A COMPARISON OF ROBUST AND RELIABILITY BASED DESIGN OPTIMIZATION[1]

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This article compares two optimization methods considering random variations in design parameters. One is reliability-based design optimization, which depends on the availability of the joint probability density function. A more practical alternative is robust optimization, which does not require the estimation of failure probability. It accounts for the random response of the structure through definitions of objective functions and constraints, incorporating mean values and response variances. An important element of the algorithm involves approximating unknown responses of the structures and employing efficient statistical moment estimation methods. The kriging method was used in this paper. Additionally, the article evaluates two experimental plan techniques: the classical random sampling plan and the OLH plan.

Keywords: deterministic optimization, robust optimization, reliability based design optimization, first order reliability method

1. Introduction

Currently, most Finite Element Method (FEM) structural design programs, popular among engineers, also include modules based on the deterministic optimization formulation. The result of optimization is a structure that is characterized by optimal features due to criteria adopted as a measure of their quality. Two factors clearly determine usefulness of the solution obtained this way. One of them is the adequacy of the numerical model itself, which must well reflect the actual physical phenomenon. Failure to meet this condition leads to serious mistakes and, consequently, bad decisions. The second factor is proper formulation of the optimization task. Inappropriate selection of the objective function, design constraints and, above all, calculation methodology may make the optimal solution completely useless.

Analysis of the influence of the random nature of parameters describing the modeled phenomenon is extremely important in the process of optimal design. Solutions that work for nominal parameter values may turn out to be unacceptable when random imperfections are taken into account. These imperfections may concern inevitable dispersion of material parameters, dimensions and external influences. The results of deterministic optimization, while maintaining previously defined coefficients of variation, may turn out to be completely useless. Striving for finding a solution that is not sensitive to imperfections in model parameters or external influences which are difficult to control, we have two options. The first one is robust optimization (Doltsinis *et al.*, 2005; Chen *et al.*, 2000; Li *et al.*, 2006; Hwang *et al.*, 2001; Sbaraglia *et al.*, 2018; Stocki, 2010). The second is optimization based on the so-called reliability based design optimization RBDO (Lopez and Beck, 2012; Aoues and Chateauneuf, 2010; Beck *et al.*, 2015; Kuschel and Rackwitz, 1997; Youn and Choi, 2004; Streicher and Rackwitz, 2002). If ensuring a high level of safety is the most important requirement for the designed structure, it is

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worth choosing RBDO. In the RBDO framework, design constraints are formulated using failure probabilities. The applicability of RBDO is strongly conditioned by the availability of the joint probability density function of random variables describing the problem. The reliability of the estimated failure probability values depends on a precise stochastic model. A formulation of non-deterministic optimization that better adapts to design realities is robust optimization. The goal of robust optimization should be to simultaneously minimize the mean value and standard deviation of the objective function. Unlike RBDO optimization, this formulation does not require the estimation of failure probabilities. The random nature of the structure response is taken into account through the definition of the objective function and constraints, containing mean values and variances. The computational complexity of this approach is related to the use of effective methods for estimating statistical moments.

The aim of the analyzed work was to compare the numerical effectiveness of robust and RBDO optimization. The Costrel module of the Strurel computing environment (http://www.Strurel.de) was used for RBDO calculations. In the Costrel module, calculations are carried out in accordance with the idea of single-level methods. The aim of these methods is to eliminate the internal loop associated with reliability analysis by expanding the set of decision variables and replacing reliability constraints with optimality criteria for design point search tasks. Calculations related to "robust" optimization were performed using Numpress Explore software (http://www.numpress.ippt.pan.pl/). Appropriate approximation of the objective function and constraints is crucial for the effectiveness and convergence of the analyzes performed. The work uses the kriging method in its approximation version along with an experimental plan based on the concept of the optimal Latin hypercube and random sampling (Simpson *et al.*, 2001; Liefvendahl and Stocki, 2006; Zabojszcza and Radoń, 2022).

