JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
62, 4, pp. 683-694, Warsaw 2024
https://doi.org/10.15632/jtam-pl/193746

METHOD FOR DETERMINING THE S-N CURVE FOR A LOW PROBABILITY OF FAILURE

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This paper presents a new method for determining the S-N curve for a low probability of failure, e.g., 5%. To apply this method, only eight fatigue tests are needed, which is fewer than standard methods require. This could be achieved because the standard deviation, which is necessary for estimating the normal distribution of fatigue life, was derived from the distribution of logarithm of the yield strength. The tensile tests necessary to get the yield strength are relatively simple and cost-effective. Verification of the method was performed for fatigue tests on S355J2+C structural steel, 1.4301 and 1.4404 stainless steels, medium carbon steel C45 and AW 6063 & AW 2017A aluminium alloys. The results showed that the proposed method gave fatigue strength for 5% failure probability with a more reliable fatigue life than the S-N curve estimated according to ASTM E-739-10, 2015. Considering that the proposed method is conservative and low-cost, it can be used in engineering practice.

Keywords: S-N curve, high-cycle fatigue, probability of failure, normal distribution, scatter of fatigue tests

Nomenclature

F_p – F-distribution value with desired confidence interval p for $n_1 = 2$ and $n_2 = k - 2$
N – number of cycles
K_{ASTM} – confidence band acc. ASTM standard
S_a, S_u, S_y – stress amplitude, ultimate tensile stress, yield stress, respectively
b, b_{inv} – intercept coefficient of S-N curve for relationship $\log(S_a) \sim \log(N)$ and $\log(N) \sim \log(S_a)$, respectively
f_X, f_Y – normal distribution of base and inverted function
$g(x), h(x)$ – base and inverted function
k – number of observed data
m, m_{inv} – slope coefficient of S-N curve for relationship $\log(S_a) \sim \log(N)$ and $\log(N) \sim \log(S_a)$, respectively
σ – standard deviation of fatigue life for small number of specimens
$\sigma_N, \sigma_R, \sigma_S$ – standard deviation of fatigue life, yield stress and fatigue strength, respectively

Indexes: i – i-th number of sample, $\overline{(\cdot)}$ – mean value, $\widehat{(\cdot)}$ – estimated value.

1. Introduction

In the design of a new machine the fatigue strength of the entire service life must be known. For this purpose, the stress-number of cycles curve (S-N curve), referring to a maximum 5% probability of failure, can be used (PN-EN 13749, 2011). Fatigue tests necessary for getting S-N curves are time-consuming, need specialised equipment and skilled personnel. For example, to obtain 10^5 cycles at a load frequency of 30 Hz, a test takes 56 minutes. However, to obtain 10^6 cycles, the test will take over 9 hours. The time of sample preparation, installation in the machine holder, setting of research parameters, etc. is not included. The costs of performing fatigue testing were calculated by Shen, 1994 to be from 500 up to 1000 dollars per specimen. Therefore, developing fatigue characteristics is very costly. For this reason, normative documents specify a minimum number of specimens necessary to determine reliable fatigue characteristics ASTM E-739-10, 2015. Unfortunately, still a significant number of samples is required. According to the standard ASTM E-739-10, 2015, 12-24 samples for reliability and design purposes and 6-12 for preliminary and exploratory tests are required.

Several analytical methods for determining S-N curves, based on tensile testing, have been developed to reduce the number of specimens needed while maintaining the reliability of the characteristics. These methods assume that S-N curves can be achieved by correlation with tensile test parameters, such as yield strength, tensile strength, or hardness (Lee *et al.*, 2005). Verification of these methods can be found in papers by Strzelecki and Sempruch (2016). According to the verification results, the fatigue life prediction error can be as high as 263%. This is because the estimation of the "fatigue limit" is done by multiplying the tensile strength or hardness by a factor appropriate for the material and assuming a constant slope factor of the regression line. "Fatigue limit" derived from tensile tests can exhibit a large scatter, as shown in Pang *et al.* (2014). Also, the slope coefficient has a large variation according to Goedel *et al.* (2018). Analytical-experimental methods have been developed, e.g., Goedel *et al.* (2018), Strzelecki and Sempruch (2016) to improve the accuracy of S-N curves. In these methods, some parameters must be additionally determined experimentally, e.g., "fatigue limit" can be determined by the Locati method. However, these methods still give a large error of up to 50% (Strzelecki *et al.*, 2015).

Another approach to the formulation of S-N curves is represented by methods based on statistical analysis of fatigue life test results. These methods utilise the relationship between the number of samples used to create an S-N curve and the level of failure probability predicted by the curve. Gope (1999) considering the number of specimens at each stress level, determined the failure probability level and confidence level of the S-N curve. He estimated that for a 10% probability of failure and a 90% confidence level, 10 samples are required. Lewis and Sadhasivini (2004) proposed 7 samples for a two-parameter Weibull distribution for 5% probability and a 95% confidence level at each stress level. Later, Gope (2012) estimated the minimum number of samples for S-N curve estimation as 13 samples. Soh Fotsing *et al.* (2010) suggested 7 samples for a 50% failure probability. As seen, there is no consensus on a guideline specifying the number of samples for a given level of damage probability. For this reason, many methods have been proposed to improve the estimation of the S-N curve from the reliability point of view.

