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HEAT TRANSFER IN A THIN METAL FILM SUBJECTED TO THE ULTRA-SHORT LASER PULSE MODELED BY A NONLINEAR TWO-TEMPERATURE MODEL[1]

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The heating of a thin metal film subjected to the ultra-short laser pulse is presented. Mathematical description of this process is based on the system of equations describing the electron and lattice temperatures and dependences between intensity of heat fluxes and temperature gradients supplemented by appropriate boundary and initial conditions. In this approach, a system of four equations needs to be solved. In this paper, another method of solution of the above formulated problem is proposed. Using appropriate mathematical manipulations, instead of four equations, two equations describing the lattice and electron temperature distributions are obtained. This system of two equations is solved using an implicit scheme of the finite difference method. The results obtained using both approaches were compared. They were almost identical, which confirms the correctness of the proposed method.

Keywords: microscale heat transfer, two-temperature model, laser, finite difference method

1. Introduction

Heat transfer in the thin metal film domain subjected to the ultra-short laser pulse can be described by different mathematical models. One of them is a two-temperature model (TTM) firstly formulated by Anisimov and co-workers (Anisimov *et al.*, 1974). In this model, two different temperatures, the electron temperature and the lattice temperature appear, and they are described by two coupled Fourier equations. The model based on the classical Fourier law is called the parabolic TTM and it has some limitations. It means, it is valid only when the characteristic space and time scales of the temperature field are much greater compared to the electrons mean free path and the relaxation time (Alexopoulou and Markopoulos, 2023). In turn, the model to be used when the characteristic space and time scales of the temperature field are comparable with the electrons mean free path and relaxation time is called the hyperbolic two-temperature model (Qiu and Tien, 1993; Chen *et al.*, 2004; Smith and Norris, 2003). This model is based on the generalized Fourier law, in which the relaxation time appears, and consists of four equations while two of them describe distributions of lattice and electron temperatures, and two of them describe relationships between intensity of heat fluxes and lattice and electron temperature gradients.

Currently, these models are used for the modeling of thermal processes occurring in the laser treated materials, see e.g. (Chen and Beraun, 2001; Majchrzak and Dziatkiewicz, 2015; Sobolev, 2016). Here one can mention the analytical or semi-analytical methods, e.g. (Oane *et al.* 2019), finite difference method, e.g. (Niu and Dai, 2009; Huang *et al.*, 2011; Dziatkiewicz *et al.*, 2014), finite volume method, e.g. (Qiu and Tien, 1993) and finite element method, e.g. (Saghebfar *et al.*, 2017).

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A broad literature review on two-temperature models can be found in the paper by Alexopoulou and Markopoulos (2023).

2. Statement of the problem

A thin metal film subjected to the ultra-short laser pulse is considered. Usually, the laser spot size is much larger than film thickness and then it is possible to treat the interactions as a one-dimensional (1D) heat transfer process, wherein the front surface $x = 0$ is irradiated by the laser pulse, and this simplification is used here.

The two-temperature model describes temporal and spatial evolution of the lattice and electron temperatures in the irradiated metal by two coupled nonlinear differential equations (Tzou, 1997; Zhang, 2007)

$$\begin{aligned} C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} &= -\frac{\partial q_e(x,t)}{\partial x} - G(T_e,T_l)[T_e(x,t) - T_l(x,t)] + Q(x,t) \\ C_l(T_l)\frac{\partial T_l(x,t)}{\partial t} &= -\frac{\partial q_l(x,t)}{\partial x} + G(T_e,T_l)[T_e(x,t) - T_l(x,t)] \end{aligned} \tag{2.1}$$

where $T_e(x,t)$, $T_l(x,t)$, $q_e(x,t)$, $q_l(x,t)$ are temperatures and heat fluxes of the electrons and lattice, respectively, $C_e(T_e)$, $C_l(T_l)$ are volumetric specific heats, $G(T_e,T_l)$ is the electron-phonon coupling factor which characterizes the energy exchange between electrons and phonons, $Q(x,t)$ is the source function associated with laser irradiation, x is the spatial coordinate and t denotes time.

The following relationships between the intensity of heat fluxes and temperature gradients proposed by Qiu and Tien (1993) are used

$$\begin{aligned} q_e(x,t+\tau_e) &= -\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x} \\ q_l(x,t+\tau_l) &= -\lambda_l(T_l)\frac{\partial T_l(x,t)}{\partial x} \end{aligned} \tag{2.2}$$

where τ_e is the relaxation time of free electrons in metals (the mean time for electrons to change their states), τ_l is the relaxation time in phonon collisions, $\lambda_e(T_e,T_l)$, $\lambda_l(T_l)$ are the thermal conductivities of electrons and lattice, respectively.

