters most frequently analyzed by researchers, a group of spatiotemporal and dynamic parameters can be distinguished. The parameters from the first group reflect the geometry and kinematics of the gait (spatial and kinematic parameters of the body and its segments), while the dynamic parameters reflect the forces and moments acting on the segments during walking. The most frequently analyzed ones include the characteristic course of ground reaction forces (GRF) measured during the limb support phase (Derlatka & Parfieniuk, 2023; Fryzowicz *et al.*, 2018; Michałowska *et al.*, 2018; Richards *et al.*, 2023). New indices are also defined to quantify walking behavior based on ground reaction force, e.g., (Park & Kim, 2022). Measured with dynamometer platforms or tensometric mats, they indicate whether the feet and joints of the lower limbs are correctly loaded, and the asymmetry occurring in the loads between the limbs. The most important parameters obtained during this type of research certainly include the parameters characterizing the course of the vertical component of the GRF, most often expressed in the percentage of the body weight of the examined person. Although it is generally known what the correct shape of the curve representing these reactions is, it depends on many factors such as body weight, gait speed, age, overall health of the person, potential dysfunctions of the musculoskeletal system, etc. Therefore, it is very difficult to determine in the studied group of

† The content of this article was presented during the 31st Conference Vibrations in Physical and Technical Systems – VIBSYS, Poznań, Poland, October 16–18, 2024.

people what a deviation from the norm is, despite the fact that the studied group is homogeneous, e.g., a group of athletes, people after a knee joint injury, or healthy people. The present paper proposes a probabilistic approach to gait analysis based on gait studies of a homogeneous group of healthy, young adults without diagnosed musculoskeletal diseases, where the relationship between key parameters describing the characteristics of the vertical component of GRF, depending on the mass of the examined person, was analyzed. Joint kinematics measurement plays the main role in describing the loads and kinematics of gait (Żuk & Trzeciak, 2017). A similar study on injuries of anterior cruciate ligament (ACL) in terms of stochastic approach was deeply examined (Lin *et al.*, 2012), where the Monte Carlo simulation (MCS) technique found application.

2. Formulation of the problem using a probabilistic approach

A finite number of deterministic solutions is necessary to carry out the probabilistic calculations. A Gaussian probability distribution of the observed design parameter is assumed. Due to the continuous distribution of the Gaussian probability density function $p_v(x)$, where x is the domain of occurrence of a given phenomenon, it is necessary to perform the continuous response function based on a finite number of deterministic results. Polynomial approximations using the least squares method (LSM) have been adopted to obtain response curves basing on a finite number of deterministic results. This allows us to determine the so-called system response fitting curves in the form of polynomials, using the LSM:

$$G = \sum_{j=0}^{n} C_j \cdot v^j. \tag{2.1}$$

This way, it is possible to express the probabilistic solution to each problem within the range determined by the coefficient of variation of the random design parameter. The semi-analytical method (SAM), the stochastic perturbation technique (SPT) and the MCS will be independently used to carry out the random analysis. Polynomials of the third order were adopted as fitting functions. The SAM is based on symbolic calculation procedures in the Maple program. All procedures of the SPT of the tenth order were carried out using the Maple program. Having this, probabilistic moments are calculated, i.e., the expectation (E), standard deviation (σ), coefficient of variation (α), skewness (β) and kurtosis (κ) (Kamiński, 2013; Kamiński *et al.* 2024):

$$\begin{gathered}
E(G) = \int_{-\infty}^{+\infty} \sum_{j=0}^{n} C_{ij} v^j p_v(x)\,\mathrm{d}x, \\
\sigma(G) = \left\{ \int_{-\alpha}^{\alpha} \left(\sum_{j=0}^{n} C_{ij} v^j - E[G] \right)^2 p_v(x)\,\mathrm{d}x \right\}^{1/2}, \\
\alpha(G) = \left| \frac{\sigma(G)}{E(G)} \right|, \qquad \beta(G) = \frac{\mu_3(G)}{\sigma^3(G)}, \qquad \kappa(G) = \frac{\mu_4(G)}{\sigma^4(G)}.
\end{gathered} \tag{2.2}$$

MCS with the number of trials equal to 10^5 was carried out using the Maple program too, with the fact that probabilistic moments are calculated from statistical formulas. The primary objective of this study is to investigate the most important gait parameters in the examined group, employing a random approach based on weight.

