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THE USE OF A GENETIC ALGORITHM IN THE PROCESS OF OPTIMIZING THE SHAPE OF A THREE-DIMENSIONAL PERIODIC BEAM

Lukasz Dolinski, Arkadiusz Zak, Wiktor Waszkowiak
Gdansk University of Technology, Digital Technologies Center, Gdansk, Poland, and
Gdansk University of Technology, Department of Biomechatronics, Gdansk, Poland
corresponding author Lukasz Dolinski, e-mail: lukasz.dolinski@pg.edu.pl
Pawel Kowalski
Gdansk University of Technology, Department of Control Engineering, Gdansk, Poland
Jacek Szkopek
Gdansk University of Technology, Department of Biomechatronics, Gdansk, Poland

Mechanical periodic structures exhibit unusual dynamic behavior thanks to the periodicity of their structures, which can be attributed to their cellular arrangement. The source of this periodicity may result from periodic variations of material properties within their cells and/or variations in the cell geometry. The authors present the results of their studies on the optimization of physical parameters of a three-dimensional axisymetrical periodic beam in order to obtain the desired vibroacoustic properties. The aim of the optimization process of the unit cell shape was to obtain band gaps of a given width and position in the frequency spectrum.

Keywords: periodic beam, shape optimization, genetic algorithm

1. Introduction

Taking into account geometric features of material systems (regardless of the scale at which these structures are built), they can be divided into two separate groups: aperiodic and periodic structures (PS). In the first case, the structures consist of elements forming disordered systems, i.e. without regularity and multiple repetitions of shapes in space. In the second case, the structures consist of basic objects (unit cells) which are repeated at precisely defined intervals in space. Although the subject of systems characterized by periodicity of their structures is extremely wide, structures of this type are invariably associated with the phenomenon of waves and vibrations. This is due to the fact that PS have mechanical properties which are counter--intuitive, especially with regard to vibroacoustic phenomena (Lee *et al.*, 2010). In a similar way to periodic metamaterials, PS exhibit the same unusual dynamic behavior thanks to the periodicity of their structures (Brillouin, 1953). The source of this periodicity may be periodic changes in the material properties within the cells (density or elastic modulus) and/or changes in the cell geometry (cross-section or the presence of certain geometric features) (Yu *et al.*, 2018). The periodicity of is manifested by the presence of the so-called bandgaps which indicate the frequency areas within which the energy cannot be transmitted through these structures (Liu *et al.*, 2011). Although the phenomenon of interaction of with waves is one of the most developed branches of modern physics, this issue is still very popular among researchers, and the range of possible issues to be analyzed is still wide. With the dynamic development and dissemination of numerical methods (especially in the field of modeling methods) and the increase in access to devices with high computing power, a renewed interest can be observed. Thanks to the finite element method, the possibilities of analyzing and designing have significantly increased.

Mechanical metamaterials and mechanical PS have recently become increasingly popular in this field. The difference between them results from scale (actually the ratio: the size of the unit cell to the size of the element), and not from topological differences. It should be noted that, unlike metamaterials, whose unit cells are generally many orders of magnitude smaller than the considered sample of such materials, cell sizes in PS are often comparable to the overall structure. The analysis of the contemporary scientific literature shows that research on metamaterials (micro- or nano-scale structures) predominates, which may be associated with technological constraints on the manufacture of such structures (it is easier to get the required minimum number of elementary cells to obtain specific dynamic properties of structures) and thus potentially greater opportunities for practical applications. Therefore, research (especially experimental) on PS is difficult. On the other hand, reducing the size of the elementary cell directly affects the dynamic properties of periodic systems. The smaller the elementary cell, the more responsive it is to waves and vibrations at higher frequencies. For this reason, there are relatively few projects investigating PS, especially in the context of optimising their shape and topology. The literature lacks comprehensive research aimed at developing and implementing procedures for designing in order to obtain the assumed dynamic properties, including: damping acoustic or elastic waves in specific frequency regimes and/or selected vibration modes. In terms of the design of vibration isolators, there is a lot of research on designs of PS with additional moving elements acting as anti-resonators. The aim of the present study is to look in a different direction at potential practical applications of passive vibroacoustic isolation or filtration of propagating vibroacoustic signals in given frequency bands. It should be noted that the precise design must exist to obtain the desired insulating or filtering properties. This process usually involves some modeling and optimization procedures which lead to predetermined widths as well as positions of individual gaps in the frequency bands. Shape and topology optimization calculations (Halkjær *et al.*, 2006) is a type of computer analysis that is often performed to identify optimal shapes of structures to achieve specific static or dynamic characteristics (e.g. assumed strength with minimum mass or filtered different types of propagation of vibroacoustic signals). Historically, the method of topology optimization in materials design was first used by Sigmund (1994) using the reverse homogenization method. In the following years, many works continued the research in this area (Xia and Breitkopf, 2015). Over the years, attempts have been made to model the dynamics of metamaterials analitically, using methods such as the plane expansion method (PEM) (Yang etal, 2004) or the transfer matrix method (TMM) (Yu *et al.*, 2006), as well as numerically, thanks to the use of sophisticated computational tools based on the classical finite element method (FEM) (Żak *et al.*, 2017; Hsu, 2011) or its variants, as well as experimentally (Xiao *et al.*, 2013). It should be noted that analytical studies were usually limited to one-dimensional (1D) or two-dimensional (2D) structures with simple geometries and boundary conditions. In contrast, FEM seems to overcome these limitations and emerges as a tool capable of solving problems of complex three-dimensional (3D) geometries, arbitrary boundary conditions, as well as material properties.

