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THE PROBABILISTIC MODEL FOR SYSTEM RELIABILITY ANALYSIS OF A STEEL PLANE AND SPATIAL TRUSSES[1]

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The article focuses on the system reliability analysis of steel trusses (plane and spatial). The computations are realized by the use of a developed by the author C++ code. The following loads are taken into account: self-weight, weight of coverings, wind and snow. The limit state function is defined as a difference between the bearing capacity and the effect of action of an element. The paper presents how effective tool is the system reliability analysis compared with traditional structural design methods. The methods of transforming Gumbel distribution into normal and generating random variables are described.

Keywords: system reliability analysis, plane steel truss, spatial steel truss, cut-sets, normal distribution transformation

1. Introduction

Nowadays, the reliability of structures is very important topic. The methods of reliability and optimization are constantly developed, improved and their meaning in design constantly increases. Nothing unusual, because exactly these methods seem to be the solution of the problem how to design with high safety, but with as costs low as possible.

The reliability methods can be divided into two groups: considering the reliability of single elements and the reliability of the whole structural systems. In the first group one can enumerate approximation methods as: FORM (Breitung, 2015; Ditlevsen, 1987; Keshtegar and Meng,2017), SORM (Cai and Elishakoff, 1994; Hu *et al.*, 2021) and simulation methods like: Monte Carlo (Rausch *et al.*, 2019; Sharma, 2020; Zaeimi and Ghoddosain, 2020), Importance Sampling (Melchers, 1989; Papaioannou *et al.*, 2018), Artificial Neural Networks (Flood, 2008; Potrzeszcz-Sut and Dudzik, 2022).

The presented article uses a system reliability method, whose basics have been well-known for decades. Instead of this fact, as the method is not easy to implement, especially for big structures (with many possible causative elements) its application is limited. Nevertheless, in the last years the system reliability analysis has been successively becomming more popular, especially for steel truss analysis (Mochocki *et al.*, 2018; Park *et al.*, 2004, Zabojszcza and Radoń, 2020).

In the article, the plane and the spatial truss were analysed. For both types of structures the main task was to identify so-called cut-sets, i.e. the possible way of transforming the structure into a mechanism. In other words, the cut-sets are such sets of elements whode failure determines the failure of the whole structure. The method of searching cut-sets for plane trusses was presented in the previous author's papers (Kubicka *et al.*, 2019; Kubicka, 2022; Kubicka and Sokol, 2023). Generally, the method is based on spectral matrix stiffness analysis and is appropriate for both persistent and accidental (fire) design situation. If the cut-sets for the analysed structure are known, it is possible to define the reliability system of a mixed type, that is a parallel-series

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system, which is a combination of two basic types of systems, which will be described in the following part of the paper.

2. Materials and methods

In the paper, the system reliability method was applied. The following steps of this method are presented in Fig. 1 with comparison with a traditional design. The red frames indicate the steps, during which the random character of variables is taken into account.

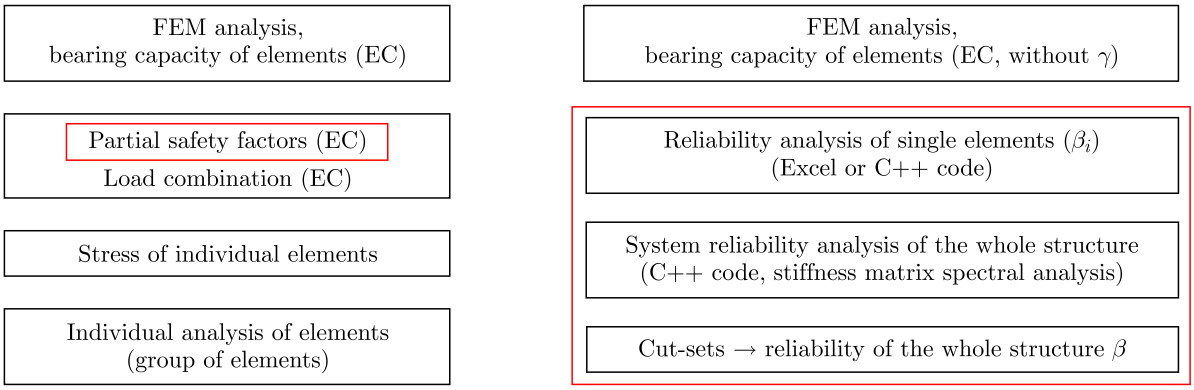


Fig. 1. Comparison of the traditional design method (a) with the proposed probabilistic method (b)

In Fig. 1, γ corresponds to the partial safety coefficient, β is the reliability index, which will be defined in the following part of the article, subscript i informs that the value is defined for i-th element.