3. Laboratory experiment

The study involved measuring the reaction forces of the ground as the test subject walked along a designated path at varying speeds in a natural gait. For this purpose, two dynamometric platforms, AMTI BP400600 with frequency of sampling 400 Hz, were used. An example of the vertical component of GRF with two characteristic maximum values: F_h – maximum force gained during heel strike, F_f – maximum force gained during terminal stance (forefoot press) for one limb during gait is shown in Fig. 1.

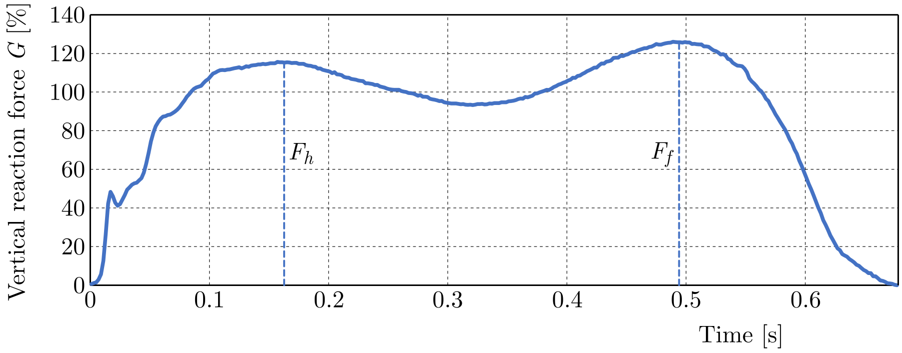

Fig. 1. Vertical component of GRF in % of body weight during gait.

The two maximum values presented in Fig. 1, characteristic of each vertical component of the GRF, were subjected to probabilistic analysis.

4. Probabilistic estimation of measured laboratory data

Loadings of the body under consideration were all found via the polynomial basis:

$$G = \sum_{j=0}^{n} C_j \cdot X^j. \tag{4.1}$$

4.1. Example 1 – heel strike maximum (F_h)

The response function is expressed as the third order polynomial:

$$G = -0.0659583377410756 - 1.20410359255101 \cdot X + 0.15008617465672 \cdot X^2 - 0.00167560644573114 \cdot X^3. \tag{4.2}$$

Figure 2 presents the results of the simulation of the expected values, kurtosis and skewness depending on the coefficient of variation, based on SPT, MSC and SAM methods.

Assuming that, according to the known rules, the desired distribution of a random variable is one in which the coefficient of variance is less than 5 %, we will notice that for the analyzed variable, this means an expected value within 110G for the studied group. In order for the distribution to be treated as normal, the kurtosis should have a value of 3 and the skewness should be as close to 0 as possible. It can be observed that in order for the conditions of normality of the distribution to be met, the deviation from the mean should not be greater than 2 %–3 %.

4.2. Example 2 – forefoot maximum press (F_f)

The response function is expressed as the third order polynomial:

$$G = 1.16382439132543 + 21.2823173261835 \cdot X - 0.664821019062622 \cdot X^2 + 0.00568578395391626 \cdot X^3. \tag{4.3}$$