One of these propositions is the backward statistical inference method presented in Xie *et al.* (2014). The method assumes that the standard deviation is constant for each stress amplitude and that the curves of the probability density function of fatigue life at each i-th stress level are the same. An improvement of the backward statistical inference method has been proposed by Li *et al.* (2020) using the modified distribution coefficients. Both methods, original and improved, require as many as 15 specimens to establish a fatigue curve. Bai *et al.* (2019) presented a new method with the similar assumption that the curves of the fatigue life probability density function at each i-th stress level were the same. An additional assumption was that the coefficient of variation had to be also the same for each stress level. This method gives similar results to the backward statistical inference method, and the improved backward statistical inference method. Liu and Sun (2020) assumed that a linear regression distribution could be obtained using non-invasive polynomial chaos. Their study plan required 16 specimens. Zu *et al.* (2020) proposed a α-S-N method based on uncertainty theory. This method gives better results than the standard ISO-12107 (2012) but still requires at least 15 specimens. In many research studies, at least three

S-N curves must be estimated for the tested material, like in multiaxial fatigue, which gives over 45 specimens.

All P-S-N methods presented above use statistical methods to improve probability calculations. In this respect, the work by Strzelecki (2021) stands out, which describes a method of increasing the accuracy of fatigue characteristics by using tensile test data. In that study, the S-N curve for the relationship $\log(S_a) \sim \log(N)$ was estimated. However, in normative documents (e.g. ASTM E-739-10, 2015) the inverse relation $\log(N) \sim \log(S_a)$ is required, as the number of cycles N varies depending on the stress amplitude S_a. Therefore, the use of the relationship $\log(S_a) \sim \log(N)$ should be considered formally incorrect.

The goal of this work is to create a new method for determining P-S-N fatigue characteristics with a low failure probability (e.g. 5%). This was done using a small number of fatigue tests and an additional series of monotonic tensile tests. The deficiency of the earlier proposal by Strzelecki (2021) was corrected in such a way that the S-N median curve was determined based on fatigue tests of a limited number of samples, and the standard deviation of fatigue life was replaced by the value from the tensile test σ_R. A value of fatigue life standard deviation σ_N was obtained by the inverse of the yield stress distribution.

2. Proposed method

The S-N curve can be described using the Basquin equation, as proposed by ASTM E-739-10 (2015)

$$\log(S_{ai}) = m\log(N_i) + b \tag{2.1}$$

The least squares method is recommended for estimating the S-N curve parameters ASTM E-739-10 (2015). The resulting estimated curve follows a normal distribution and is expressed as

$$f(\log(S_{ai})) = \frac{1}{\sqrt{2\pi\sigma_S^2}} \exp\Big(\frac{[\log(S_{ai}) - (m\log(N_i) + b)]^2}{\sigma_S^2}\Big) \tag{2.2}$$

The experimental results of the yield stress determination can also be expressed by the normal distribution function (Fig. 1)

$$f(\log(S_y)) = \frac{1}{\sqrt{2\pi\sigma_R^2}} \exp\Big(\frac{[\log(S_{yi}) - \overline{\log(S_y)}]^2}{\sigma_R^2}\Big) \tag{2.3}$$

The proposed method assumes that the S-N curve for 50% probability of failure is estimated for a small number of specimens, e.g. 8 (Eq. (2.2)), and the standard deviation is taken from the distribution of yield strength (Eq. (2.3)). Such a distribution was written below and was named P-S-N (Fig. 1)

$$f(\log(S_{ai})) = \frac{1}{\sqrt{2\pi\sigma_R^2}} \exp\Big(\frac{[\log(S_{ai}) - (m\log(N_i) + b)]^2}{\sigma_R^2}\Big) \tag{2.4}$$

Because, the number of cycles N is a dependent variable, so Eq. (2.1) must be given by an inverse function. The inverse of function (2.1) is as follows

$$\log(N_i) = \frac{1}{m}[\log(S_{ai}) - b] \qquad \log(N_i) = m_{inv}\log(S_{ai}) + b_{inv} \tag{2.5}$$

Let us assume that the base function is $g(x)$ and the inverted function can be denoted as $h(y)$. If $g(x)$ is monotonic and differentiable, one can get a distribution for the inverted function. It must be calculated distribution acc. to the following equation, Walpole *et al.* (2012)

$$f_Y(y) = f_X(h(y))\Big|\frac{dx(y)}{dy}\Big| \tag{2.6}$$

Calculating the inverse distribution according to Eq. (2.6) and assuming linear regression according to Eq. (2.1), the following equation is obtained

$$f_Y(y) = -\frac{1}{m} f_X\Big(\frac{y-b}{m}\Big) \tag{2.7}$$

where: f_Y – normal distribution of the inverted function, f_X – normal distribution of the base function.

After substituting Eq. (2.7) into Eq. (2.4), the resulting equation is as follows

$$f(\log(N_i)) = \frac{1}{m_{inv}\sqrt{2\pi\sigma_R^2}} \exp\Big(\frac{[\log(N_i) - (m_{inv}\log(S_{ai}) + b_{inv})]^2}{m_{inv}^2\sigma_R^2}\Big) \tag{2.8}$$

The following substitution can be made in Eq. (2.8)

$$\sigma_N = m_{inv}\sigma_R \tag{2.9}$$

Then, Eq. (2.8) has the following form (Fig. 1)

$$f(\log(N_i)) = \frac{1}{\sqrt{2\pi\sigma_N^2}} \exp\Big(\frac{[\log(N_i) - (m_{inv}\log(S_{ai}) + b_{inv})]^2}{\sigma_N^2}\Big) \tag{2.10}$$