Expanding the left-hand sides of equations (2.2) into the Taylor series with an accuracy of two terms, one obtains

$$\begin{aligned} q_e(x,t) + \tau_e\frac{\partial q_e(x,t)}{\partial t} &= -\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x} \\ q_l(x,t) + \tau_l\frac{\partial q_l(x,t)}{\partial t} &= -\lambda_l(T_l)\frac{\partial T_l(x,t)}{\partial x} \end{aligned} \tag{2.3}$$

The source function $Q(x,t)$ is associated with laser irradiation (Chen and Beraun, 2001; Majchrzak and Dziatkiewicz, 2019)

$$Q(x,t) = \sqrt{\frac{\beta}{\pi}}\frac{1-R}{t_p\delta}I\exp\Big[-\frac{x}{\delta} - \beta\frac{(t-2t_p)^2}{t_p^2}\Big] \tag{2.4}$$

where I is the laser intensity, t_p is the characteristic time of the laser pulse, δ is the optical penetration depth, R is the reflectivity of the irradiated surface and $\beta = 4\ln 2$.

For $x = 0$ and $x = L$ the non-flux conditions are assumed, and the initial condition $T_e(x,0) = T_l(x,0) = T_p$, where T_p is the initial temperature of electrons and lattice, is also known.

In literature, the algorithms that involve simultaneous solution of equations (2.1) and (2.3) using a staggered grid are presented, e.g. (Huang *et al.*, 2009; Majchrzak *et al.*, 2017; Wang *et al.*, 2006, 2008). It means that for even nodes the temperatures are calculated, and for odd nodes the intensity of the heat fluxes are determined (1D problem). In this paper, another method of solution of the above formulated problem is proposed. Using appropriate mathematical manipulations, instead of four equations, two equations describing the lattice and electron temperature distributions are obtained. This system of two equations is solved using an implicit scheme of the finite difference method.

3. Mathematical model

In this Section, the mathematical manipulations leading to two equations describing the heat transfer in a thin metal layer subjected to the ultra-short laser pulse are presented.

From equation $(2.3)_1$ it follows that

$$-q_e(x,t) = \tau_e \frac{\partial q_e(x,t)}{\partial t} + \lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x} \tag{3.1}$$

therefore

$$-\frac{\partial q_e(x,t)}{\partial x} = \tau_e \frac{\partial^2 q_e(x,t)}{\partial t \partial x} + \frac{\partial}{\partial x}\Big[\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x}\Big] \tag{3.2}$$

Formula (3.2) is introduced into equation $(2.1)_1$, and then

$$\begin{aligned} C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} &= \tau_e \frac{\partial^2 q_e(x,t)}{\partial t \partial x} + \frac{\partial}{\partial x}\Big[\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x}\Big] \\ &\quad - G(T_e,T_l)[T_e(x,t)-T_l(x,t)] + Q(x,t) \end{aligned} \tag{3.3}$$

It follows from equation $(2.1)_1$ that

$$\frac{\partial q_e(x,t)}{\partial x} = -C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} - G(T_e,T_l)[T_e(x,t)-T_l(x,t)] + Q(x,t) \tag{3.4}$$

Introducing (3.4) into equation (3.3), one has

$$\begin{aligned} C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} &= \tau_e \frac{\partial}{\partial t}\Big[-C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} - G(T_e,T_l)[T_e(x,t)-T_l(x,t)] + Q(x,t)\Big] \\ &\quad + \frac{\partial}{\partial x}\Big[\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x}\Big] - G(T_e,T_l)[T_e(x,t)-T_l(x,t)] + Q(x,t) \end{aligned} \tag{3.5}$$

or

$$\begin{aligned} C_e(T_e)\frac{\partial T_e(x,t)}{\partial t} + \tau_e\frac{\partial}{\partial t}\Big[C_e(T_e)\frac{\partial T_e(x,t)}{\partial t}\Big] &= \frac{\partial}{\partial x}\Big[\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x}\Big] \\ &\quad - G(T_e,T_l)[T_e(x,t)-T_l(x,t)] - \tau_e\frac{\partial}{\partial t}\Big\{G(T_e,T_l)[T_e(x,t)-T_l(x,t)]\Big\} \\ &\quad + Q(x,t) + \tau_e\frac{\partial Q(x,t)}{\partial t} \end{aligned} \tag{3.6}$$