From the point of view of modern materials science, a constant challenge is to create such artificial structures, whose properties (in particular, dynamic responses) resulting from their periodicity can not only be predictable but, above all, programmed in a purposeful and fully controlled manner. Additionally, it should be mentioned that both the fabrication and testing of such elements in three dimensions are still at a relatively early stage. Therefore, it seems necessary to modify the classic and to develop new methods of optimising the shape and topology of PS, which can not just manage the level of complexity and non-linearity of the models, but can also take into account the issues of manufacturing and experimental verification of such structures. The main idea of the presented research was to develop a novel approach to the design of passive vibration isolators or mechanical vibration filters (without anti-resonators) with optimised dynamic characteristics. It was assumed that a precisely designed shape of a periodic beam, affecting its dynamic properties, would enable vibration filtering not only in terms of the selected frequency range but above all in terms of the types of vibration to be filtered. Depending on the engineering problem being solved related to vibration isolation, it would be possible to design a vibration-damping filter in a selected frequency band for any combination of mechanical vibration types (longitudinal, transverse, torsional). At the same time, the dimensions of PS enable damping of vibrations at much lower frequencies than metamaterials, with no moving or active elements.

2. Methodology

Analysing the current state of knowledge, it can be seen that the key issue that remains insufficiently explored is the optimisation of topology and shape (Chen *et al.*, 2010). Optimizing the shape of the unit cell of a structure is a challenge because structural analysis of the three-dimensional elasticity problem is computationally intensive. Additionally, external boundary conditions and loads can affect each component differently, making their performance heterogeneous (Tantikom *et al.*, 2005). Algorithms in use today focusing on structural optimization can generally be assigned to one of three basic groups: gradient algorithms, population algorithms and artificial intelligence algorithms (Goldberg and Holland, 1988; Ji *et al.*, 2023). These methods are aimed to achieve the assumed dynamic parameters of designed structural elements (such as damping of mechanical vibrations in specific frequency regimes and/or for selected modes of vibrations). Due to the complexity of numerical models of periodic systems, their non-linear nature and the number and unknown connections of parameters that may affect the dynamic properties of the structure, it seems that the process of designing elements with given dynamic properties should be implemented through a multi-criteria optimization process.

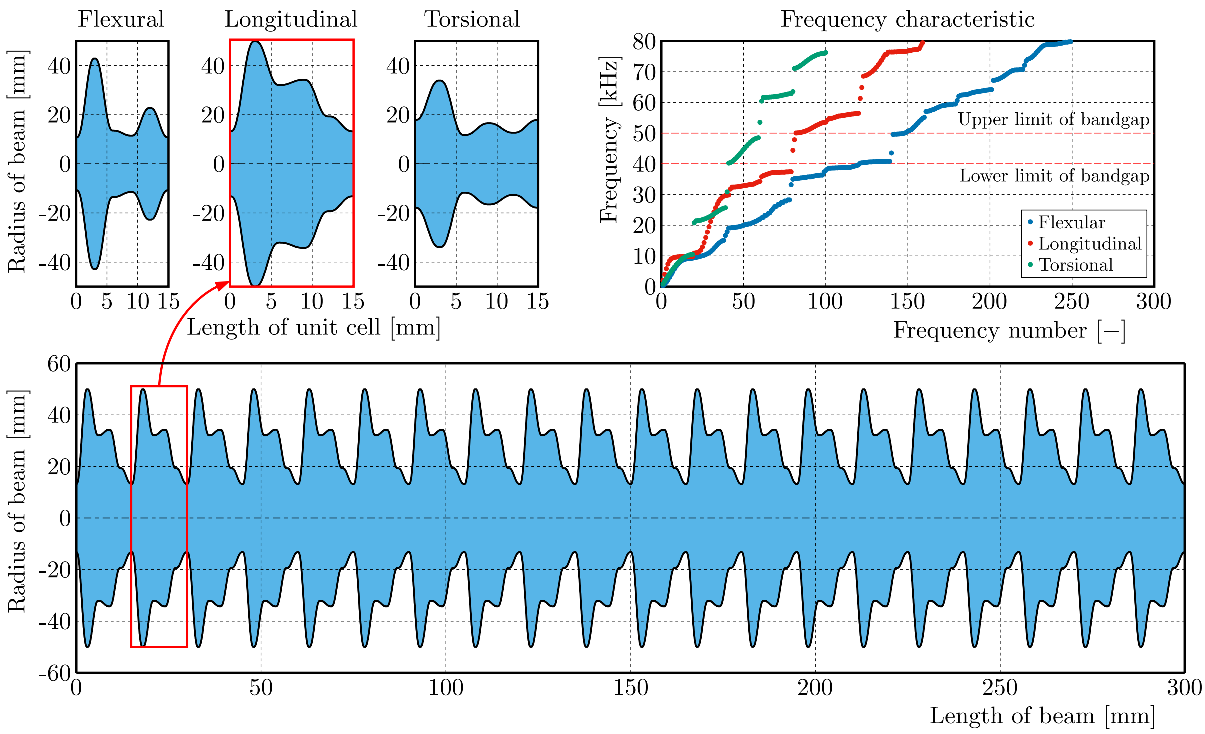

Fig. 1. Periodic beam diagram

The results presented in this paper concern implementation of a genetic algorithm (GA) to optimize the structural parameters (the shape of the unit cell of a tested structure) of a selected axisimmetric isotropic beam representing a periodic mechanical structure. Optimization calculations were carried out in order to obtain frequency characteristics in which the position and width of the forbidden bands were imposed from above, for each of the three basic types of vibrations (longitudinal, rotational and flexural). Since the dynamic properties of are directly related to the shape of the unit cell, the searched parameters (subject to the optimization process) were the five radii of disks forming the shape of the unit cell, as shown in Fig. 1. The target shape of the cell cross-section is created by a curve passing through subsequent points using interpolating polynomials of the 5th degree. The procedure presented in the authors' earlier research was used here (Żak *et al.*, 2019).