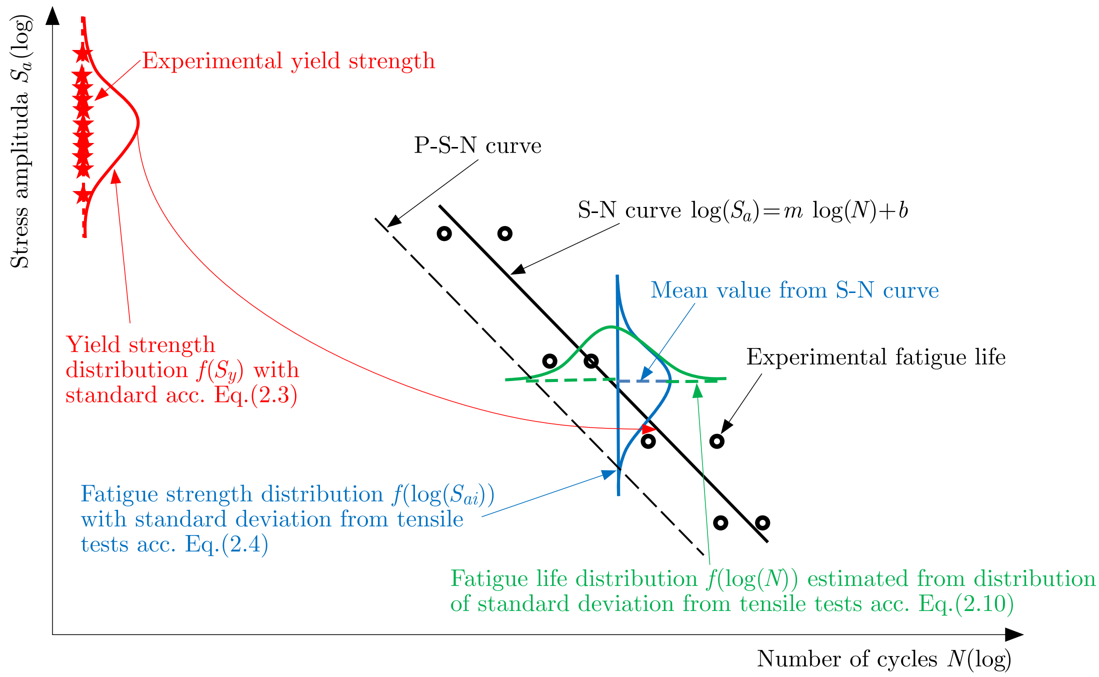

Fig. 1. Scheme of an S-N curve obtained by the proposed method

3. Method proposed in ASTM standard

According to standard ASTM E-739-10 (2015), the confidence band of median S-N curve can be estimated for desired probability. It is assumed that the logarithm of cycles N is a dependent variable and the logarithm of stress amplitude S_a is independent. For this statement, Eq. (2.5) is valid. The following equation is proposed for estimating the P-S-N curve

$$\log(N) = \widehat{m_{inv}} \log(S_a) + \widehat{b_{inv}} \pm K_{ASTM} \tag{3.1}$$

where

$$K_{ASTM} = \sqrt{2F_p}\sigma\sqrt{\frac{1}{k} + \frac{[\log(S_a) - \overline{\log(S_a)}]^2}{\sum_{i=1}^{k}[\log(S_{ai}) - \overline{\log(S_a)}]^2}} \tag{3.2}$$

where: F_p – F-distribution value with the desired confidence interval p for $n_1 = 2$ and $n_2 = k-2$, k – number of observed data, σ – standard deviation for a small number of specimens.

The positive value of K_{ASTM} in Eq. (3.1) is for the upper confidence band, and the negative value is for the lower confidence band. Because the value of stress amplitude S_a in Eq. (3.2) is substituted for each level of the load, K_{ASTM} must be calculated for each stress level separately, Lee *et al.* (2005). A line regression is provided through the estimated values of KASTM. Scheme of the ASTM method was presented in Fig. 2.

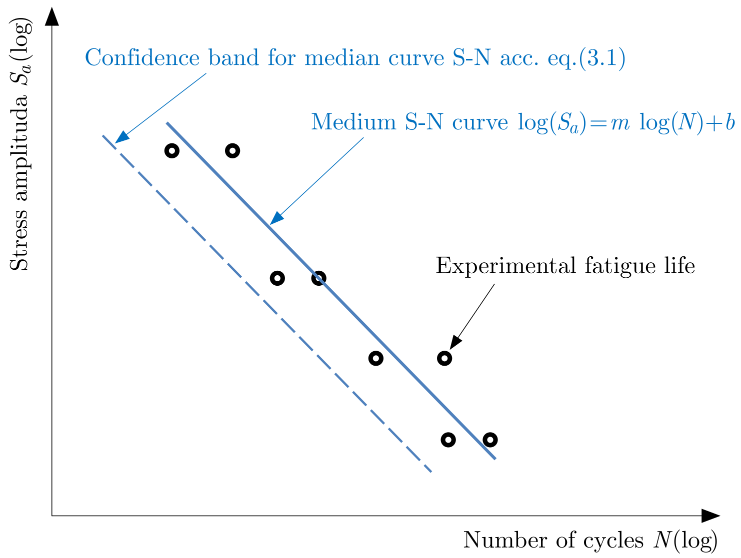

Fig. 2. Scheme of an S-N curve obtained by the ASTM method

4. Verification of the proposed method

The use of the proposed method requires knowledge of histograms and distributions of the yield strength. The tensile tests were performed by Instron 8874 testing machine for S355J2+C (Strzelecki, 2018), 1.4301 (Strzelecki, 2021), 1.4404 (Skibicki *et al.*, 2014) amd AW 6063 T6 (Strzelecki and Wachowski, 2022). The histograms and distributions of yield strength were presented in Figs. 3c-6c and Table 1. The Shapiro-Wilk test of normality was performed for those data. The hypothesis of normality of the data can be rejected if p-value is higher than 0.1 acc. to R Core Team (2023). Whereas the tensile test for C45 and AW 2017A T4 was performed by Instron 8501. Unfortunately, the histograms of yield strength for C45 and AW 2017A T4 are