In a similar way, the equation describing the lattice temperature can be derived

$$\begin{aligned} C_l(T_l)\frac{\partial T_l(x,t)}{\partial t} + \tau_l\frac{\partial}{\partial t}\Big[C_l(T_l)\frac{\partial T_l(x,t)}{\partial t}\Big] &= \frac{\partial}{\partial x}\Big[\lambda_l(T_l)\frac{\partial T_l(x,t)}{\partial x}\Big] \\ &\quad + G(T_e,T_l)[T_e(x,t)-T_l(x,t)] + \tau_l\frac{\partial}{\partial t}\{G(T_e,T_l)[T_e(x,t)-T_l(x,t)]\} \end{aligned} \tag{3.7}$$

Equations (3.6) and (3.7) can be written in the form

$$\begin{aligned}
&C_e(T_e)\Big[\frac{\partial T_e(x,t)}{\partial t}+\tau_e\frac{\partial^2 T_e(x,t)}{\partial t^2}\Big]+\tau_e\frac{\partial C_e(T_e)}{\partial t}\frac{\partial T_e(x,t)}{\partial t}=\frac{\partial}{\partial x}\Big[\lambda_e(T_e,T_l)\frac{\partial T_e(x,t)}{\partial x}\Big]\\
&\quad -G(T_e,T_l)[T_e(x,t)-T_l(x,t)]-\tau_e\frac{\partial G(T_e,T_l)}{\partial t}[T_e(x,t)-T_l(x,t)]\\
&\quad -\tau_e G(T_e,T_l)\Big[\frac{\partial T_e(x,t)}{\partial t}-\frac{\partial T_l(x,t)}{\partial t}\Big]+Q(x,t)+\tau_e\frac{\partial Q(x,t)}{\partial t}\\
&C_l(T_l)\Big[\frac{\partial T_l(x,t)}{\partial t}+\tau_l\frac{\partial^2 T_l(x,t)}{\partial t^2}\Big]+\tau_l\frac{\partial C_l(T_l)}{\partial t}\frac{\partial T_l(x,t)}{\partial t}=\frac{\partial}{\partial x}\Big[\lambda_l(T_l)\frac{\partial T_l(x,t)}{\partial x}\Big]\\
&\quad +G(T_e,T_l)[T_e(x,t)-T_l(x,t)]+\tau_l\frac{\partial G(T_e,T_l)}{\partial t}[T_e(x,t)-T_l(x,t)]\\
&\quad +\tau_l G(T_e,T_l)\Big[\frac{\partial T_e(x,t)}{\partial t}-\frac{\partial T_l(x,t)}{\partial t}\Big]
\end{aligned}\tag{3.8}$$

By performing derivative operations, one has (arguments omitted for simplicity)

$$\begin{aligned}
&C_e(T_e)\Big(\frac{\partial T_e}{\partial t}+\tau_e\frac{\partial^2 T_e}{\partial t^2}\Big)+\tau_e\frac{dC_e(T_e)}{dT_e}\Big(\frac{\partial T_e}{\partial t}\Big)^2=\lambda_e(T_e,T_l)\frac{\partial^2 T_e}{\partial x^2}\\
&\quad +\Big[\frac{\partial\lambda_e(T_e,T_l)}{\partial T_e}\frac{\partial T_e}{\partial x}+\frac{\partial\lambda_e(T_e,T_l)}{\partial T_l}\frac{\partial T_l}{\partial x}\Big]\frac{\partial T_e}{\partial x}-G(T_e,T_l)(T_e-T_l)\\
&\quad -\tau_e\Big[\frac{\partial G(T_e,T_l)}{\partial T_e}\frac{\partial T_e}{\partial t}+\frac{\partial G(T_e,T_l)}{\partial T_l}\frac{\partial T_l}{\partial t}\Big](T_e-T_l)\\
&\quad -\tau_e G(T_e,T_l)\Big(\frac{\partial T_e}{\partial t}-\frac{\partial T_l}{\partial t}\Big)+Q+\tau_e\frac{\partial Q}{\partial t}\\
&C_l(T_l)\Big(\frac{\partial T_l}{\partial t}+\tau_l\frac{\partial^2 T_l}{\partial t^2}\Big)+\tau_l\frac{dC_l(T_l)}{dT_l}\Big(\frac{\partial T_l}{\partial t}\Big)^2=\lambda_l(T_l)\frac{\partial^2 T_l}{\partial x^2}+\frac{d\lambda_l(T_l)}{dT_l}\Big(\frac{\partial T_l}{\partial x}\Big)^2\\
&\quad +G(T_e,T_l)(T_e-T_l)+\tau_l\Big[\frac{\partial G(T_e,T_l)}{\partial T_e}\frac{\partial T_e}{\partial t}+\frac{\partial G(T_e,T_l)}{\partial T_l}\frac{\partial T_l}{\partial t}\Big](T_e-T_l)\\
&\quad +\tau_l G(T_e,T_l)\Big(\frac{\partial T_e}{\partial t}-\frac{\partial T_l}{\partial t}\Big)
\end{aligned}\tag{3.9}$$

Summing up, in the proposed approach instead of solving four equations (2.1) and (2.3) it is enough to solve two equations (3.9) supplemented by appropriate boundary and initial conditions.