It was assumed that the periodic beam was divided into 20 identical cells of a continuously varying radius in the range between $R_{min} = 10\,\text{mm}$ and $R_{max} = 50\,\text{mm}$. The total length of the beam was $L = 300\,\text{mm}$, therefore the length of a particular cell was $l = 15\,\text{mm}$. Additionally, it was assumed that each cell was subdivided into 5 segments equal in length (3 mm each). Additionally, during the numerical calculation stage, it was necessary to take into account the limitations resulting primarily from the assumption that at the experimental research stage, the samples would be made using 3D printing from PLA material of the following material properties: Young's modulus $E = 2.35\,\text{GPa}$, Poisson's ratio $\mu = 0.36$, mass density $\rho = 1045\,\text{kg/m}^3$ (Witkowski *et al.*, 2021). It should be emphasised that additive manufacturing technologies are characterised by features that should be taken into account both at the stage of developing numerical models and in the optimisation process. For this reason, the length of the beam was determined to be 300 mm (the size of the 3D printer built plate). At this point, the attention should be paid to an extremely important aspect of modelling PS, which is the choice of boundary conditions. In the presented study, the authors assumed that both ends of the beam are free, and the beam has finite length. These parameters were determined primarily in terms of the possibility of manufacturing samples for verification tests. Indeed, the choice of aperiodic boundary conditions with a small number of cells at the same time may affect the dynamic properties of PS (Ashcroft and Mermin, 2022). However, the results of the authors' research (both numerical and experimental) allow the conclusion that certain periodicity features for large PS are already revealed with a smaller number of repeating cells (Żak *et al.*, 2019).

Figure 2 shows the general scheme of the optimization algorithm used in the study. The optimization process can be defined as an iterative mathematical procedure aimed at minimizing the error between actual and predefined parameter values over a discrete range. In order to characterize the optimal shape of the cell within the structure, it is first necessary to build a numerical model of a periodic structure, which can undergo the optimization process. To facilitate the optimisation process, a numerical model of the beam was applied using the spectral finite element method in the time domain (TD-SFEM) combined with a GA. This numerical model has been developed, tested and experimentally verified by the authors for an aluminum beam in earlier studies, the results of which can be found in (Żak *et al.*, 2019).

In the first step of optimization calculations, the algorithm was initialized by creating first-generation individuals. For this purpose, 768 sets of 5 numbers were generated with an accuracy of four decimal places. These numbers represented the successive radii of disks that formed the cross-sectional shape of the unit cell. The population size was dictated by the capabilities of the computing equipment. The calculations were performed on a multi-processor High Power Computer using Matlab software and parallel calculations on 768 processors. In the next step, it was decided to carry out the chromosome generation process according to the value coding scheme, in which the gene is represented by a sequence of values. The chromosome of each individual consisted of 5 pairs of genes (10 genes in total). Each pair defined radii (R_1-R_5) (Fig. 3). The first gene of the given pair determined an integer part of the decimal number (which could take values from 1 to 4), whereas the second gene was its fractional part (accurate to 4 decimal places – the number range from 0 to 9999). Coding the decimal number with division into an integer and a decimal part allowed, first of all, to search for better solutions