2. Deterministic optimization

In the currently dominant design practice, a building should not only be safe, but also optimal. The behaviour of a building under a given load is closely related to strength parameters of the materials used and stiffness of the structure. The designer decides whether the response of the structure is satisfactory, which depends on the assumptions and requirements introduced.

The need to take into account the variability or uncertainty of design parameters is suggested in most proposed design and construction standards (Standards and Eurocodes). Strict adherence to standard instructions is the simplest course of action, and such a treatment of the problem is called the deterministic approach.

A typical formulation of the deterministic optimization problem can be expressed as follows: find values of the variables $\mathbf{X}_d$, minimizing $f(\mathbf{X}_d)$ with constrains

$$
\begin{aligned}
&g_i(\mathbf{X}_d) \geqslant 0 && i = 1, \ldots, k_g && \text{– unequal constraints} \\
&h_i(\mathbf{X}_d) = 0 && i = 1, \ldots, k_g && \text{– equality constraints} \\
&X_{dj}^l \leqslant X_{dj} \leqslant X_{dj}^u && j = 1, \ldots, n_d && \text{– simple constraints}
\end{aligned}
\tag{2.1}
$$

where: $\mathbf{X}_d$ – design variables, $f(\mathbf{X}_d)$ – objective function.

In the above formulation, both design variables, as well as all parameters defining the structure model, as well as objective and constraint functions, are deterministic, i.e., they are represented by one nominal value. The dominant methods of solving the task are linear or nonlinear programming methods. The optimal solution is most often searched for in an iterative manner. The most popular algorithms include: gradient algorithms, such as the conjugate gradient method, the sequential quadratic programming method, and the sequential linear programming method. An interesting comparison of various methods used in optimization was made in (Schittkowski *et al.*, 1994).

3. Reliability Based Design Optimization (RBDO)

The formulation of the RBDO consists in minimizing the objective function under probabilistic constraints. This formulation is written as: find $\mathbf{d}$, $\boldsymbol{\mu}_x$, minimize $f(\mathbf{d}, \boldsymbol{\mu}_X, \boldsymbol{\mu}_P)$ with constraints

$$
\begin{aligned}
&p[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leqslant 0] - \Phi(-\beta_i^t) \leqslant 0 && i = 1, \ldots, k_g \\
&d_j^l \leqslant d_j \leqslant d_j^u && j = 1, \ldots, n_d \\
&\mu_{x_r}^{\ l} \leqslant \mu_{x_r} \leqslant \mu_{x_r}^{\ u} && r = 1, \ldots, n_x
\end{aligned}
\tag{3.1}
$$

where: $p_f^i = p[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leqslant 0]$ – failure probability corresponding to the i-th limit function $g_i(\cdot)$, $\Phi(\cdot)$ – cumulative distribution function of the standard normal distribution, $\mathbf{X}$, $\mathbf{P}$ – vectors of random variables with expected values, respectively $\boldsymbol{\mu}_X$ and $\boldsymbol{\mu}_P$, β_i^t, $i = 1, \ldots, k_g$ – minimum reliability indices established by the designer. Variables $\mathbf{d}$ describe deterministic values.
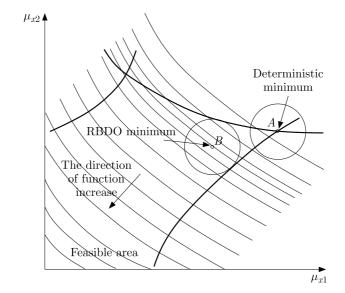

Fig. 1. Comparison of the optimal solution with deterministic optimization – point A and reliability based design optimization – point B

The idea of reliability optimization is presented in Fig. 1. In the case of a hypothetical optimization problem with two design variables and three constraints, the solution to this task in the deterministic version is point A. At the optimal point located on the border of the feasible area, two constraints are active. Let us assume that the design variables are not deterministic quantities but are characterized by a certain dispersion, and the coordinates of point A create a vector of expected values of the appropriate random variables. In such a case, most of the possible realizations of these variables will fall within some limited region around the deterministic optimum. For simplicity of presentation in Fig. 1, this area is marked as a circle centered at point A. It is easy to observe that a large part of the implementation of design variables is outside the feasible area. To ensure the required level of reliability, the circle surrounding point A should be moved inside the feasible area so that its new center B determines the solution guaranteeing higher reliability. This operation, of course, leads to an increase in the value of the objective function. How far solution B must be from the boundary of the permissible area is determined by the assumed safety margin.