4. Method of solution

The problem formulated is solved using an implicit scheme of the finite difference method (Majchrzak and Dziatkiewicz, 2015; Niu and Dai, 2009; Wang *et al.*, 2008). Let us denote $T_i^f=T(ih,f\Delta t)$, where h is the mesh step, Δt is the time step, $i=0,1,2,\ldots,n$. Using the appropriate difference quotients, the following approximation of equation $(3.9)_1$ is proposed

$$\begin{aligned}
&C_{ei}^{f-1}\Big(\frac{T_{ei}^f-T_{ei}^{f-1}}{\Delta t}+\tau_e\frac{T_{ei}^f-2T_{ei}^{f-1}+T_{ei}^{f-2}}{(\Delta t)^2}\Big)+\tau_e\Big[\frac{dC_e(T_e)}{dT_e}\Big]_i^{f-1}\Big(\frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t}\Big)^2\\
&\quad =\lambda_{ei}^{f-1}\frac{T_{ei-1}^f-2T_{ei}^f+T_{ei+1}^f}{h^2}+D_{ei}^{f-1}-G_i^{f-1}(T_{ei}^{f-1}-T_{li}^{f-1})-E_{ei}^{f-1}\\
&\quad -\tau_e G_i^{f-1}\Big(\frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t}-\frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t}\Big)+Q_i^f+\tau_e\Big(\frac{\partial Q}{\partial t}\Big)_i^f
\end{aligned}\tag{4.1}$$

where

$$
\begin{aligned}
D_{ei}^{f-1} &= \Big\{ \Big[\frac{\partial \lambda_e(T_e,T_l)}{\partial T_e}\Big]_i^{f-1} \frac{T_{ei+1}^{f-1}-T_{ei-1}^{f-1}}{2h} + \Big[\frac{\partial \lambda_e(T_e,T_l)}{\partial T_l}\Big]_i^{f-1} \frac{T_{li+1}^{f-1}-T_{li-1}^{f-1}}{2h} \Big\} \frac{T_{ei+1}^{f-1}-T_{ei-1}^{f-1}}{2h} \\
E_{ei}^{f-1} &= \tau_e \Big\{ \Big[\frac{\partial G(T_e,T_l)}{\partial T_e}\Big]_i^{f-1} \frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t} + \Big[\frac{\partial G(T_e,T_l)}{\partial T_l}\Big]_i^{f-1} \frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t} \Big\} (T_{ei}^{f-1}-T_{li}^{f-1})
\end{aligned}
\tag{4.2}
$$

Equation (4.1) can be written in the form

$$
T_{ei}^f = \frac{\lambda_{ei}^{f-1}}{A_{ei}^{f-1}h^2}(T_{ei-1}^f + T_{ei+1}^f) + \frac{F_{ei}^{f-1}}{A_{ei}^{f-1}} + \frac{1}{A_{ei}^{f-1}}\Big[Q_i^f + \tau_e\Big(\frac{\partial Q}{\partial t}\Big)_i^f\Big]
\tag{4.3}
$$

where

$$
\begin{aligned}
A_{ei}^{f-1} &= C_{ei}^{f-1}\frac{\Delta t+\tau_e}{(\Delta t)^2} + \frac{2\lambda_{ei}^{f-1}}{h^2} \\
F_{ei}^{f-1} &= C_{ei}^{f-1}\frac{\Delta t+2\tau_e}{(\Delta t)^2}T_{ei}^{f-1} - C_{ei}^{f-1}\frac{\tau_e}{(\Delta t)^2}T_{ei}^{f-2} - \tau_e\Big[\frac{dC_e(T_e)}{dT_e}\Big]_i^{f-1}\Big(\frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t}\Big)^2 \\
&\quad + D_{ei}^{f-1} - G_i^{f-1}(T_{ei}^{f-1}-T_{li}^{f-1}) - E_{ei}^{f-1} - \tau_e G_i^{f-1}\Big(\frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t} - \frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t}\Big)
\end{aligned}
\tag{4.4}
$$