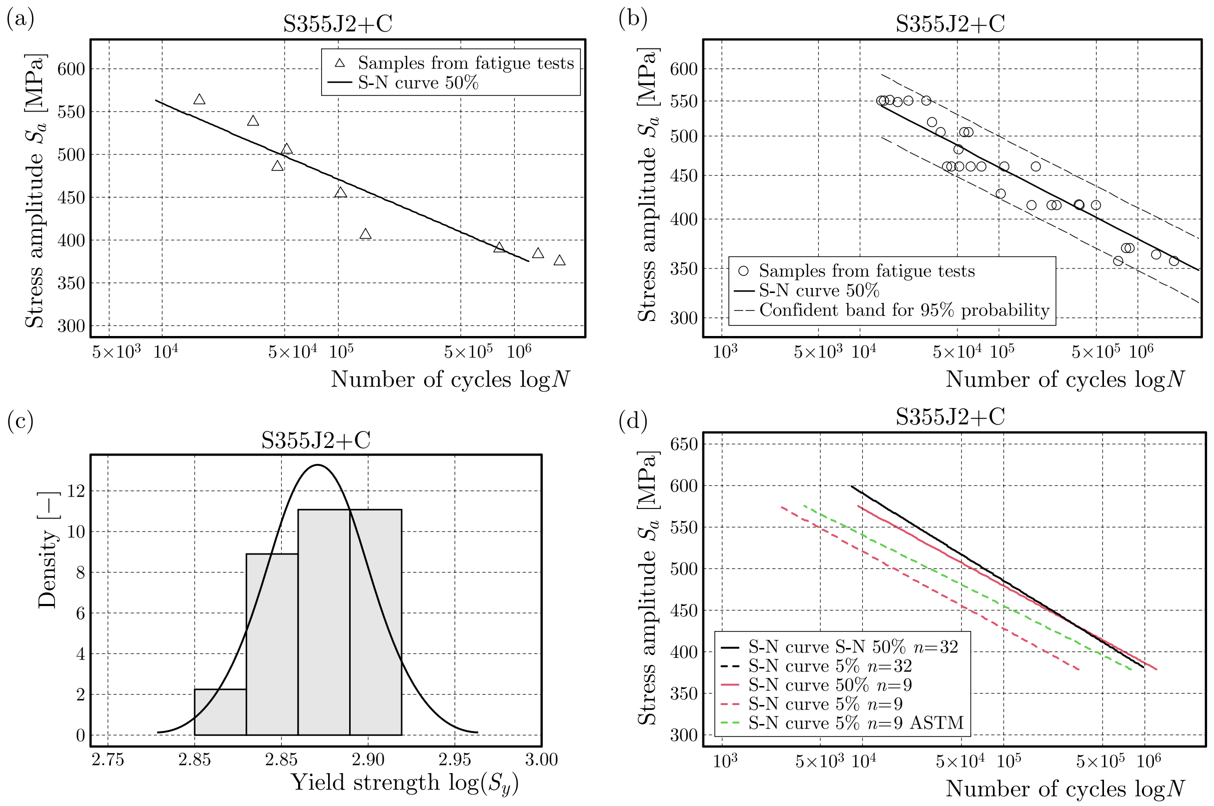

Fig. 3. S-N curves for S355J2+C steel for (a) 9 specimens and (b) 32 specimens, (c) histogram of the yield strength with normal distribution and (d) S-N curves on one diagram for 32 specimens and 9 specimens with standard deviation from the tensile test