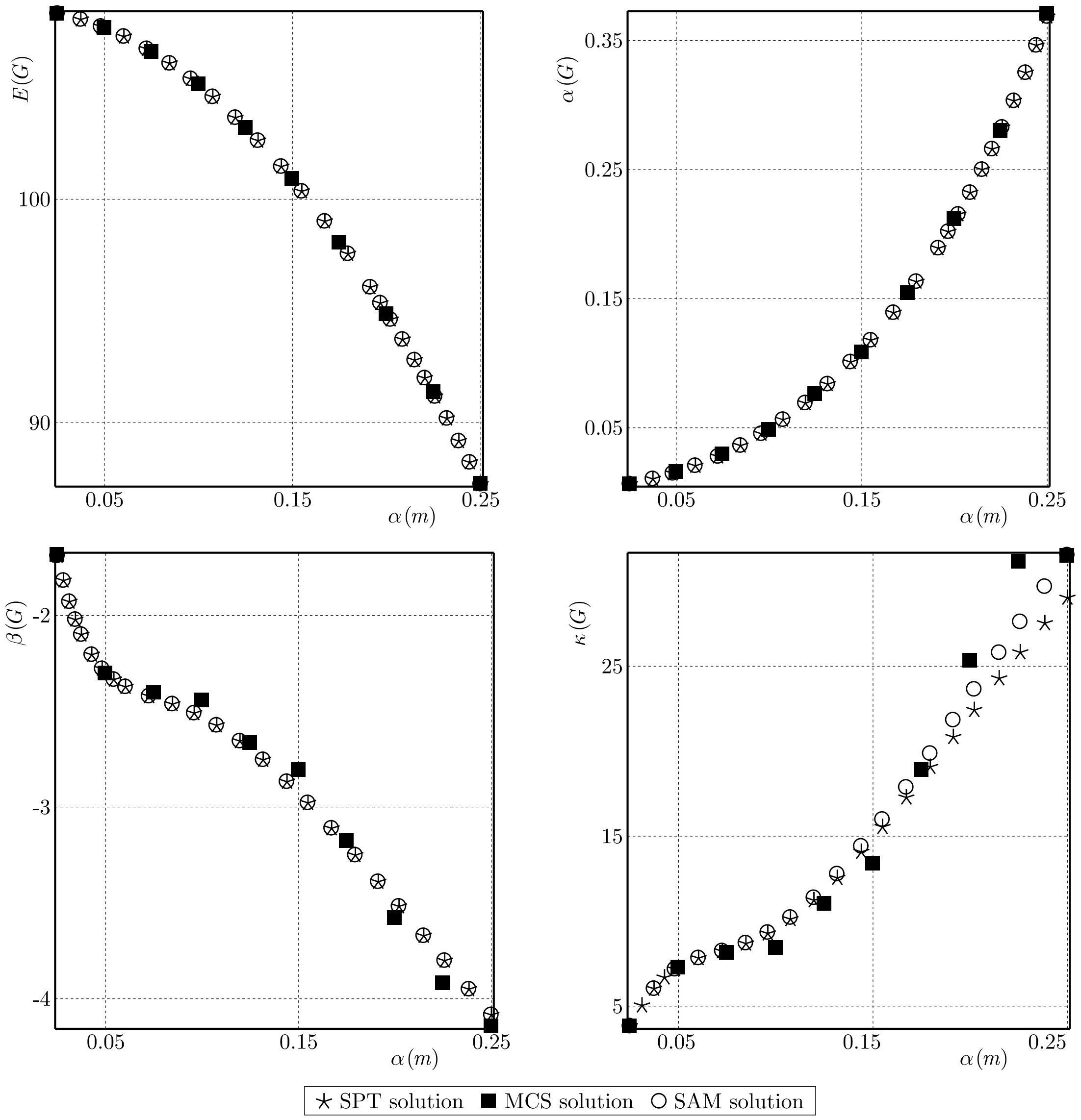

Fig. 2. Results of probabilistic calculations for the randomly distributed mass location.

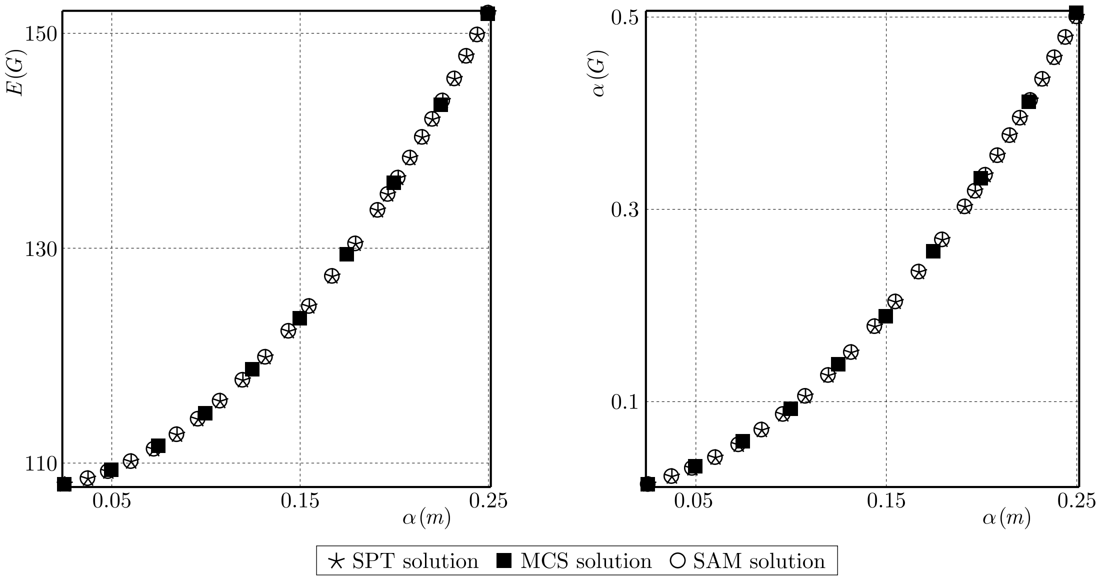

Fig. 3. Results of probabilistic calculations for the randomly distributed mass location.