In a similar way, equation $(3.9)_2$ is approximated, namely

$$
\begin{aligned}
&C_{li}^{f-1}\Big(\frac{T_{li}^f - T_{li}^{f-1}}{\Delta t} + \tau_l\frac{T_{li}^f - 2T_{li}^{f-1} + T_{li}^{f-2}}{(\Delta t)^2}\Big) + \tau_l\Big[\frac{dC_l(T_l)}{dT_l}\Big]_i^{f-1}\Big(\frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t}\Big)^2 \\
&\quad = \lambda_{li}^{f-1}\frac{T_{li-1}^f - 2T_{li}^f + T_{li+1}^f}{h^2} + \Big[\frac{d\lambda_l(T_l)}{dT_l}\Big]_i^{f-1}\Big(\frac{T_{li+1}^{f-1}-T_{li-1}^{f-1}}{2h}\Big)^2 \\
&\quad + G_i^{f-1}(T_{ei}^{f-1}-T_{li}^{f-1}) + E_{li}^{f-1} + \tau_l G_i^{f-1}\Big(\frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t} - \frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t}\Big)
\end{aligned}
\tag{4.5}
$$

where

$$
E_{li}^{f-1} = \tau_l \Big\{ \Big[\frac{\partial G(T_e,T_l)}{\partial T_e}\Big]_i^{f-1} \frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t} + \Big[\frac{\partial G(T_e,T_l)}{\partial T_l}\Big]_i^{f-1} \frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t} \Big\} (T_{ei}^{f-1}-T_{li}^{f-1})
\tag{4.6}
$$

Equation (4.5) can be written in the form

$$
T_{li}^f = \frac{\lambda_{li}^{f-1}}{A_{li}^{f-1}h^2}(T_{li-1}^f + T_{li+1}^f) + \frac{F_{li}^{f-1}}{A_{li}^{f-1}}
\tag{4.7}
$$

where

$$
\begin{aligned}
A_{li}^{f-1} &= C_{li}^{f-1}\frac{\Delta t+\tau_l}{(\Delta t)^2} + \frac{2\lambda_{li}^{f-1}}{h^2} \\
F_{li}^{f-1} &= C_{li}^{f-1}\frac{\Delta t+2\tau_l}{(\Delta t)^2}T_{li}^{f-1} - C_{li}^{f-1}\frac{\tau_l}{(\Delta t)^2}T_{li}^{f-2} - \tau_l\Big[\frac{dC_l(T_l)}{dT_l}\Big]_i^{f-1}\Big(\frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t}\Big)^2 \\
&\quad + \Big[\frac{d\lambda_l(T_l)}{dT_l}\Big]_i^{f-1}\Big(\frac{T_{li+1}^{f-1}-T_{li-1}^{f-1}}{2h}\Big)^2 + G_i^{f-1}(T_{ei}^{f-1}-T_{li}^{f-1}) + E_{li}^{f-1} \\
&\quad + \tau_l G_i^{f-1}\Big(\frac{T_{ei}^{f-1}-T_{ei}^{f-2}}{\Delta t} - \frac{T_{li}^{f-1}-T_{li}^{f-2}}{\Delta t}\Big)
\end{aligned}
\tag{4.8}
$$

The non-flux boundary conditions are also approximated

$$
\begin{aligned}
&x = 0: \quad \frac{T_{e1}^f - T_{e0}^f}{h} = 0 \qquad && x = L: \quad \frac{T_{en}^f - T_{en-1}^f}{h} = 0 \\
&x = 0: \quad \frac{T_{l1}^f - T_{l0}^f}{h} = 0 \qquad && x = L: \quad \frac{T_{ln}^f - T_{ln-1}^f}{h} = 0
\end{aligned}
\tag{4.9}
$$

that is

$$
T_{e0}^f = T_{e1}^f \qquad T_{en}^f = T_{en-1}^f \qquad T_{l0}^f = T_{l1}^f \qquad T_{ln}^f = T_{ln-1}^f \tag{4.10}
$$

From the initial condition, it follows that

$$
T_{ei}^0 = T_{ei}^1 = T_p \qquad T_{li}^0 = T_{li}^1 = T_p \qquad i = 0, 1, \dots, n \tag{4.11}
$$

For each transition $t^{f-1} \to t^f$, the system of equations (4.3), (4.7), (4.10) is solved using e.g. the Gauss-Seidel iterative method.

5. Results of computations

A gold film of thickness $L = 100\,\mathrm{nm}$ ($1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$) is considered. The initial temperature is equal to $T_p = 300\,\mathrm{K}$.

