# INVESTIGATION OF OUT-OF-PLANE COMPRESSION MECHANICAL PROPERTY FOR NOVEL HIERARCHICAL REENTRANT HONEYCOMB STRUCTURES

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In this study, three novel types of hierarchical reentrant honeycomb structures were designed, which comprised subunits with a semi-reentrant, reentrant, and hexagonal honeycomb. These subunits exhibited zero/negative/positive Poisson's ratios, respectively. Geometric relationships of those structures were established. The out of plane compression behavior and deformation characteristics were investigated through finite element simulations and experiments. It was shown that the hierarchical structure composed of reentrant subunits had the best mechanical properties among three structures. The hierarchical structure composed of the classic hexagonal honeycomb subunit exhibited plastic deformation characteristic spreading from the middle and upper layers to the whole region during compression.

*Keywords:* hierarchical reentrant structure, honeycomb structures, out of plane, 3D printing, compression performance

### 1. Introduction

Honeycomb structures frequently appear in porous materials and are commonly regarded as effective energy-absorbing materials in many critical engineering applications. In the past research, various types of honeycombs such as hexagonal (Li *et al.*, 2019; Liu *et al.*, 2021; Ahmed and Xue, 2019), Kagome (Zhang and Zhang, 2013), circle (Ahmed and Xue, 2019) and Nomex honeycomb (Liu *et al.*, 2015) have been proposed and extensively studied according to specific application scenarios. Scholar Jamal *et al.* performed experimental and numerical modelling investigations of a typical four-point bending test of single and multi-layer honeycomb sandwich structures (Arbaoui *et al.*, 2014). Due to their high out-of-plane stiffness, low mass, and excellent mechanical properties, honeycomb structures have found wide application in traditional architectural design (Lei *et al.*, 2022; Davalos *et al.*, 2001), various airfoil aircraft(Gong *et al.*, 2022; Heo *et al.*, 2013; Solak *et al.*, 2023), and fields of nanotechnology and biomedicine Gao *et al.*, 2022; Gadkaree, 1998; Masuda and Fukuda, 1995).

To further optimize the out-of-plane mechanical performance of the traditional hexagonal honeycomb structure, researchers restructured the hexagonal honeycomb into a reentrant honeycomb with negative Poisson's ratio characteristics by concaving the cell walls on both sides. Different from hexagonal honeycomb structures, auxetic structures with negative Poisson's ratios, such as reentrant structures, exhibit synclastic curvature and many superior mechanical properties (Wu *et al.*, 2023). This structure is characterized by lateral contraction when compressed longitudinally. Due to its superior crashworthiness, it has garnered significant attention in recent years. Studies have shown that reentrant honeycombs outperform traditional honeycombs in terms of structural strength, stiffness, and energy absorption capacity (Garg *et al.*, 2023; Günaydin *et al.*, 2022; Ha *et al.*, 2022; Teng *et al.*, 2022; Hamzehei *et al.*, 2022).

However, all the aforementioned design concepts are limited to honeycomb structures with single-level cellular configurations, which have the disadvantage of low structural stiffness, restricting their potential applications. In this study, to further enhance their out-of-plane mechanical properties, hierarchical structures inspired by nature are introduced (Wang et al., 2021; Yang et al., 2021), such as hierarchical structures of osteons and lamellae with their length scale varying from macroscopic to microscopic. In a hierarchical structure, each level of the honeycomb structure can bear part of the load, thereby increasing the overall strength and stiffness of the structure. Different from traditional honeycomb structures, the novel self-similar hierarchical honeycomb (CVH) proposed by Liang et al. (2021) exhibits high energy absorption capacity and stable deformation characteristics under quasi-static compression. Unlike the traditional five--variable shear deformation theory, the four-variable hierarchical plate shear theory proposed by Bouazza et al. (2019) based on the principle of virtual work no longer requires shear correction factors and has been shown to have similarities with classical plate theories in many aspects. Ha et al. (2021) investigated energy absorption characteristics of bionic hierarchical multi-cell square tubes under axial compression, while Wang et al. (2021) examined compressive behavior of reinforced hierarchical lattice structures through both experiments and simulations. These studies demonstrate that hierarchical structures significantly improve mechanical performance and energy absorption capacity.