In this formulation, an important element of the algorithm is calculation of the probability of meeting the design constraints. Numerical methods for determining the probability of failure and reliability indicators have been the subject of many publications in the field of various structure analyses. The articles (Mochocki and Radoń, 2019; Mochocki *et al.*, 2020) concern the reliability

analysis of lattice towers using a systems approach. The articles analyzed the impact of the wind load probability distribution and the type of connections in towers on their reliability. The paper (Kubicka and Radoń, 2018) is devoted to a unique design situation, which is undoubtedly the occurrence of fire in building structures. The authors examine the change in reliability indicators during fire using the example of lattice trusses. The article (Dudzik and Potrzeszcz--Sut, 2021) presents two approaches to the analysis of structural reliability. The primary research method was the First Order Reliability Method (FORM). The second analysis proposed a hybrid approach enabling the introduction of explicit forms of the limit state function into the reliability program. Neural networks and the proprietary MES module were used to create descriptions of this formula. The reliability of single-layer steel domes using FORM and Monte Carlo was the main subject of papers (Zabojszcza and Radoń, 2019, 2020; Radoń *et al.*, 2021).

In this paper, reliability-based design optimization is calculated using Costrel module of Strurel software (http://www.Strurel.de). The optimization task was solved by a constrained sequential quadratic programming procedure. The optimization scheme is a first order scheme. It requires twice-differentiability of the state function and uses only first-order approximations for failure probabilities (FORM). In Costrel, two search algorithms, Joint5 and NLPQL, respectively, are implemented. Both are multi-constraint optimizers with special driving routines.

4.   Robust optimization

The possibility of using reliability optimization in design practice depends on the availability of the joint probability density function of the structure and load parameters. Unfortunately, due to the lack of appropriate statistical data, the use of this formulation becomes impossible. A formulation that better adapts to design realities is robust optimization. This formulation does not require an estimate of the failure probability. The random nature of the structure response is taken into account through the definitions of the objective function and constraints, which include mean values and response variances. The typical robust optimization formulation is written as: find d, $\boldsymbol{\mu}_x$, minimize $\{E[f(\mathbf{d}, \mathbf{X}, \mathbf{P})], \sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]\}$ with constraints

$$
\begin{aligned}
&E[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})] - \widetilde{\beta}_i \sigma[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})] \geqslant 0 && i = 1, \ldots, k_g \\
&\sigma[c_k(\mathbf{d}, \mathbf{X}, \mathbf{P})] \leqslant \sigma_k^u && k = 1, \ldots, k_c \\
&d_j^l \leqslant d_j \leqslant d_j^u && j = 1, \ldots, n_d \\
&\mu_{x_r}{}^l \leqslant \mu_{x_r} \leqslant \mu_{x_r}{}^u && r = 1, \ldots, n_x
\end{aligned}
\tag{4.1}
$$

where: $\mathbf{d}$ – deterministic design variables, $\mathbf{X}$, $\mathbf{P}$ – vectors of random variables with expected values of μ_x, μ_p, f – objective function, g_i – functions of constraints, c_k – functions, the standard deviations of which must not exceed the allowable values σ_k^u, $\widetilde{\beta}_i > 0$ – coefficients corresponding to the constraints $g_i \geqslant 0$ which represent the safety margin with which these constraints must be met.