For a high laser intensity, the following formula describing temperature-dependent volumetric specific heat of electrons is proposed (Huang *et al.*, 2009, 2011; Majchrzak and Dziatkiewicz, 2019)

$$
C_e(T_e) = \begin{cases}
AT_e & \text{for} \quad T_e < \dfrac{T_F}{\pi^2} \\[2mm]
A\dfrac{T_F}{\pi^2} + \dfrac{Nk_B - AT_F/\pi^2}{2T_F/\pi^2}\Big(T_e - \dfrac{T_F}{\pi^2}\Big) & \text{for} \quad \dfrac{T_F}{\pi^2} \leqslant T_e < 3\dfrac{T_F}{\pi^2} \\[2mm]
Nk_B + \dfrac{Nk_B/2}{T_F - 3T_F/\pi^2}\Big(T_e - 3\dfrac{T_F}{\pi^2}\Big) & \text{for} \quad 3\dfrac{T_F}{\pi^2} \leqslant T_e < T_F \\[2mm]
3N\dfrac{k_B}{2} & \text{for} \quad T_e \geqslant T_F
\end{cases}
\tag{5.1}
$$

where $N = 5.9 \cdot 10^{28}\,\mathrm{m}^{-1}$ is the electron concentration, $T_F = 64\,200\,\mathrm{K}$ is the Fermi temperature, k_B is the Boltzmann constant and A is given by formula $A = \pi^2 N k_B/(2T_F) = 62.7\,\mathrm{J/(m^3K)}$.

The electrons thermal conductivity is described by the formula (Huang, 2011; Majchrzak and Dziatkiewicz, 2015)

$$
\lambda_e(T_e, T_l) = \chi \frac{[(T_e/T_F)^2 + 0.16]^{5/4}[(T_e/T_F)^2 + 0.44](T_e/T_F)}{[(T_e/T_F)^2 + 0.092]^{1/2}[(T_e/T_F)^2 + \eta(T_l/T_F)]} \tag{5.2}
$$

and the coupling factor

$$
G(T_e, T_l) = G_{rt}\Big[\frac{A_e}{B_l}(T_e + T_l) + 1\Big] \tag{5.3}
$$

where $\chi = 353\,\mathrm{W/(mK)}$, $\eta = 0.16$, $A_e = 1.2 \cdot 10^7\,1/(\mathrm{K^2s})$, $B_l = 1.23 \cdot 10^{11}\,1/(\mathrm{Ks})$ and $G_{rt} = 2.2 \cdot 10^{16}\,\mathrm{W/(m^3K)}$ (Majchrzak and Dziatkiewicz, 2015).

Temperature dependent thermal conductivity and volumetric specific heat of gold are taken from (Huang *et al.*, 2009, 2011)

$$
\lambda_l(T_l)\ \Big[\frac{\mathrm{W}}{\mathrm{m\,K}}\Big] = \begin{cases}
320.973 - 0.0111T_l - 2.747 \cdot 10^{-5}T_l^2 - 4.048 \cdot 10^{-9}T_l^3 & T_l \leqslant 1336\,\mathrm{K} \\
37.72 + 0.0711T_l - 1.721 \cdot 10^{-5}T_l^2 + 1.064 \cdot 10^{-9}T_l^3 & T_l > 1336\,\mathrm{K}
\end{cases}
\tag{5.4}
$$

and

$$C_l(T_l)\left[\frac{\mathrm{J}}{\mathrm{m^3K}}\right]=\begin{cases}(105.1+0.2941T_l-8.731\cdot 10^{-4}T_l^2+1.787\cdot 10^{-6}T_l^3 & \\ \quad -7.051\cdot 10^{-10}T_l^4+1.538\cdot 10^{-13}T_l^5)19300 & T_l\leqslant 1336\,\mathrm{K} \\ 163.205\cdot 17280 & T_l>1336\,\mathrm{K}\end{cases} \tag{5.5}$$

The other parameters are as follows: electrons relaxation time $\tau_e = 0.04\,\mathrm{ps}$, phonons relaxation time $\tau_l = 0.8\,\mathrm{ps}$ (Chen and Beraun, 2001), reflectivity $R = 0.93$, optical penetration depth $\delta = 15.3\,\mathrm{nm}$.

The problem is solved using the finite difference method on the assumption that $\varDelta t = 0.002\,\mathrm{ps}$ and $h = 1\,\mathrm{nm}$.

First, calculations were performed for the laser intensity $I = 4182\,\mathrm{J/m^2}$ and the characteristic time of laser pulse $t_p = 0.1\,\mathrm{ps}$. In Figs. 1 and 2, the electrons and lattice temperature histories on the irradiated surface are presented. These temperatures were compared with the results obtained using a repeatedly verified algorithm based on the simultaneous solution of four equations (2.1) and (2.3) using the staggered grid and thoroughly described, among others in (Majchrzak and Dziatkiewicz, 2015, 2019). As can be seen, the results are almost identical, which confirms the correctness of the algorithm and the authors' computer program presented in this paper.