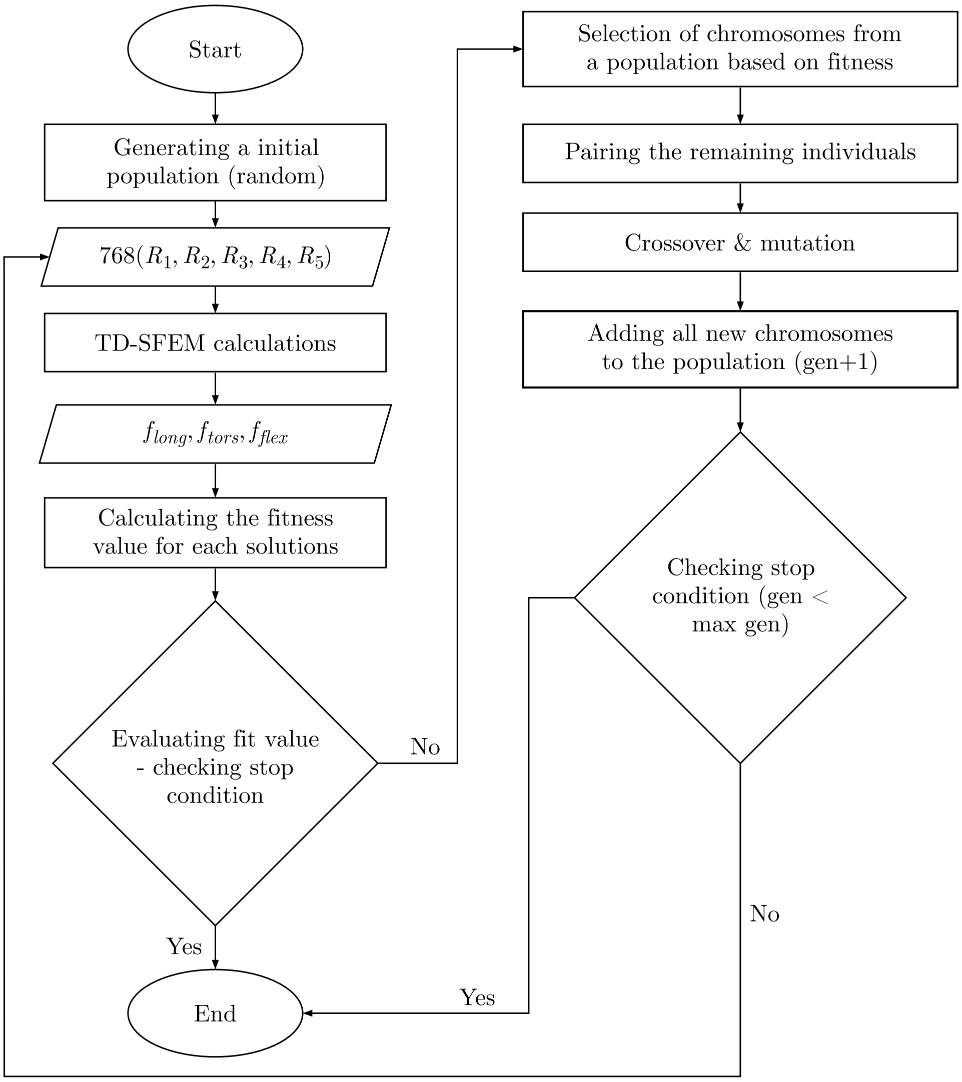

Fig. 2. Scheme of the optimization process using the evolutionary method and the TD-SFEM model

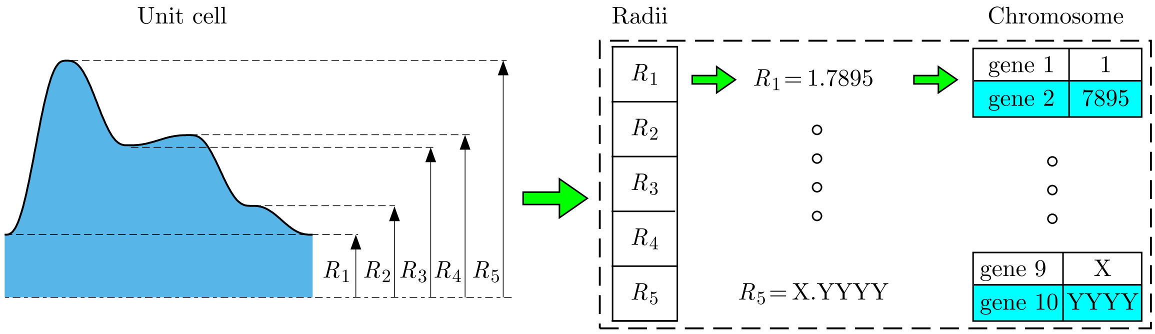

Fig. 3. Scheme for encoding the values of radii creating the shape of the unit cell

around the optimum while maintaining high variability in the first phase of searching for the optimal solution.

The generated population of random solutions was introduced into the FEM numerical model, which generated independent frequency characteristics for each considered type of vibration.

Due to the multi-criteria optimization problem, the basic challenge was to determine the objective function used to evaluate the results of FEM calculations. The aim of the optimization process was to obtain the shape of the unit cell of the periodic beam, which could ensure the appearance of bandgap characteristics in a specific place for all selected types of vibrations.

Table 1. Considered cases of optimization goals

Case No.	Flexural	Longitudinal	Torsional
1	forbiden[1]	allowed[2]	allowed
2	allowed	forbiden	allowed
3	allowed	allowed	forbiden
4	allowed	forbiden	forbiden
5	forbiden	forbiden	allowed
6	forbiden	allowed	forbiden
7	forbiden	forbiden	forbiden

[1] unable to propagate waves in a given frequency range
[2] able to propagate waves in a given frequency range

For mechanical vibration excitations generated in the bandgap range, the beam is a kind of mechanical band-stop filter that does not allow the propagation of waves of selected frequencies. Taking into account 3 basic types of beam vibrations, 7 cases of optimization goals were adopted, the summary of which is presented in Table 1. Since changing one of the radii R_i affects all types of vibrations simultaneously (including the position and width of the frequency bandgaps), the optimization process cannot be carried out by assessing the fit of only one vibration characteristic to the target without simultaneously checking the condition for the other types. Therefore, we can write that the objective function is defined from three separate functions

$$G = \begin{cases} G_{long} = g_1(f_{long}(R_1, R_2, R_3, R_4, R_5)) \\ G_{tors} = g_2(f_{tors}(R_1, R_2, R_3, R_4, R_5)) \\ G_{flex} = g_3(f_{flex}(R_1, R_2, R_3, R_4, R_5)) \end{cases} \tag{2.1}$$

where $f_{(long,tors,flex)}(R_1, R_2, R_3, R_4, R_5)$ is the frequency characteristic of a given type of structure natural vibrations (long – longitudinal, tors – torsional, flex – flexular). The following constrains were imposed

$$R_{min} \leqslant r_i \leqslant R_{max} \qquad (i = 1, 2, ..., 5) \qquad \text{and} \qquad f_L \leqslant f_i \leqslant f_U \qquad (i = 1, 2, 3) \tag{2.2}$$

where r_i represents the parameter value of the disc radius, and R_{min} and R_{max} represent the lower and upper limit of disc radius, respectively. The value of r_i of the initial generation population is generated randomly. The f_L and f_U represent the lower and upper limit of the bandgap. At the same time, it was assumed that all three types of vibrations have an equal impact on the overall fitting function, and the goal of optimization is to maximize the function G

$$\max(G) = \max\Big(\frac{1}{3}G_{long} + \frac{1}{3}G_{tors} + \frac{1}{3}G_{flex}\Big) \tag{2.3}$$