The robust optimization task is a multi-objective optimization task. In addition to the average value of the objective function, its dispersion is also minimized. The task can be modified to the following scalar optimization task: find values of variables $\mathbf{d}$, $\boldsymbol{\mu}_x$, that minimize

$$
\widetilde{f} = \frac{1 - \gamma}{\mu^*} E[f(\mathbf{d}, \mathbf{X}, \mathbf{P})] + \frac{\gamma}{\sigma^*} \sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]
\tag{4.2}
$$

with constraints (4.1).

The weighting factor $\gamma \in [0, 1]$ defines the importance of each criterion. Values μ^* and σ^* are normalizing constants.

The computational complexity of the task requires the use of appropriate approximations of the unknown responses of the structure, as well as the use of effective methods for estimating statistical moments. In the paper, the kriging algorithm with optimal Latin hypercubes and random sampling was used. Additionally, in order to verify the correctness of the obtained results, calculations were performed using the second order method. The calculations were conducted using Numpress Explore software (http://www.numpress.ippt.pan.pl).

5. Numerical results and discussion

5.1. Geometry

In this paper, a steel single-storey frame with dimensions $h = 600$ cm and $L = 2h = 1200$ cm (Fig. 2) is analysed. The columns were originally modeled using square tubes with dimensions $D = 26$ cm and $d = 18$ cm, Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$, yield strength $f_y = 235$ MPa. The stiffness of the beam is very high compared to the stiffness of the columns. In further calculations we assume $EI_b = \infty$. The structure is loaded with a horizontal force $P = 120$ kN. The initial column mass is $f_{M1} = 1658$ kg.
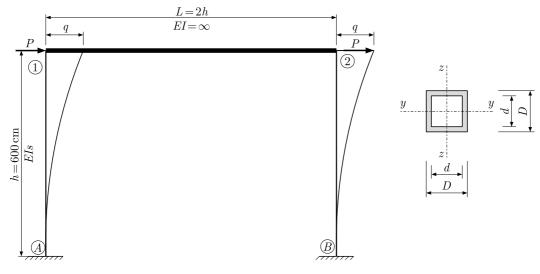

Fig. 2. Frame geometry and load

For the structure presented above, a series of analyzes is performed in subsequent stages. The first stage aims to verify the condition of the basic structure by performing a reliability analysis in Numpress Explore using the FORM method. The subsequent stages are related to the optimization of the structure. Deterministic optimization (in the Numpress Explore), reliability based design optimization (in the Costrel) and robust optimization (in the Numpress Explore) are performed successively.

5.2. Static analysis

When using the displacement method in terms of the first-order theory, the frame is once geometrically indeterminate. In the basic diagram of the displacement method, only one translational displacement is active, i.e. the horizontal displacement of the beam q. From the sum of projections on the X axis, we get

$$\Sigma X = P + P + T_{1A} + T_{2B} = 0 \qquad 2P = -T_{1A} - T_{2B} \tag{5.1}$$

After using the transformation formulas of the displacement method, the horizontal displacement of the frame is

$$\frac{24EJ}{h^3}q = 2P \quad \rightarrow \quad q = \frac{2Ph^3}{24EJ} = \frac{2Ph^3 \cdot 12}{24E(D^4 - d^4)} = \frac{Ph^3}{E(D^4 - d^4)} \tag{5.2}$$

In the example, in order to compare two optimization methods that take into account the random nature of design parameters, we only analyze the serviceability limit state. It expresses the difference between the permissible displacement and the displacement obtained as a result of calculations.

5.3. Reliability analysis

The reliability analysis of the structure was carried out using the FORM method. For the example under consideration, random variables were assumed as: D – the external dimension of the cross-section, d – the internal dimension of the cross-section, E – Young's modulus and P – force. Random variables are not correlated. The mean values of random variables and the coefficient of variation are listed in Table 1.