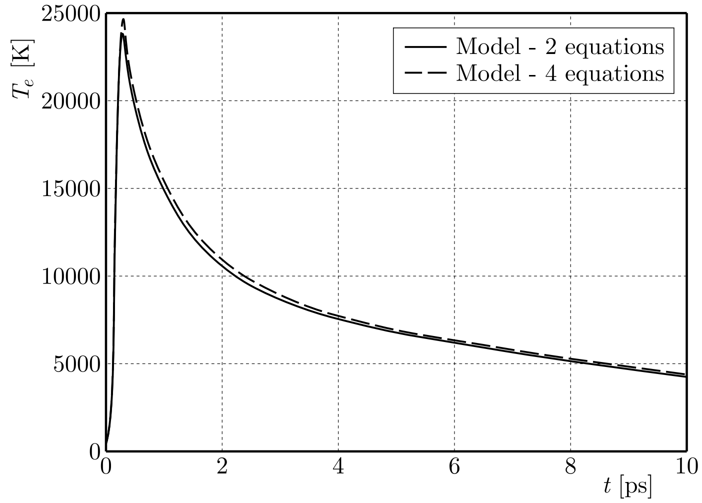

Fig. 1. Comparison of calculated electron temperature

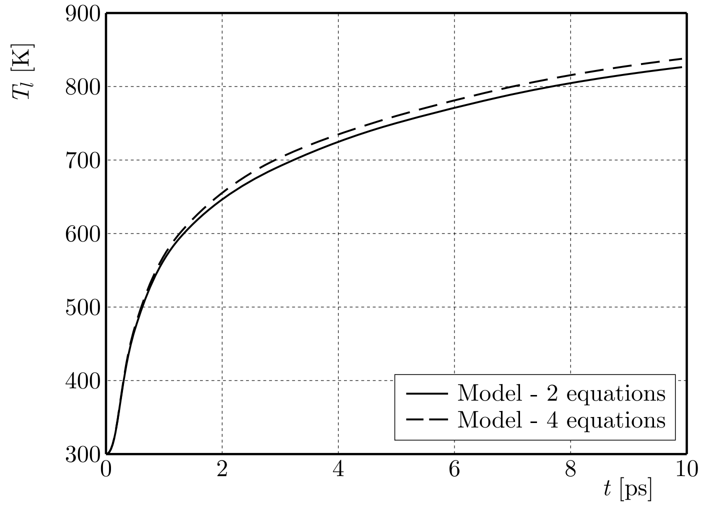

Fig. 2. Comparison of calculated lattice temperature

In Figs. 3 and 4, the temperature distributions of electrons and lattice for selected moments of time are presented. The solid line shows the calculation results for the model with two equations and the dashed line for the model with four equations. It can be seen that the obtained results are very cosistent.