For each type of vibration, the function can take two forms (G_+ or G_-), depending on the adopted optimization goal (existence of a bandgap in a given range or not). The objective function G takes the form of the G_+ function when the expected result of a perfect match is a full coverage of the given frequency range, which means that the waves can propagate freely. Otherwise (in a given frequency range wave propagation is not allowed) the objective function takes the form of the G_-. The G_+ function (aimed at filling a given band) takes the form of a percentage coverage of the given range by the available natural frequencies, Eq. $(2.4)_1$. Due to naturally occurring bandgap ranges, the form of the G_+ function is also dependent on the form of currently assessed vibration characteristics of a given type. Figure 4 shows the most common cases of filling the band with natural frequencies of the structure. By modifying the geometric parameters of the unit cell, it is possible to control both the width and position of the bandgaps. If the goal is to fill the entire band with natural frequencies of a given type of vibration, the optimization algorithm can strive to reduce all existing forbidden bands to zero. Hence, the sum of the widths of all frequencies appears in the objective function. The function G_-, Eq. $(2.4)_2$, whose purpose is to remove the frequency from the band, is determined by width of the existing bandgap (or not) in the given frequency band

$$G_+ = \frac{\sum d_i}{\Delta f_{BG}} \qquad G_- = \Big(\frac{R}{\Delta f_{BG}}\Big)^2 \tag{2.4}$$

where: d_i – natural frequencies of a given type existing in the considered frequency band, R – existing initial bandgap width, Δf_{BG} – bandgap width.

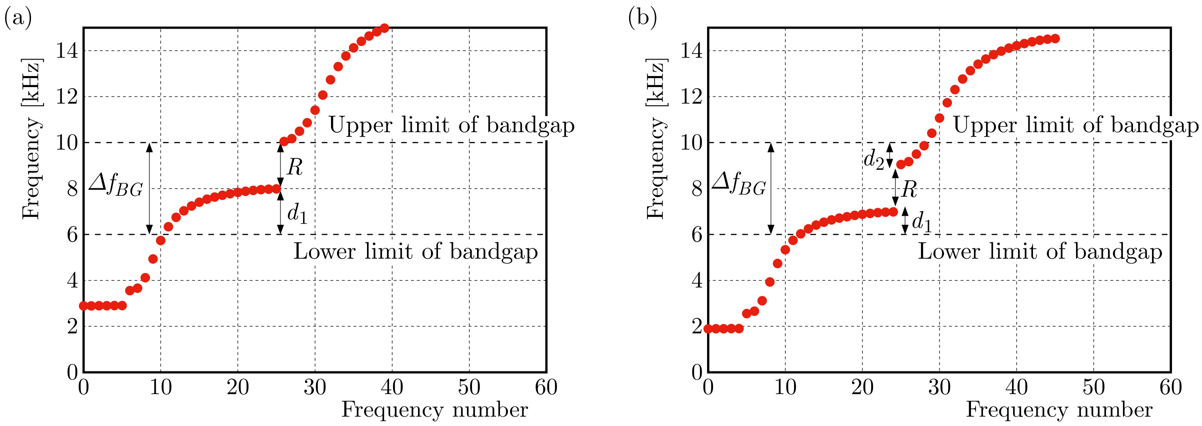

Fig. 4. The width of the existing bandgap R and the width of the natural frequency coverage d_i within the optimization goal: (a) the existing bandgap extends beyond the bandgap limit, (b) the existing bandgap is within the assumed bandgap

After assessing the fit of a given generation, a sorting and selection process takes place. In the selection process, half of the population with the best fit is kept unchanged, while the other half is discarded and new individuals are generated in its place by interbreeding the surviving individuals. For this purpose, pairs of individuals were randomly created and genes were exchanged using the 2-point method. This method involves random crossover points being selected, and the genetic information of parents is swapped as per the segments that have been created. What is important is that in this method the order of genes in the chromosome before and after the crossing operation is preserved. This ensures that there is no possibility of the error of replacing the gene representing the integer part of the number with the decimal part gene.

After replenishing the population with new individuals, a mutation operation was performed, which concerned only new individuals. Taking into account the 60% probability, it was checked whether a given individual would undergo mutation. If so, the gene to be changed was selected at random. The new value of a gene was randomized while maintaining the restrictions for a given gene type (1-4 for an odd gene and 0000 to 9999 for an even gene). In this way, the new population generated was reintroduced to the FEM model, where new frequency characteristics were generated. The process was carried out until a perfect match was achieved or 120 generations were exceeded.

3. Results

As a result of the optimization calculations performed for all assumed cases (Table 1), 7 different shapes of unit cells were obtained. The calculations were divided into three stages depending on the optimization goal. In the first stage, the calculations aimed to obtain the shape of the unit cell for beams in which the wave propagation filtering was assumed for only one form of vibration. The summary of the results is shown in Fig. 5 (a-c: flexural vibrations, d-f: longitudinal vibrations, g-i: torsional vibrations). The first column presents the optimized shapes of the unit cell meeting the optimization condition. The second column shows the resulting frequency characteristics with the marked location of the band gaps for the three vibration types. Analyzing the obtained frequency characteristics, one can notice individual frequencies appearing within the band gaps. These are anti-resonant frequencies, so their occurrence in the assumed frequency band does not affect the result of the optimization process.