Figure 3 presents, as an example, only the expected value depending on the coefficient of variation.

As in the case of the analysis of the first maximum, it can be noted that in order for the spread of the value of the analyzed variable not to be too large from the average, the expected value should oscillate within 110G. The other analyzed parameters have similar characteristics as in Fig. 2. It can be concluded that in order for the analyzed variables to have a normal distribution in the studied group, it would be necessary to remove those cases that generate "tails" in the distribution.

5. Conclusions

The calculation results presented in the previous chapter allow the following conclusions:

- Laboratory experiments and computational studies presented in this work clearly demonstrate that the common application of three probabilistic approaches – SAM, SPT, and MCS techniques – enables the accurate determination of the probabilistic coefficients of external loading in the presence of input Gaussian uncertainty. In most cases, very good agreement has been noticed between these three methods. The simplest random approach is the semi-analytical one, and it allows us to derive random moments in analytical terms. It seems to be relatively easy for future implementations in much more complex biomechanical issues.
- The Monte Carlo method is characterized by a relatively long computation time depending on the number of trials. The real number of trials that can provide correct results is 100 000.
- The presented analysis can provide valuable statistical information on the parameters determined in the analyzed group of people, and will allow the assessment of whether statistical inference is justified.
- In biomedical research, a common situation is when the size of the research group is too small. The presented approach can be useful in a process of supplementing the data in such a situation.
- The method will allow observing and eliminating from the group units that statistically differ from the rest.
- The work is a contribution to further research and presents preliminary analyses of the random approach.

Acknowledgments

The presented research results were funded with the grant 0612/SBAD/3576 allocated by the Ministry of Science and Higher Education in Poland.

References

1. Derlatka, M., & Parfieniuk, M. (2023). Real-world measurements of ground reaction forces of normal gait of young adults wearing various footwear. *Scientific Data*, *10*, Article 60. https://doi.org/10.1038/s41597-023-01964-z
2. Fryzowicz, A., Murawa, M., Kabaciński, J., Rzepnicka, A., & Dworak, L.B. (2018). Reference values of spatiotemporal parameters, joint angles, ground reaction forces, and plantar pressure distribution during normal gait in young women. *Acta of Bioengineering and Biomechanics*, *20*(1), 49–57.
3. Kamiński, M. (2013). *The stochastic perturbation method for computational mechanics*. Wiley.
4. Kamiński, M., Przychodzki, M., Łasecka-Plura, M., Guminiak, M., & Lenartowicz, A. (2024). Random eigenvibrations of beams with viscoelastic layers. *Journal of Theoretical and Applied Mechanics*, *62*(4), 763–767. https://doi.org/10.15632/jtam-pl/194242

5. Lin, C.-F., Liu, H., Gros, M.T., Weinhold, P., Garret, W.E., & Yu, B. (2012). Biomechanical risk factors of non-contact ACL injuries: A stochastic biomechanical modeling study. *Journal of Sport and Health Science*, *1*(1), 36–42. https://doi.org/10.1016/j.jshs.2012.01.001

6. Michałowska, M., Walczak, T., Grabski, J.K., & Cieślak, M. (2018). People identification based on dynamic determinants of human gait. *Vibrations in Physical Systems*, *29*, Article 2018012.

7. Park, J.S., & Kim, C.H. (2022). Ground-reaction-force-based gait analysis and its application to gait disorder assessment: New indices for quantifying walking behavior. *Sensors*, *22*(19), Article 7558. https://doi.org/10.3390/s22197558

8. Richards, J., Levine, D., & Whittle, M.W. (2023). *Whittle's gait analysis* (6th ed.). Elsevier.

9. Żuk, M., & Trzeciak, M. (2017). Anatomical protocol for gait analysis: Joint kinematics measurement and its repeatability. *Journal of Theoretical and Applied Mechanics*, *55*(1), 369–376. https://doi.org/10.15632/jtam-pl.55.1.369

Manuscript received December 8, 2024; accepted for publication March 6, 2025;
published online September 10, 2025.