Previous studies have primarily focused on mechanical properties of honeycomb structures with negative Poisson's ratios and hierarchical structures, but those studies mostly concentrated on single-type honeycomb structures. This paper innovatively integrates nested honeycomb subunits with negative, zero, and positive Poisson's ratios, proposing a novel hierarchical reentrant honeycomb structure with superior mechanical performance, filling the gap in current research. Through a combination of finite element simulations and 3D printing experiments, the mechanical properties and deformation mechanisms of the proposed structure under out-of-plane compression were systematically studied. Through optimized design, the proposed hierarchical reentrant honeycomb structure not only enhances the mechanical properties of the material but also exhibits significant energy absorption and deformation control capabilities, making it highly relevant for practical applications. The results reveal the potential applications of this novel structure in the aerospace and civil engineering fields, providing new insights for the design of high-strength, lightweight structures.

### Nomenclature of abbreviation

- SEA specific energy absorption,
- RH novel hierarchical reentrant honeycomb structures with reentrant honeycomb subunit,
- SRH novel hierarchical reentrant honeycomb structures with semi-reentrant honeycomb subunit
- CH novel hierarchical reentrant honeycomb structures with classic hexagonal honeycomb subunit.

#### 2. Novel hierarchical reentrant honeycomb structure

### 2.1. Geometric description and material

In this study, novel hierarchical reentrant honeycomb structures are constructed by using nested substructures in place of the honeycomb walls of conventional recessed honeycombs. Here, three types of hierarchical inner-concave honeycombs are considered consisting of a reentrant honeycomb, a semi-concave honeycomb, and a classical hexagonal honeycomb substructure. The specific dimensions of the structure as well as the sub-cells were determined at the beginning

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of the study. The three novel hierarchical reentrant honeycomb structures are  $c = 142 \text{ mm} \log and b = 141 \text{ mm}$  wide, and the cell wall thickness of all three cellular metamaterials is 0.6 mm, as shown in Fig. 1.



Fig. 1. Novel hierarchical reentrant honeycomb structure

A conventional hexagonal positive Poisson's ratio honeycomb is usually positive in the x-direction when pressurized in the y-direction. A positive Poisson's ratio indicates that the material tends to expand transversely when stretched in the longitudinal direction. In this study, the classical hexagonal honeycomb is utilized as a substructure, and the wall length h = 2 mm, l = 1.8 mm, turning angle  $\alpha = 33.69^{\circ}$ , and thickness h = 2 mm of this substructure are determined.

The reentrant honeycomb structure exhibits negative Poisson's ratio (NPR) properties as a tensile extension material. In contrast to conventional materials, NPR materials contract in the transverse direction when they are stretched in the longitudinal direction. The reentrant honeycomb substructure designed in this study has wall length h = 4 mm, l = 1.8 mm, turning angle  $\beta = 33.69^{\circ}$ , and thickness h = 2 mm. The semi-reentrant honeycomb structure is a new type of hexagonal honeycomb structure that combines the characteristics of the traditional hexagonal honeycomb structure and the reentrant honeycomb structure, which exhibits a semireentrant appearance. Unlike the previous two structures, this structure exhibits zero Poisson's ratio (ZPR) properties (Ingrole etal, 2017; Farrokhabadi *et al.*, 1022) in tensile experiments, and this unique behavior is attributed to the fact that the structure combines the specific deformation mechanisms of the conventional hexagonal honeycomb with a positive Poisson's ratio (PPR) and the concave honeycomb negative Poisson's ratio (NPR) (Shukla and Behera, 2022). The semi-reentrant honeycomb substructure designed in this study has a wall length h = 3 mm, l = 1.8 mm, and thickness d = 30 mm. In this study, PLA (polylactic acid) was used as the main material to provide a feasible basis for the study with its good plasticity and biodegradation properties, and the specific material property parameters are shown in Table 1.

Materials	Elastic modulus	Yield strength	Fracture	Poison's	Mass density	
	[MPa]	[MPa]	$\operatorname{strain}$	ratio	$ ho_A  [{ m kg/m^3}]$	
PLA	1451.14	39.14	0.18	0.3	1378	

 Table 1. Mechanical properties of PLA

### 2.2. Specimen manufacturing

In this study, a polylactic acid (PLA) filament was chosen as the printing material based on the fused deposition principle. It has a low melting point, which allows it to melt and flow rapidly at a certain printing temperature, contributing to efficient printing. Three novel hierarchical reentrant honeycomb structures were used to print. Meanwhile, in order to ensure the quality of printed specimens, the printing speed of the printer was set to 50 mm/s, the nozzle diameter was set to 0.3 mm and the printing temperature was set to  $210^{\circ}$ C, while the temperature of the printing platform was set to  $40^{\circ}$ C in this study. The solid experimental samples of the three novel hierarchical reentrant honeycomb substructures are shown in Fig. 2.