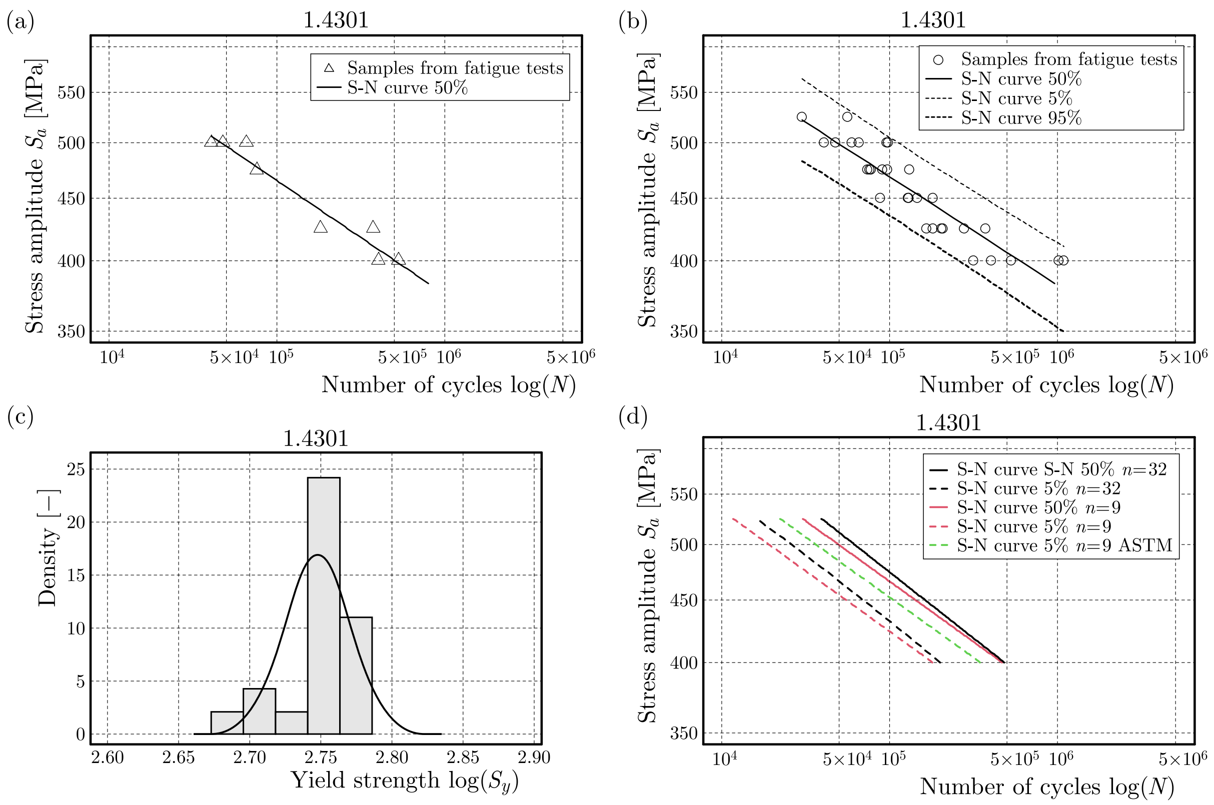

Fig. 4. S-N curves for 1.4301 steel for (a) 9 specimens and (b) 32 specimens, (c) histogram of the yield strength with normal distribution and (d) S-N curves on one diagram for 32 specimens and 9 specimens with standard deviation from the tensile test

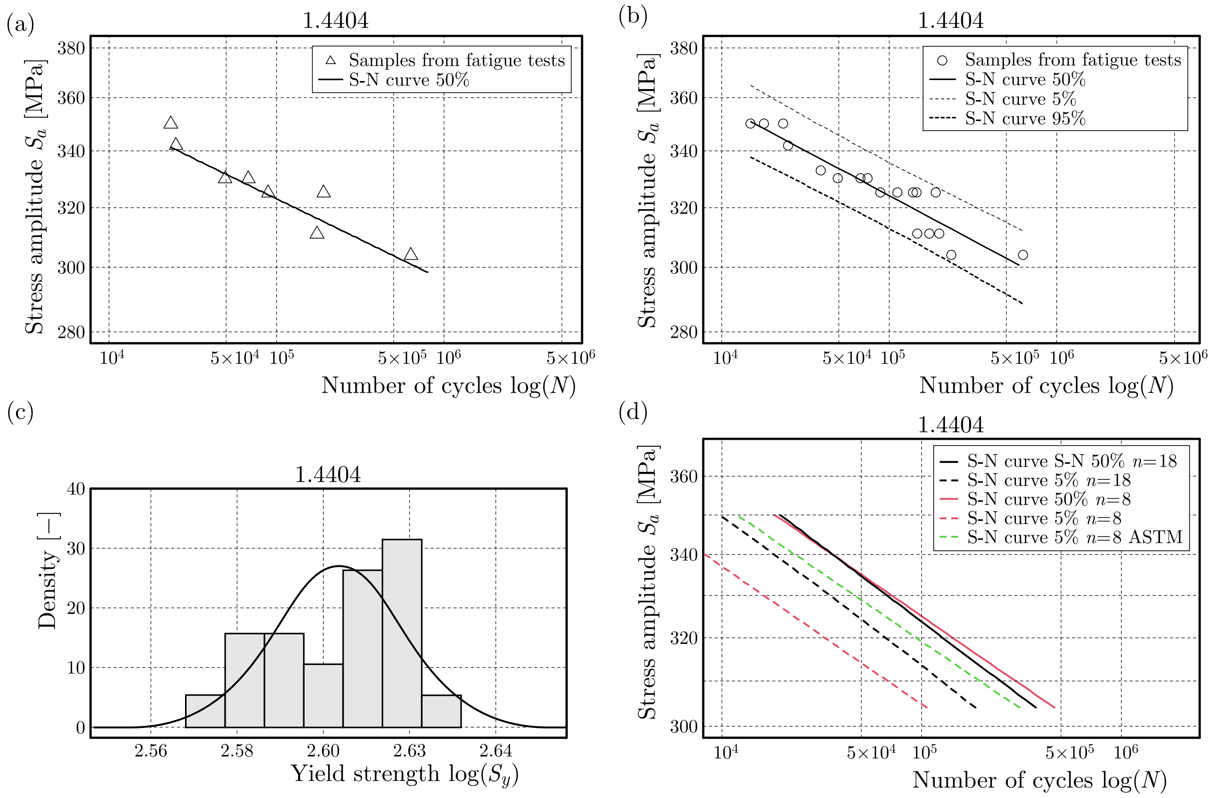

Fig. 5. S-N curves for 1.4404 steel for (a) 8 specimens and (b) 18 specimens, (c) histogram of the yield strength with normal distribution and (d) S-N curves on one diagram for 18 specimens and 8 specimens with standard deviation from the tensile test