The proposed method has a few advantages, including:

- Possibility to decide which values are random and which are deterministic
- Coefficient of variation can be defined individually for each random variable
- Possibility to use different types of distribution

After computing the bearing capacity and conducting FEM analysis, knowing the effect of action in individual bars, it is possible to compute reliability for each individual element according to Table 1. In this method, all variables (N, Eff) must have normal distribution, in other case the transformation to normal distribution is essential.

Table 1. The method of computing reliability for individual elements

Number of element	Bearing capacity (N) Effect of action (Eff)	Safety margin (M)	Reliability index of element (β_i)	Probability of elements failure (P_{fi}) Reliability of element (R_i)
i K	$N_i(\mu_{Ni}, \mu_{Ni})$ $\mathit{Eff}_i(\mu_{\mathit{Eff}_i}, \sigma_{\mathit{Eff}_i})^*$	$M_i = N_i - \mathit{Eff}_i$ $\mu_{Mi} = \mu_{Ni} - \mu_{\mathit{Eff}_i}{}^*$ $\sigma_{Mi} = \sqrt{\sigma_{Ni}^2 + \sigma_{\mathit{Eff}_i}^2}{}^*$	$\beta_i = \dfrac{\mu_{Mi}}{\sigma_{Mi}}$	$P_{fi} = \Phi(-\beta_i)$ $R_i = 1 - P_{fi}$ Φ – Laplace function

* μ_{Xi} – mean value of X value for i-th element,
σ_{Xi} – standard deviation of X value for i-th element

Knowing the reliability of single elements, it is possible to compute the reliability of the whole structure. To realize this task, it is essential to find so-called cut-sets, i.e. the way the structure can transform into a mechanism. On this basis, a mixed system is created, whose elements are connected in groups in a parallel way, and then are connected in series. Therefore, the parallel systems have to be computed and than the series system. The series system is reliable as long as all of its elements are reliable. In other words, the failure of a single element determines the failure of the whole structure. The reliability of a series system is computed according to the following formula

$$R = \prod_{i=1}^{n} R_i \tag{2.1}$$

The parallel system is such a system which is reliable as long as at least one of its element is reliable. The reliability of such a type of system is computed according to

$$R = 1 - \prod_{i=1}^{n} (1 - R_i) \tag{2.2}$$

In the paper, plane and spatial truss were analysed according to the algorithm presented in Fig. 1 with a simple and more advanced probabilistic model described in the following part. FEM analysis was conducted in Robot Structural Analysis program, computation connected with reliability was realized in the author C++ code.

2.1. Simplified probabilistic model

Simplified probabilistic model was used in some of the previous author's papers. In this model, it is assumed that the truss element may fail if the bearing capacity of the element exceeds the effect of action. Therefore, the limit state function g can be written as

$$g = N - E_{eff} \tag{2.3}$$

where N is defined as in Eq. $(2.4)_1$ for compressed elements, and in Eq. $(2.4)_2$ for element in tension

$$N_{b,fi} = \chi_{fi} A f_y \qquad N_{c,fi} = A f_y \tag{2.4}$$

In this model, it is assumed that the buckling coefficient χ_{fi} is a deterministic value computed according to Eurocode. In Table 2, characteristics of all variables are presented, all of them are assumed to have normal distribution.

Table 2. The characteristic of variables used in the simplified model

Value	Deterministic or probabilistic	Coefficient of variation ν [%]	Distribution type
Buckling coefficient χ_{fi}	deterministic	–	–
Cross-sectional area A	probabilistic	8	normal
Yield strength f_y	probabilistic	6	normal
Effect of action E_{eff}	probabilistic	6	normal

Because of the formulation of the limit state function (product of random variables) and assumption about normal distribution, the coefficient of variation for bearing capacity can be approximated according to the following formula (Biegus, 1999)

$$\nu_N = \sqrt{\nu_{fy}^2 + \nu_A^2} = \sqrt{0.06^2 + 0.08^2} = 0.1 = 10\% \tag{2.5}$$