Fig. 2. 3D printed specimen of a novel hierarchical reentrant honeycomb substructure

### 2.3. Experiment and simulation setting

In the experimental phase, quasi-static compression tests were conducted using an Instron--5892 testing machine. A VIC-3D strain measurement system (model ProsilicaGained900) recorded deformation characteristics with a loading rate of 2 mm/min and compression images captured every 4 seconds until densification.

In the numerical simulation phase, detailed analysis was performed using Abaqus 2023 based on finite element theory. The compression and deformation mechanisms of three novel hierarchical reentrant honeycomb structures were studied using Abaqus 2023. Shell models were constructed based on previously designed geometric structures and meshed using four-node surface shell elements (S4R). To replicate the experimental compression state, rigid plates were placed on the front and rear surfaces of the structure, as shown in Fig. 1. To ensure simulation accuracy, a displacement boundary condition in the out-of-plane compression direction (Z direction) was applied to the front rigid plate, while the rear rigid plate was completely fixed. General contact was used as the interaction condition, with "hard contact" set in the normal direction and a friction coefficient of 0.3 set in the tangential direction. The mesh element size was set to 2 mm, with dynamic linear element types and element deletion enabled. Ultimately, each hierarchical structure was divided into approximately 90 000 elements.

### 3. Out of plane compression of novel hierarchical reentrant honeycomb structures

### 3.1. Simulation analysis

The stresses are calculated by the reaction forces F of the rigid grip section S is the area of grip section, so the stress is  $\sigma_1$  or  $\sigma_2 = F/S$ . Strains  $\varepsilon = U/d$  are computed from the displacements U of top grip divided by thickness d of the structure.

In this study, four compressive strain nodes of 0, 0.2, 0.4 and 0.6 are intercepted in order to investigate the deformation characteristics of the structure. The compressive deformation images of the overall structure are shown in Fig. 3. At a compressive strain of 0.2, the stress concentration of the RH structure is obviously in the inner recess of the novel hierarchical reentrant honeycomb structure, while the stress concentration range of the SRH structure and the CH structure is mainly characterized in the two sides of the wall of the hierarchical reentrant honeycomb structure, and it is worth noting that at this time, the three structures do not have a significant wave-shaped region as shown in Fig. 3 variable rows. At a compressive strain of 0.4, the reentrant of the novel hierarchical reentrant honeycomb structure that forms the whole has a tendency to shrink towards the center, whereas the shrinkage tendency of the RH structure is the smallest, which represents that the structure has a much higher stiffness and stability during the compression process. When the compressive strain reaches 0.6, all three structures show the phenomenon of collapse and fracture, and the compression of the overall structure tends to densify stability.



Fig. 3. Deformation contours of (a) entire structure, (b) substructure

To deeply investigate the deformation characteristics, this study was carried out to analyze characteristics of the novel hierarchical reentrant honeycomb structure. As shown in Fig. 3, the deformation characteristics of the substructure at four compressive strain nodes of 0, 0.2, 0.4 and 0.6 are plotted. The results show that the substructure has the same strain concentration region as the overall structure at a compressive strain of 0.2, while there is a phenomenon that the honeycomb subunits in the reentrant shrink towards the center region. In particular, in

observing the front views process, both the RH and SRH structures have a stress concentration from the center-lower region and extend to the whole wall during compression. In contrast, the range of stress concentration for the CH structure is in the upper-middle region.

As shown in Fig. 4, the out of plane compressive stress-strain curves are shown for the overall structure and the substructure, respectively. The expression for the deformation energy per unit volume E corresponding to any point ( $\varepsilon_a, \sigma_a$ ) in this curve is given by

$$E = \int_{0}^{\varepsilon_{a}} \sigma_{a} \, d\varepsilon \tag{3.1}$$

The expression for calculating the energy absorption efficiency  $\eta(\varepsilon_a)$  at any point is

$$\eta(\varepsilon_a) = \int_0^{\varepsilon_a} \frac{\sigma(\varepsilon)}{\sigma_a} \, d\varepsilon \qquad 0 \le \varepsilon_a \le 1 \tag{3.2}$$

The densification strain of this structure in out-of-plane compression was obtained by intercepting the highest point of the energy absorption efficiency-strain curve as shown in Fig. 4. Based on this value and the stress-strain curve, the mechanical property parameters associated with this structure were obtained as shown in Table 2.



Fig. 4. Simulated out-of-plane compressive stress-energy absorption efficiency-strain curves

The results of the study show that the overall structure and the substructures are highly correlated and synchronized in the conclusive characterization of the mechanical property parameters and that the densification strains of the three structures are approximate and have characteristics of RH>SRH>CH in representation of the plateau stresses, the specific absorbed energies, and the modulus of elasticity, which means that the RH structure has an even better load-bearing capacity as well as stability. Meanwhile, the results of the study show that there is a positive relationship between the platform stress and the specific absorption energy.