Table 1. Description of random variables

Random variables X_i	Mean values	Standard deviation	Coefficient of variation
D	26 cm	0.26 cm	1 %
d	18 cm	0.18 cm	1 %
E	21 000 kN/cm^2	630 kN/cm^2	3 %
P	120 kN	3.6 kN	3 %

The limit function is the limitation of the permissible horizontal displacement q_d of the node (SLS)

$$f_{SLS}(x) = q_d - q = 4 - \frac{Ph^3}{E(D^4 - d^4)} \tag{5.3}$$

where: q – horizontal displacement of the frame bolt, q_d – maximum horizontal displacement equal to $L/150 = 4$ cm.

The reliability index is $\beta^{SLS} = 1.909$ and probability of failure $p_f = 2.812\text{E-}02$.

5.4. Deterministic optimization

To emphasize the advisability of using the uncertainty of design parameters, in the first stage we performed deterministic optimization along with the assessment of structure reliability. In this optimization method, we look for optimal cross-section dimensions, using the classic deterministic optimization algorithm. The objective function is the mass of the single column

$$f_c = \min(D^2 - d^2)h\rho = \min(\text{mass}) \tag{5.4}$$

where: $\rho = 0.00785$ kg/cm^3 – steel density, h [cm] – column height.

Simple bounds are described in Table 2. They are the upper and lower limits of the searched design variables.

For this case 1% tolerance of the cross-sectional dimensions of the square tubes has been assumed. Inequality limits are formulated as conditions for not exceeding the permissible frame displacement

$$f_{SLS}(x) = q_d - q = 4 - \frac{Ph^3}{E(D^4 - d^4)} \tag{5.5}$$

Table 2. Simple constraints of the design variables

Design variable	Lower limit	Upper limit
D	25 cm	27 cm
d	17 cm	19 cm

Additionally, the feasible area is shown in Fig. 3. The vertical lines (green dotted) and horizontal lines (blue dashed) represent the simple constraints (for D and d) used in the considered example. The red line marks the limitation of the permissible horizontal node displacement of the frame. The permissible area is the result of individual restrictions and is marked in grey.
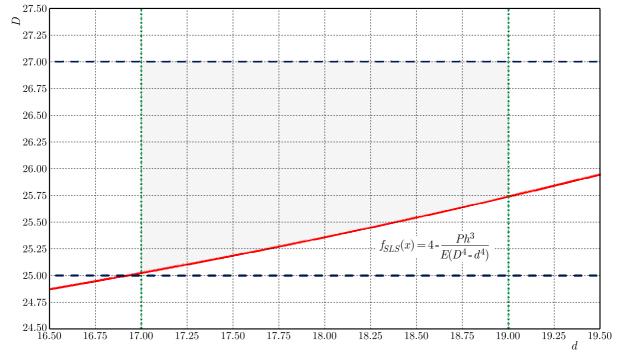

Fig. 3. Feasible area

The resulting cross-sectional dimensions are summarized in Table 3. The value of the objective function is 1420 kg.

Table 3. Values of the design variables obtained in deterministic optimization

Design variable	Optimal value
D	25.74 cm
d	19.00 cm

The probability of failure and the reliability index have also been verified, $p^{SLS} = 0.5$, $\beta^{SLS} = 0.0004$.

5.5. Reliability based design optimization

In the next approach, reliability based design optimization was used.

The task of RBDO takes the form: find μ_D, μ_d, minimizing $f_c = (D^2 - d^2)h\rho = \min(\text{mass})$ with constrains

$$p\left(4 - \frac{Ph^3}{E(D^4 - d^4)}\right) - \Phi(-1.8) \leqslant 0 \qquad 25 \leqslant \mu_D \leqslant 27 \qquad 17 \leqslant \mu_d \leqslant 19 \qquad (5.6)$$

In the case of reliability optimization, it is necessary to assume a limit reliability index (failure probability). In the case under consideration, the limit was set at $\beta = 1.8$. After performing reliability optimization, the values of width and height of the cross-section were obtained as: $D = 26.34$ cm and $d = 19.00$ cm. The probability of failure and the reliability index for RBDO approach were $\beta^{SGU} = 1.8$. $p_f^{SGU} = 3.6\text{E-}02$.