2.2. "Full" probabilistic model

In the previous work (Kubicka and Radoń, 2018) it was demonstrated that treating the buckling coefficient as a deterministic value leads to getting significantly different results than in the case of considering it random. What is more, the buckling coefficient is a function of few variables, where some of them (A, f_y) were defined random

$$\chi_{fi} = f(f_y, A, E, I_y, L) \tag{2.6}$$

what also suggest that the value χ_{fi} should be defined probabilistic. So, the "full" probabilistic model was extended by the assumption that the buckling coefficient is a random value. In the function of buckling coefficient only length of element L was assumed to be deterministic. All other variables, i.e. cross-sectional area A, yield strength f_y, Young's modulus E and moment of inertia I_y, were treated probabilistic. The assumption that all random variables have normal distribution was abandoned, especially it was assumed that atmospheric loads (wind, snow) have Gumbell distribution. Therefore, the transformation to normal distribution had to be made.

2.2.1. Transformation from Gumbell to normal distribution

In the article, the transformation from Gumbell to normal distribution was realized according to two methods, namely the method of moments and collocation point method (Murzewski, 1989):

— method of moments

$$\overline{x} = \widetilde{x} + Cu_x \qquad C = 0.57721 \qquad \mu_x = \frac{\pi}{\sqrt{6}}u_x \tag{2.7}$$

— collocation point method

$$\overline{x} = \widetilde{x} + t_N u_x \qquad t_N = 0.3457 \qquad \mu_x = \frac{u_x}{\theta_N} \qquad \theta_N = 0.9762 \tag{2.8}$$

where $\overline{x}$, μ_x are characteristics of normal distribution and symbols $\widetilde{x}$, u_x correspond to Gumbell distribution.

2.2.2. Generation of random samples using the Box-Müller algorithm

In the presented method, the set of random variables was generated for each probabilistic value and, on this base, the mean value and standard deviation were computed. A part of code is presented in Fig. 2. The generator of random variables available in C++ generates variables with uniform distribution. To conduct reliability analysis, it is necessary to have variables with normal distribution. One of the method that enables transforming uniform variables into variables with normal distribution is using the Box-Müller algorithm (https://mathworld.wolfram.com/Box-MullerTransformation.html).

In this method, the variable with normal distribution Y_1 is generated on the base of two random variables with uniform distribution (y_1, y_2), according to the formula

$$Y_1 = \cos(2\pi y_2)\sqrt{-2\ln y_1} \tag{2.9}$$

After generation of variables with normal distribution, the loop that generates random values of variables (A, f_y, E, I_y) is started. It is realized n_s (number of simulation) times, in the presented article $n_s = 100$. In each realization of the loop, the initially assumed standard deviation of a random variable was multiplied by a random value generated previously, and it was added to the mean value of the random variable. Symbol A[i] and Iy[i] means the cross-sectional area and yield strength of the i-th structure element. sA, sfy, sE, sIy define the initial standard deviation.

Having 100 samples of rA, rfy, rE, rIy, the mean value and the standars deviation were computed for each random value, and these characteristics were taken into account in the further reliability analysis.

```
double normalRandom()
{
  double y1=RandomGenerator();
  double y2=RandomGenerator();
  return cos(2*M_PI*y2)*sqrt(-2.*log(y1));
}
   for (int z=1; z<=ns; z++)
   {
       double rA = normalRandom()*sA+A[i];
       double rfy = normalRandom()*sfy+fy;
       double rE = normalRandom()*sE+E;
       double rIy = normalRandom()*sIy+Iy[i];
```

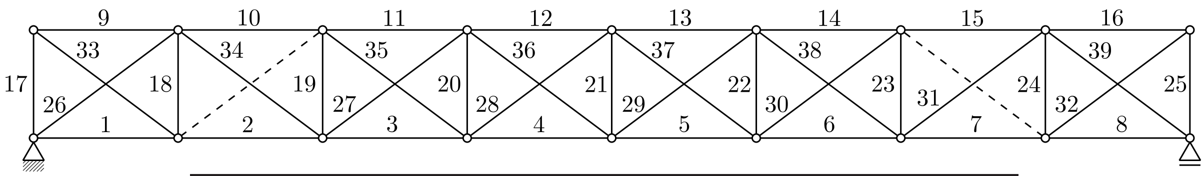
Fig. 2. The part of C++ code generating random samples of variables using the Box-Müller algorithm

3. Results

In the paper, two types of trusses (plane and spatial) were analysed. Both of them are statically indeterminate, because in the case of such types of structure the advantages of system reliability is most visible.