	Plateau stress		Energy absorption		Densification		Elastic modulus	
Type	Type $[MPa]$ $[J/s]$		[J/g]	$\operatorname{strain}$		[MPa]		
	Entire	Sub	Entire	Sub	Entire	Sub	Entire	Sub
RH	4.24	6.13	9.01	9.17	0.66	0.58	218.03	312.14
SRH	4.19	5.88	8.19	8.51	0.67	0.51	196.16	305.38
CH	4.15	5.35	7.24	8.02	0.67	0.51	174.21	255.59

**Table 2.** Mechanical properties of the novel hierarchical reentrant honeycomb structure in outof-plane compression (simulation analysis)

## **3.2.** Mechanical experiment

As shown in Fig. 5, the deformation characteristics at compressive strains of 0, 0.2, 0.4 and 0.6 reveal distinct stages of deformation, including initial plastic buckling, collapse fracture, and densification. At a compressive strain of 0.2, all three structures exhibit milky-white, worm-like plastic regions. The plastic regions of the RH and SRH structures are localized in the lower layers of the structure, while the plastic regions of the CH structure are more uniformly distributed. Upon reaching a compressive strain of 0.4, all three structures display varying degrees of collapse and fracture phenomena, with the plastic and fracture regions of the CH structure situated in the middle and upper layers of the structure. When the compressive strain reaches 0.6, the overall structure exhibits a tendency towards densification. This observation aligns entirely with the deformation characteristics identified in the simulation analysis above, providing a conclusive evidence of the synchronicity between finite element simulation and physical experiments in terms of deformation behavior.



Fig. 5. Deformation contours of the novel hierarchical reentrant honeycomb structure

Plots of the stress-energy absorption efficiency-strain curves for this solid experiment are shown in Fig. 6. Based on the densification strain corresponding to the highest point of energy absorption efficiency, a detailed analysis of the mechanical property parameters was carried out in this study, and the results are summarized in Table 3. The results show that the three novel hierarchical reentrant honeycomb structures perform close to each other in terms of the values of densification strain. In the characterization of platform stress, modulus of elasticity, and specific absorption energy, the order is RH>SRH>CH, which indicates that the RH structure possesses superior mechanical properties in the out-of-plane compression solid experiment. In addition, the platform stress and specific absorption energy showed a positive proportional relationship in this out-of-plane compression solid experiment. Finally, this study verifies the synchronicity and consistency between the solid experiment and the finite element simulation analysis in the characterization of mechanical properties.



Fig. 6. Physical experiment for the out-of-plane compressive stress-energy absorption efficiency-strain curves

**Table 3.** Mechanical properties of the novel hierarchical reentrant honeycomb structure in outof-plane compression (physical experiment)

Type	Plateau stress	Energy absorption	Densification	Elastic modulus
	[MPa]	[J/g]	$\operatorname{strain}$	[MPa]
RH	11.07	9.16	0.47	427.89
SRH	10.44	8.71	0.42	401.39
СН	9.45	8.5	0.47	366.53

### 4. Conclusion

In this study, we designed novel hierarchical reentrant honeycomb structures with different Poisson's ratio characteristics, including positive, zero, and negative Poisson's ratios. Using finite element simulations and 3D printing experiments based on fused deposition technology, we investigated the mechanical properties and deformation characteristics of those structures under out-of-plane compression. The results showed that the stress concentration in RH structure was located in the inner recess, while in SRH and CH structures, the stress concentration was mainly on the sides of the walls. The RH structure exhibited the smallest contraction tendency, indicating higher stiffness and stability.

Both experiments and simulations confirmed that the substructures had the same strain concentration regions as the overall structure. From the front view, the reentrant honeycomb subunits also showed a tendency to shrink towards the center. During compression, the RH and SRH structures generated worm-like plastic regions from the lower center, extending to the entire wall, while the plastic regions in the CH structure were located in the upper center.

Mechanical performance analysis indicated that the RH structure had the highest stiffness, stability, load-bearing capacity, and energy absorption capability, followed by the SRH and CH structures. This study verifies the synchronicity between simulation and experimental results in characterizing the mechanical properties under out-of-plane compression.

The enhanced mechanical properties and energy absorption capabilities of these novel hierarchical reentrant honeycomb structures suggest significant practical applications in aerospace, automotive, and civil engineering. They are suitable for lightweight, high-strength components such as fuselage panels in aerospace, impact-resistant structures in automotive applications, and improved building materials in civil engineering to withstand dynamic loads like earthquakes. This study highlights the potential of these innovative honeycomb structures in advancing engineering design and materials science.

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