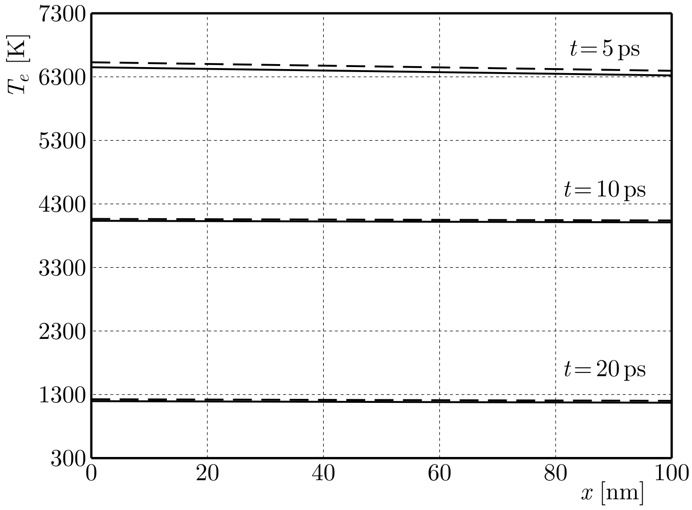

Fig. 3. Comparison of calculated electron temperature for different times

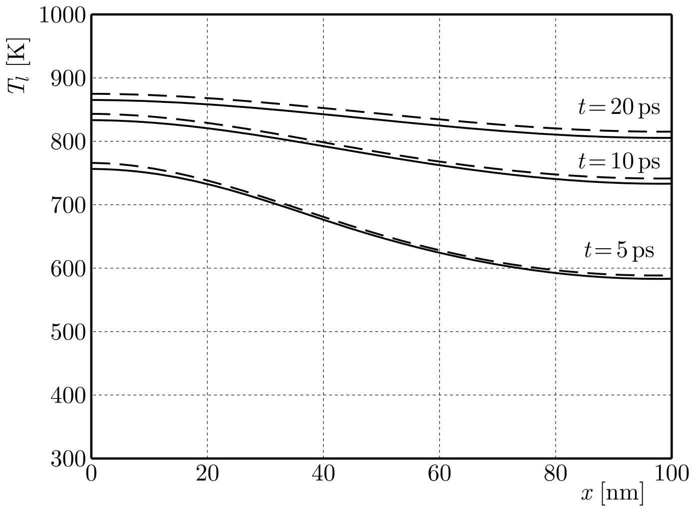

Fig. 4. Comparison of calculated lattice temperature for different times

Then, the influence of the parameters C_l and λ_l on the obtained results was checked. Calculations were prepared for constant values of C_l and λ_l equal to $2.5 \cdot 10^6$ J/(m^{3}K) and 315 W/(mK), respectively, and for values obtained from formulas (5.4) and (5.5).

Figures 5 and 6 show the distribution of temperature of electrons and lattice over time. The solid line shows the results obtained for variable parametrs and the dashed line for constant values.

Calculations were performed for low and high laser intensities. It can be noticed that as the laser intensity increases, the use of constant parameters C_l and λ_l is inappropriate and the obtained results differ significantly. For low intensities, the results are almost identical.

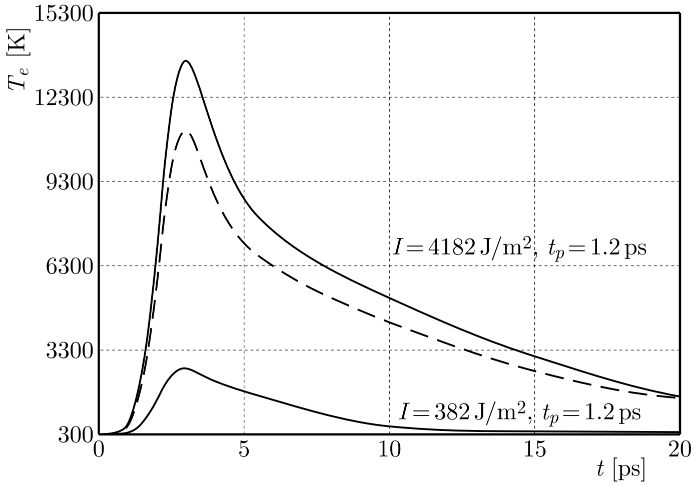

Fig. 5. Electron temperature distribution

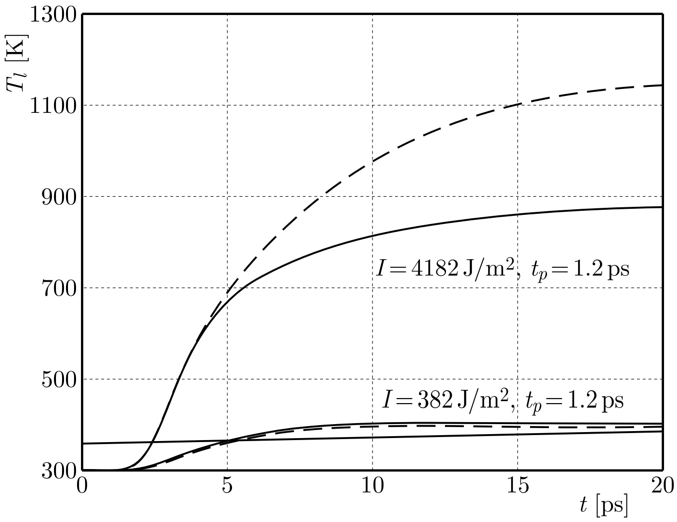

Fig. 6. Lattice temperature distribution

6. Conclusions

In this paper, the laser heating of a thin metal film made of gold is analyzed. A two-temperature model containing four equations (2.1) and (2.3) and the proposed approach with two equations (3.9) were considered. These problems were solved using the finite difference method. The problem with two equations was solved by an implicit the scheme of finite difference method, while the problem with four equations was solved by an explicit scheme of the finite difference method with the staggered grid. The results were compared, and it was shown that they are almost identical, which confirms the correctness of the proposed approach based on the two equations.

In the future, the presented approach can be extended to an axisymmetric (spatial) task, which better reflects the course of the analyzed phenomenon.

The developed algorithm and computer program should be supplemented with procedures that take into account melting, evaporation and ablation processes (Alexopoulou and Markopoulos, 2023). This will allow one to analyse thermal processes occurring in thin metal layers under the influence of higher laser powers.

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