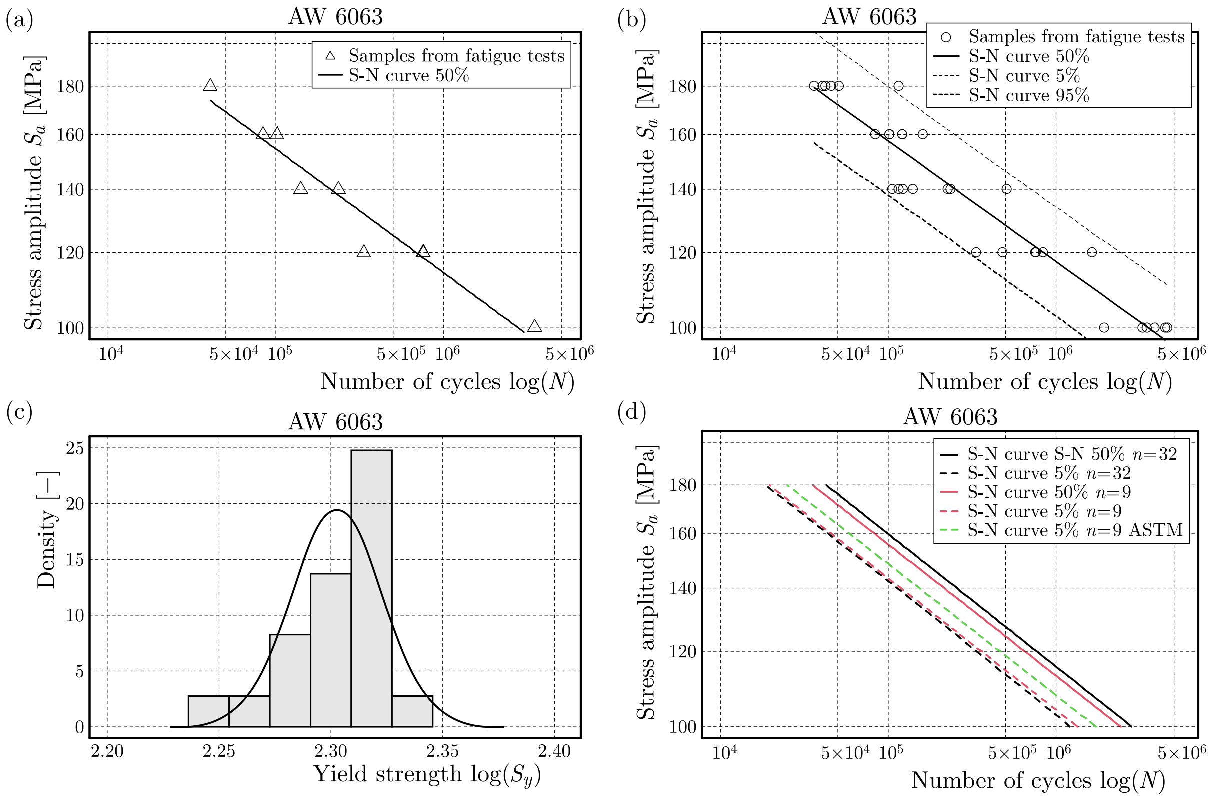

Fig. 6. S-S-N curves for AW 6063 T6 steel for (a) 9 specimens and (b) 32 specimens, (c) histogram of the yield strength with normal distribution and (d) S-N curves on one diagram for 32 specimens and 9 specimens with standard deviation from the tensile test

Table 1. Value of parameters for the distribution acc. to Eq. (2.3)

Material	Speci-mens No.	$\overline{\log(S_y)}$	σ_R	Shapiro-Wilk statistic W	Shapiro-Wilk p-value
1	2	3	4	5	6
S355J2+C (Strzelecki, 2018)	15	2.871	0.030	0.977	0.08
1.4301 (Strzelecki, 2021)	20	2.748	0.023	0.886	0.02
1.4404 (Skibicki *et al.*, 2014)	21	2.603	0.014	0.915	0.07
C45 (Ligaj and Szala, 2013)	–	2.633	0.045	–	–
AW 6063 (Strzelecki and Wachowski, 2022)	20	2.302	0.021	0.884	0.02
AW 2017A (Ligaj and Szala, 2013)	–	2.499	0.028	–	–

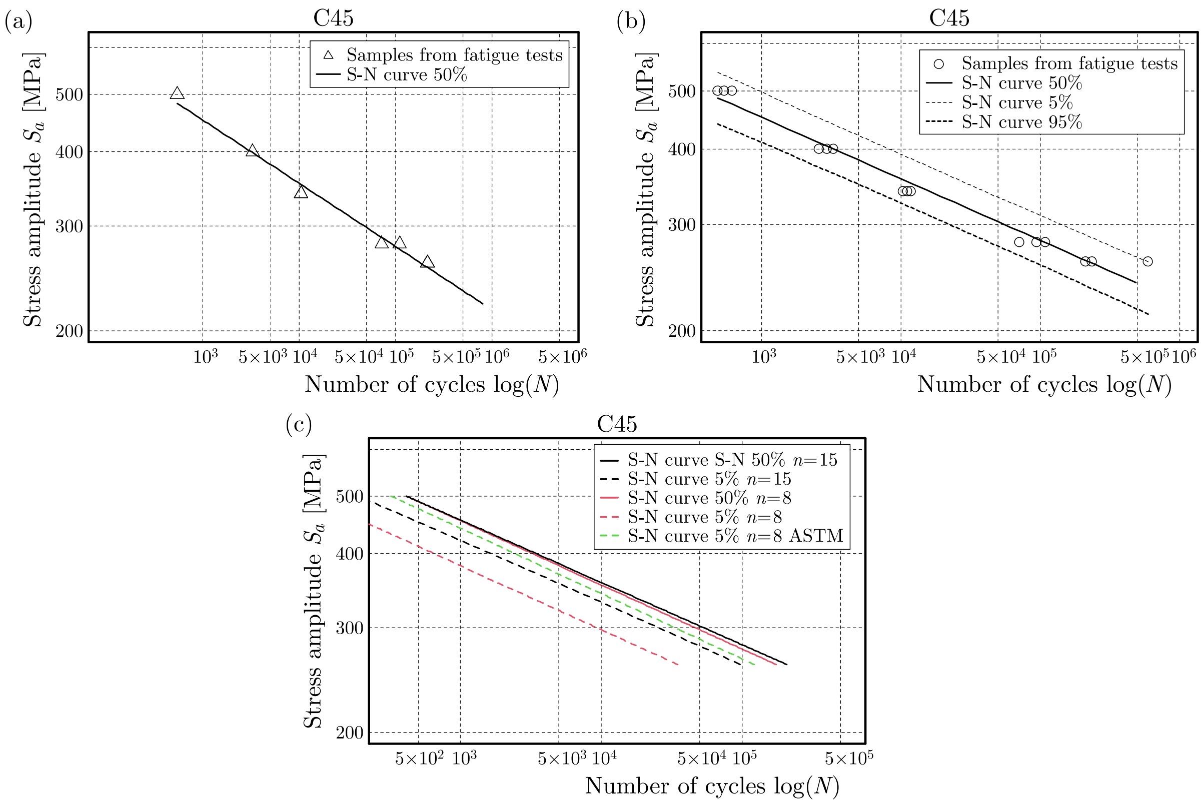

Fig. 7. S-N curves for C45 steel for (a) 8 specimens and (b) 15 specimens, (c) and (d) S-N curves on one diagram for 15 specimens and 8 specimens with standard deviation from the tensile test

not presented, because only the value of standard deviation is known from the literature. All calculations were made in R software ver. 4.3.1 (R Core Team, 2023).