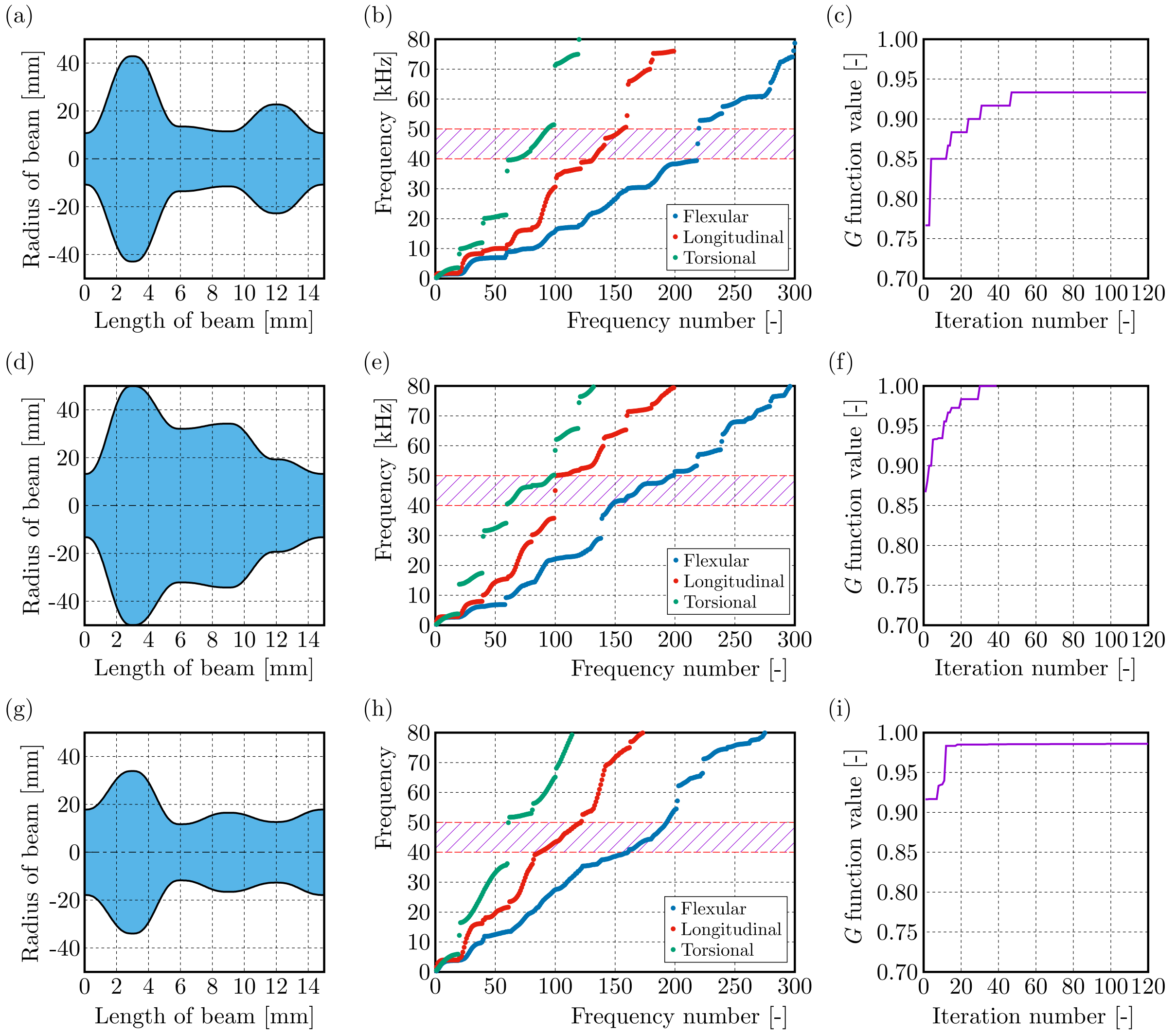

Fig. 5. Results of the optimization process for the first three cases (1st column – the shape of the optimized unit cell, 2nd column – the location of the band gaps for the three modes of vibration, 3rd column – convergence diagram): (a), (b), (c) – filtration of only flexural vibrations; (d), (e), (f) – filtration of only longitudinal vibrations; (g), (h), (i) – filtration of only torsional vibrations

The last column of Fig. 5 contains the convergence graphs obtained during the optimization process. The value of 1 means 100% fulfilment of the objective function. As can be seen, almost full matching was achieved for longitudinal and torsional vibrations (100% and 98.6%, respectively). Only in the case of flexural vibrations, a convergence of 93.33% was achieved. It should be noted, however, that this is due to the lack of fulfilment of the parallel condition regarding longitudinal vibrations and the full filling of the band gap with natural frequencies (Fig. 5b). The condition of the existence of a band gap in the assumed frequency range for the bending mode of vibrations has been met.

Figure 6 shows the results of the second stage of calculations, in which the cases of attenuation wave propagation in the beam were considered simultaneously for two selected forms of vibration (a-c: bending and longitudinal, d-f: longitudinal and torsional, g-i: bending and torsion). Due to the need to meet the condition of the existence of a bandgap for two types of vibrations simultaneously, the achieved matching levels for all cases are below 100%. The last case of the optimization process considered was damping wave propagation simultaneously for all forms of vibrations in the assumed frequency range.