The weight of the optimized structure was $f_c = 1567$ kg.

5.6. Robust optimization

The objective function is mass of the structure, but assuming that it takes into account the weighting factor γ, it determines the meaning of each criterion. Design variables are the expected values of the external and internal dimensions of the cross-section: μ_D, μ_d. The value of the coefficient of variation was set at 1%.

The robust optimization task takes the form: find: μ_D, μ_d, minimizing $f_C = (1-\gamma)E(\text{mass}) + \gamma\sigma(\text{mass})$ with constrains

$$E\left(4 - \frac{Ph^3}{E(D^4 - d^4)}\right) - \widetilde{\beta}_i\sigma\left(4 - \frac{Ph^3}{E(D^4 - d^4)}\right) \geqslant 0$$
$$25 \leqslant \mu_D \leqslant 27 \qquad 17 \leqslant \mu_d \leqslant 19$$

(5.7)

where $\gamma \in [0,1]$ – weighting factor determines the importance of each criterion.

Structural optimization was performed using the kriging response surface. Experiments are generated according to the plan of optimal Latin cubes and random sampling (Fig. 4). The parameters are $\gamma = 0.5$, $\widetilde{\beta^{SGU}} = 2.0$.

The values of the design variables are summarized in Table 4.

Table 4. Values of the random variables obtained in robust optimization

Design variable	OLH sampling	Random sampling
D [cm]	26.33	26.29
d [cm]	18.91	18.89

An increase in the cross-section height and an increase in the weight of the structure result in a significant change in the value of the reliability index and the probability of failure, which in this case are, respectively, for OLH and random sampling $\beta_R^{\text{OLH}} = 1.868$ and $\beta_R^{\text{RS}} = 1.775$, while the mass of structure is $f_{fR}^{\text{OLH}} = 1581$ kg and $f_{fR}^{\text{RS}} = 1577$ kg.

5.7. Summary

Table 5 presents a summary of the results, including cross-sectional dimensions, structure weight, reliability index and failure probability. An additional aspect involves comparing the necessary number of iterations and the calculation time for each case.

The optimization results (Table 5) show that the best optimized design in terms of weight (a change of almost 240 kg compared to the initial value) does not meet the safety requirements. For deterministic optimization, the reliability index tended to 0. The results of Robust optimization, both in the case of generating experiments using the Optima Latin Hypercube (OLH) and Random Sampling (RS) plans, gave similar results. However, the use of the assumed better and much more effective method of generating experimental points (OLH) allowed the result to be provided much faster than in the case of RS. The Latin hypercube concept ensures an even distribution of points in the experiment plan (Fig. 4). This avoids clustering in certain areas and leaving other areas unexplored. The formation of such clusters has a particularly negative impact on the operation of the starting point selection procedure. Obtaining the final result obviously
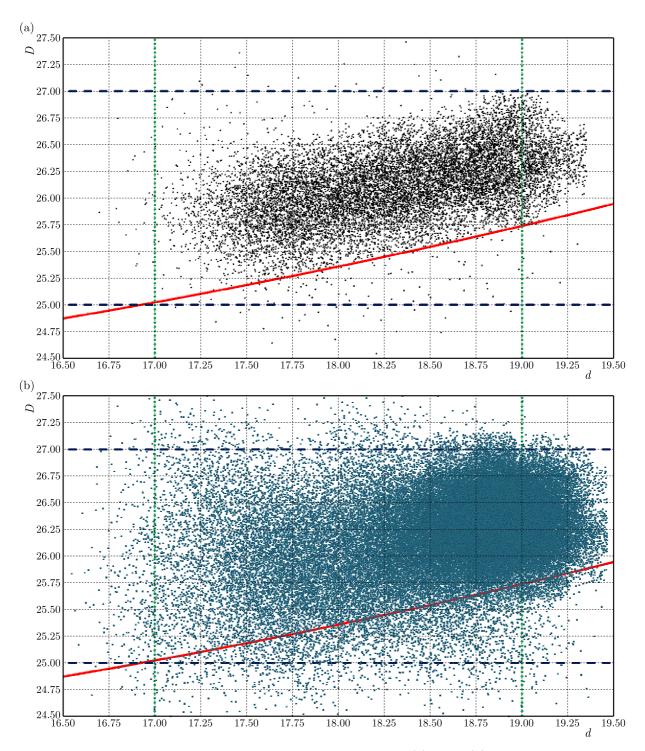
(a)