The proposed method was verified for six structural materials, four steels: S355J2+C (Strzelecki, 2018), 1.4301 (Strzelecki, 2021), 1.4404 (Skibicki *et al.*, 2014), C45 (Ligaj and Szala, 2013), and two aluminium alloys: AW 6063 T6 (Strzelecki and Wachowski, 2022) and AW 2017A T4 (Ligaj and Szala, 2013). Fatigue tests were performed by rotating the bending machine for S355J2+C, 1.4301 and AW 6063 T6. Loads were applied with 28.5 Hz frequencies for S355J2+C and 50 Hz frequency for 1.4301 and AW 6063. The fatigue test of stainless steel 1.4404 was performed by fatigue machine Instron 8874. Instron 8501 was used to test the rest two materials: C45, AW 2017A T4. Axial loading was applied with a stress ratio $R = 1$. Test results are presented in Figs. 3b-8b.

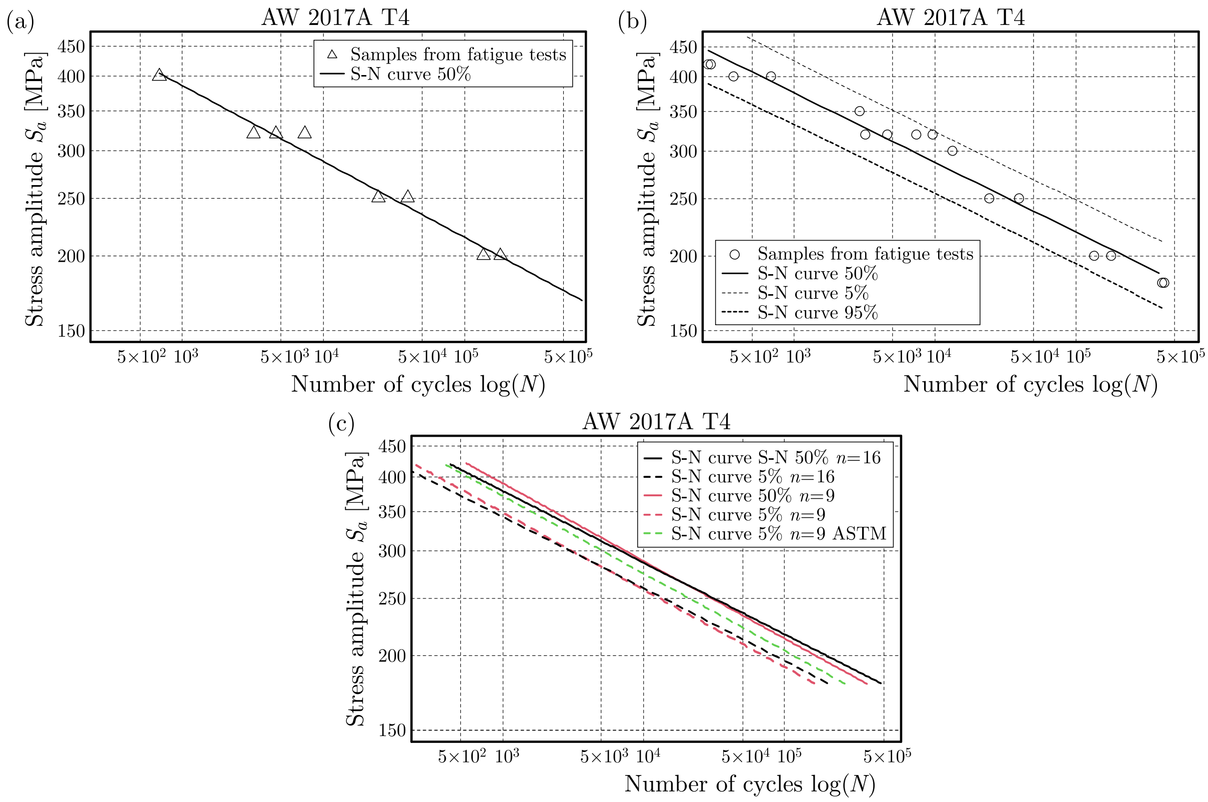

Fig. 8. S-N curves for AW 2017A T4 for (a) 9 specimens and (b) 16 specimens, (c) and (d) S-N curves on one diagram for 16 specimens and 9 specimens with standard deviation from the tensile test