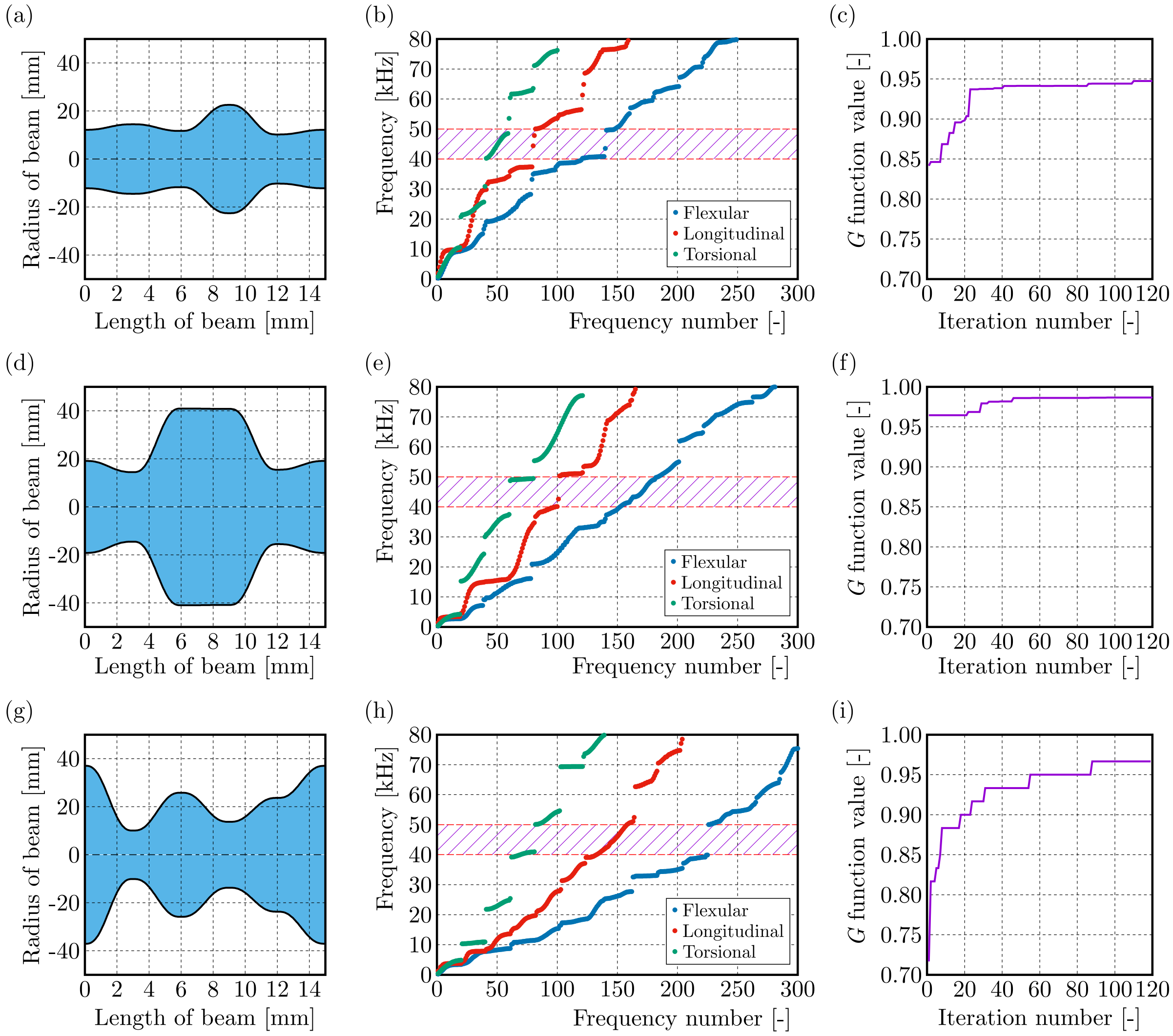

Fig. 6. Results of the optimization process (1st column – the shape of the unit cell, 2nd column – the location of the band gaps, 3rd column – convergence diagram): (a), (b), (c) – filtration of flexural and longitudinal vibrations; (d), (e), (f) – filtration of longitudinal and torsional vibrations; (g), (h), (i) – filtration of flexural and torsional vibrations

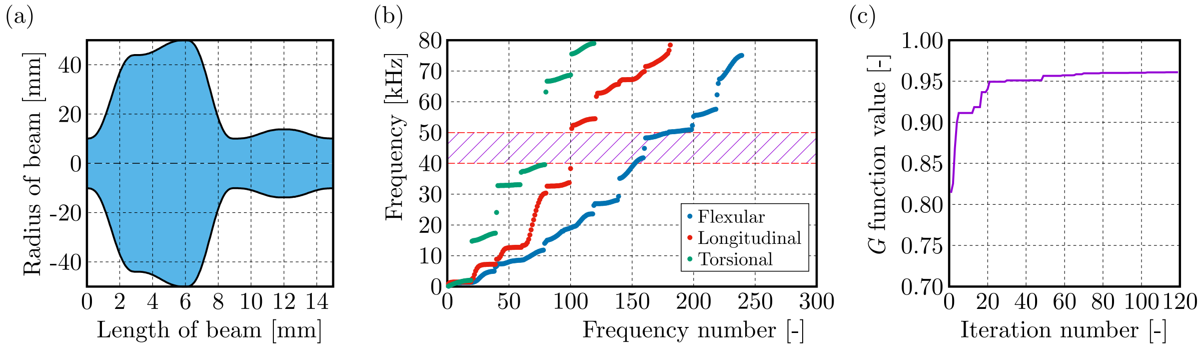

Fig. 7. Results of the optimization process for filtration all types of vibrations: (a) the shape of the optimized unit cell, (b) the location of the band gaps for the three modes of vibration, (c) convergence diagram

The calculation results are shown in Fig. 7. In the case under consideration, despite the need to simultaneously meet the three bandgap conditions, a matching degree of 96.1% was achieved, which is a satisfactory result considering the imposed geometric constraints (minimum and maximum diameter of the unit cell disks).