(b)

Fig. 4. An example of implementing a random sample using: (a) OLH, (b) random sampling

depends on the choice of the starting point for the iterative process. In the analyzed task, we see this in the example of comparing computation times using OLH and random sampling. For calculations using OLH it is 49 seconds, while for random sampling it is 8 hours 49 minutes and 49 seconds. The weight of the optimized structure is lower by approximately 80 kg (less than 5%) while still having a satisfactory reliability index. A similar Reliability Based Design Optimization analysis resulted in a slightly better optimized design, assuming a similar reliability index (at the level of 1.8). However, the obtained result is within the assumed limit of the permissible

area. Adopting such a solution may have a negative impact on possible additional unforeseen aspects of the analysis, i.e. inaccurate adoption of analysis parameters (standard deviation of the adopted variables, availability of the joint probability density function, etc.).

Table 5. Summary of results for individual analyses

Variable	Initial Values	Determi- nistic	Robust (Kriging)		RBDO
			OLH sampling	Random sampling	
D [cm]	26.00	25.74	26.38	26.29	26.34
d [cm]	18.00	19.00	18.97	18.89	19
Mass [kg]	1658	1420	1581	1577	1567
Reliability index	1.909	0.0004	1.868	1.775	$\beta = 1.8$
p_f	0.0281	0.5	0.0309	0.0380	0.036
No. Iterations	–	–	6	9	5
Time of calculation	–	1 s	49 s	8 h 49 min 49 s	0.2 s

Figure 5 shows the results of design optimization for the four analyses performed. The results of deterministic optimization and RBDO were on the border of the acceptable range. Only the Robust optimization results were within the acceptable range.
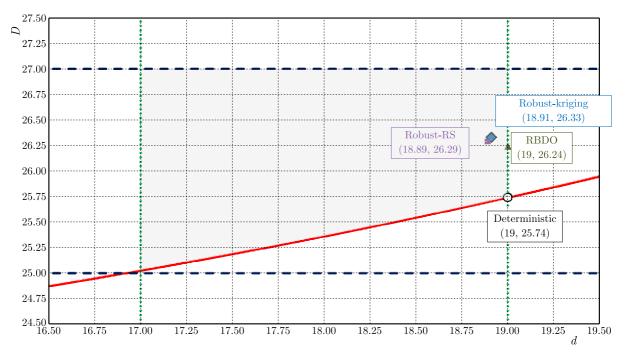

Fig. 5. Results for individual analyses with the feasible area

6. Conclusions

In the traditional deterministic approach, the random nature of the design variables and other parameters involved in the optimization formulation is accounted for by partial safety factors, which are typically calibrated to be applicable to the widest range of design tasks.

In order to find a solution that is insensitive to imperfections of model parameters or external influences that are difficult to control, we have two options. The first one is robust optimization. The second one is optimization based on the reliability of the so-called RBDO.

If guaranteeing a high level of safety is the most important requirement for the designed structure, it is worth choosing RBDO. Within RBDO, design constraints are formulated using failure probabilities. The applicability of RBDO is strongly dependent on the availability of the joint probability density function. The reliability of the estimated failure probability values depends on the precise stochastic model.

A formulation of non-deterministic optimization that better adapts to design realities is robust optimization. Unlike RBDO optimization, this formulation does not require estimation of failure probabilities. The random nature of the structure response is taken into account by defining the objective function and constraints, including mean values and variances. The computational complexity of this approach is related to the use of effective methods of estimating statistical moments.

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