Table 2. Value of parameters for the estimated S-N curves

Material	Type of load	Para-meter	Standard method	Method with small no. of specimens	Proposed method	$\mathcal{A}$	$\mathcal{B}$
1	2	3	4	5	6	7	8
S355J2+C (Strzelecki, 2018)	Rotating bending	m_{inv} b_{inv} σ_N	−9.917 31.325 0.1748	−9.922 31.300 0.2745	−9.922 31.300 0.3373	36.32%	48.18%
1.4301 (Strzelecki, 2021)	Rotating bending	m_{inv} b_{inv} σ_N	−9.190 29.595 0.2135	−10.041 31.795 0.1164	−10.041 31.795 0.2513	−83.42%	15.04%
1.4404 (Skibicki *et al.*, 2014)	Axial	m_{inv} b_{inv} σ_N	−20.952 57.594 0.1587	−22.890 62.501 0.1740	−22.890 62.501 0.3889	8.79%	59.19%
C45 (Ligaj and Szala, 2013)	Axial	m_{inv} b_{inv} σ_N	−9.526 28.326 0.1738	−9.256 27.595 0.1225	−9.256 27.595 0.4196	−41.88%	58.58%
AW 6063*	Rotating bending	m_{inv} b_{inv} σ_N	−7.108 20.664 0.2051	−7.176 20.735 0.1534	−7.176 20.735 0.1567	−33.70%	−30.89%
AW 2017A (Ligaj and Szala, 2013)	Axial	m_{inv} b_{inv} σ_N	−8.288 24.375 0.1985	−7.691 22.918 0.1317	−7.691 22.918 0.2221	−50.72%	10.63%

$\mathcal{A}$ – Difference standard method – small no. of specimens $E = (\sigma_{N5} - \sigma_{N4})/\sigma_{N5}$
$\mathcal{B}$ – Difference standard method – proposed method $E = (\sigma_{N6} - \sigma_{N4})/\sigma_{N6}$
* (Strzelecki and Wachowski, 2022)

The estimated parameters for Eq. (2.5) are shown in Table 2. The least squares method was used to determine the parameters for median S-N curves. The S-N curves obtained acc. to ASTM E-739-10 (2015) were named "standard method". In column 4, the parameters for "standard method" for reliability and design purposes were placed. That case involved testing at least 15 specimens for each material. In column 5, the parameters for preliminary tests were placed. In that case a small number, namely 8-9 specimens, were tested. Such values were chosen, because they are in the middle range of preliminary tests according to ASTM E-739-10 (2015). Additionally, standard ISO-12107 (2003) requires a minimum of eight specimens for exploratory testing.

As seen, there are little differences between the "standard method" for reliability and design purposes versus preliminary tests. The biggest differences are for standard deviation σ_N. The parameters for the proposed method, where the standard deviation was calculated according to Eq. (2.9), are shown in the 6th column of Table 2. In the 7th and 8th columns, differences between the standard method, method with a small number of specimens and the proposed method are presented. In all cases, the proposed method gets higher values of standard deviation than the small number of specimens method. This means that the method is conservative, except for material AW6063. However, the proposed method got higher values of standard deviation than the method with a small number of specimens.

5. Discussion

It can be seen in Figs. 3d-6d and Figs. 7c-8c that the S-N found by the proposed method is shifted and has a different angle compared to the S-N curve acc. to the standard method for all materials. This is expressed in different values of parameters m_{inv} and b_{inv}, as shown in Table 2. However, these differences are little and are expected according to widely scattered fatigue tests results. Dispersion of the fatigue test results is caused by microstructural inhomogeneity in material properties, geometry of specimen, differences in the surface roughness, test conditions, environment, and personal aspects (skill of laboratory technicians).

The influence of surface roughness was investigated by Nanninga and White (2009). They found a little difference in the fatigue life for roughness line in the transverse and longitudinal direction. However, they stated that "while it is probably not statistically significant, some polished transverse specimens exhibited fatigue lives that were higher than those of the longitudinal specimens". Additionally, Kurek *et al.* (2017) found that the type of load causes a different value of standard deviation. They stated that the lower scatter of the fatigue test is for tension-compression than for repeated bending. Another factor of the scatter of test results is an error of the applied load, which was analysed in the paper by Strzelecki (2018). It was found that the applied load could cause up to 46% of standard deviation. Gope (2012) found that the error of estimation fatigue life for salt solution was around half smaller than that for air condition. Standard deviation for different types of materials was tested by Wormsen *et al.* (2015). Standard deviation for the same type of material AISI 8630 M, but from different suppliers, have values 0.133, 0.188, 0.126, 0.186 and 0.106 for the axial load with asymmetry of cycles $R = -1$.

Taking these factors into account, it is impossible to estimate the exact value of standard deviation of the fatigue life. It can only be stated that it is from 0.1 to 0.25, see Wormsen *et al.* (2015). Values in this range, namely 0.159 to 0.214, were estimated while analysing materials, Table 2. So, the scatter of fatigue tests can be estimated only from experimental results.

It is worth the notice, that the relation of hardness, tensile strength or yield strength has been used to develop analytical methods for many years (e.g. presented in Pang *et al.* (2014)). However, the authors have not found such correlations for the scatter of fatigue life. The test results presented above show good correlations of standard deviation for the yield strength with fatigue life. Despite of different damage mechanisms in these tests, in both cases, they are dominated by the elastic strain. So, it can be assumed that the standard deviation can be the same, which has been proven for the presented materials.

6. Conclusion

Upon analysing the obtained results for standard deviation using the standard method, it becomes clear that the standard deviation for rotary bending was higher than that for axial loading. For stainless steels (1.4301 and 1.4404) differences of the standard deviations were significant and equaled 26%. However, those differences for aluminium alloy (AW 6063 T6 and AW 2017A T4) were little and equaled 3%. For most cases, the standard deviation for a small number of specimens was smaller than the standard deviation σ_N from the standard method. An exception was S355J2+C steel.

The proposed method of determination of the P-S-N curve has a lower value fatigue life than the ASTM method. An overestimated fatigue life was obtained by the ASTM method (except for S355J2+C). The proposed method gives underestimated values (except for aluminium alloys). Thus, the proposed method can be used by engineers.

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Manuscript received December 8, 2023; accepted for publication September 25, 2024