4. Discussion

The article presents the results of optimization calculations aimed at adjusting geometric parameters of an elementary cell of an axisymmetric mechanical periodic structure. The optimization procedure used by the authors, using a GA and a numerical beam model, made it possible to obtain a structure with the given dynamic properties of the beam in terms of the position and width of the common frequency bandgap. Based on the authors' research described in this article, the following general conclusions can be drawn:

- The GA optimization procedure used by the authors made it possible to adjust the geometric parameters within a single cell defining the periodic beam. Thus, the width and position of the resulting common band gap can be treated aspreconfiguration at the beam modelling stage, allowing the designer to freely select both the position and width of the band gap for longitudinal, bending and torsional vibrations within the natural frequency spectrum of the periodic beam under study. For all cases considered, a level of fit to the objective function in the range of 93.33%-100% was obtained.
- The approach presented by the authors in this work, along with the optimization procedure, can be used in the design of with specific dynamic properties, i.e. widths and positions of the frequency bands in their vibration spectra. Such modelling could complement the inherent properties of high-damping materials such as PLA by extending the frequency range of damped vibrations (vibroacoustic isolators) or designing elements that damp only selected types of vibrations (mechanical filters).
- The main limitation of using the optimization procedures in conjunction with numerical models based on the classical finite element method is their size and complexity (large number of degrees of freedom). The use of the TD-SFEM modelling method, the size of which is tens times smaller than the FEA model, while maintaining the accuracy of the results, allows faster calculations and more robust results, which allows them to be combined with various optimization algorithms.

The growing availability of computers with high computing power has created new paths for the rapid development of optimization methods, including evolutionary methods and machine learning techniques, which enable their application to larger and more complex problems, including modelling nonlinear problems. The use of advanced numerical methods to predict the physical properties of PS allows one the design and optimization of structures before their experimental implementation, but also the understanding of relationships between physical parameters describing mechanical (topology, shape, size of unit cells, mechanical properties of the material) and their physical properties. The understanding of these relationships is important in relation to the issue of modelling structures characterized by specific mechanical properties. We are convinced that the combination of AI technology and numerical modeling issues may allow us to develop procedures for optimizing mechanical structures to achieve the assumed dynamic properties, including damping mechanical vibrations. At this point, attention should be paid to another aspect related to the complex study of PS. There is no doubt that the interest in the issues of this type of structures is closely related to the possibilities of their production. Taking into account the fact that the authors plan to conduct experimental verification tests in the next stage, it was necessary to take into account the limitations resulting primarily from the assumption that the samples are made using the 3D printing method. The recent advances in additive manufacturing techniques combined with the topological optimization issues enable the design of PS with controlled anisotropy. In other words, 3D printing can be used to produce cellular metamaterials and PS of arbitrary shapes. On the other hand, it should be understood that, despite undeniable advantages, additive technologies have features that should be taken into account both at the stage of developing numerical models and in the optimization process, such as the damping coefficient of the material used, unintended anisotropy and heterogeneity of the material or the limited size of the printed object (the need to combine larger structures with separately printed elements, which introduces additional discontinuities that may affect dynamic properties of the system, in particular wave propagation).

5. Conclusions

This paper presents the results of a study of a 3D axisymmetric periodic beam, which can be used as a vibroacoustic filter due to its precisely defined dynamic properties. The study combines evolutionary optimisation methods with numerical modelling. The use of a spectral finite element method and a GA allowed the behavior of the structure over a wide frequency spectrum to be investigated with a high degree of accuracy at a low computational cost. The optimisation process focused on determining the dimensions and shape of the elemental cell to obtain both the desired width and position of bandgaps present in the frequency spectrum of the beam, for each type of mechanical vibrations (flexural, longitudinal and torsional). The results for all combinations of vibration types were analysed and obtained, resulting in 7 passive mechanical vibration filters allowing the filtering of all or selected types of vibrations in the assumed frequency band (40-50 kHz). In this aspect, the presented method is closer to the topological modelling than to a classical optimisation approach. It is worth noting that the calculations were carried out taking into account the potential technology for making the actual filters, i.e. 3D printing using the PLA material. This was primarily related to limitations on the minimum and maximum dimensions of the specimen. The calculations carried out proved that it is possible to successfully design vibration/vibration isolation elements with precisely tuned dynamic characteristics. This was achieved by combining the properties of periodic structures, mechanical properties of the thermoplastic material and methods for optimising the shape of the elementary cell. At the same time, the challenges and limitations of using such algorithms in the design and shape optimisation of periodic structures were analysed. In conclusion, it can be stated that the design and shape optimisation of periodic structures using GA is a promising research area and has shown great potential in overcoming the limitations of traditional methods to enable the design of periodic structures with improved performance.

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