3.1. Plane truss

The plane truss analysed in the paper is presented in Fig. 3. Two of cross-braces are drawn by the dashed line. They were not taken into account during reliability analysis, because FEM analysis indicated that the bearing capacity in this elements is exceeded. Nevertheless, reliability analysis indicated that the structure as a whole is safe despite the fact that these two elements are unreliable.



No. of element	1-8	9-16	17,25,26,39	27-38
Profile	IPE 160	IPE 300	CHS 108x5	CHS 70x5

Fig. 3. The analysed plane truss

It was assumed that the structure was loaded by: self-weight (sw), cover (c), snow-3rd zone(s) and wind-1st zone(w). Each type of load was considered individually during the reliability analysis.

The cut-sets identified for the truss are presented in Table 3. During searching cut-sets, only posts (17-25) and cross-braces (26-39) were taken into account as the most probable elements of failure. As from the system reliability analysis point of view, the most important are initial cut-sets, consisting of few elements. The searching for cut-sets was limited to 4-elements. Usually, in the case of cut-sets consisting of a higher number of elements the reliability is equal to one. The more elements in the cut-set, the more probably some of them will have reliability equal 1.0. According to Eq. (2.2), in such a case, the reliability of a parallel subsystem is equal 1, what does not change the final reliability of the structure.

The reliability of each element under a single type of load: self-weight (sw), cover (c), snow (s) or wind (w) was computed according to the procedure presented in Table 1. Then reliabilities of the whole structure under the individual type of load (R_{sw}, R_c, R_s, R_w) were computed according to the cut-sets presented in Table 3. Elements in the brackets are connected in parallel (for example $\{17, 18\}$, $\{19, 20, 28\}$, $\{19, 20, 21, 29\}$), so the reliabilities of such a set of elements are computed according to Eq. (2.2). Then all set of elements in the brackets are connected in series, what is computed according to Eq. (2.1). The final reliability R_{fin} is computed according to Eq. (3.1) as a product of reliabilities under individual loads. This corresponds to a series system and is correct for the most unsafe situation, when all types of loads act together

$$R_{fin} = R_{SW} R_C R_S R_W \tag{3.1}$$

Table 3. The cut-sets identified for a plane truss

1-element cut-sets	2-element cut-sets	3-element cut-sets	4-element cut-sets
{31}	{17,18}	{19,20,28}	{19,20,21,29}
{34}	{17,26}	{19,20,36}	{19,20,21,37}
	{17,33}	{20,27,28}	{20,21,27,29}
	{18,26}	{20,27,36}	{20,21,27,37}
	{18,33}	{20,28,35}	{20,21,29,35}
	{19,27}	{20,35,36}	{20,21,35,37}
	{19,35}	{21,28,29}	{21,22,23,28}
	{23,30}	{21,28,37}	{21,22,23,36}
	{23,38}	{21,29,36}	{21,22,28,30}
	{24,25}	{21,36,37}	{21,22,28,38}
	{24,32}	{22,23,29}	{21,22,30,36}
	{24,39}	{22,23,37}	{21,22,36,38}
	{25,32}	{22,29,30}	
	{25,39}	{22,29,38}	
	{26,33}	{22,30,37}	
	{27,35}	{22,37,38}	
	{28,36}		
	{29,37}		
	{30,38}		
	{32,39}		

In the paper during computing, the final reliability simplified assumption was made, one type of load for snow and wind was taken into account. To be more precise, these reliabilities should be considered as the following functions

$$R_S = R(S_1, S_2) \qquad R_w = R(W_1, \dots, W_n) \tag{3.2}$$

The reliability of the truss presented in Fig. 3 was estimated in few attempts. The first attempt was completely conducted according to a simplified probabilistic model, described in Section 2.1. In the second attempt, the model was extended to take into account different types of distribution (for wind and snow Gumbell distribution was assumed). So, some transformation method to normal distribution had to be applied (method of moments and collocation point method). In the last attempt, the (3rd) "full" model was applied. Characteristics of random variables are presented in Table 4.

In Table 5, the results of system reliability analysis, conducted according to the previously described method are presented.

It is noticeable that the reliability index β is slightly different in the subsequent attempts. But the structure, according to each attempt, is reliable because $\beta > 3.8$, what is the minimum value recommended by Eurocode (EN-1990, 2002).

Table 4. The variables used in the "full" model for the plane truss

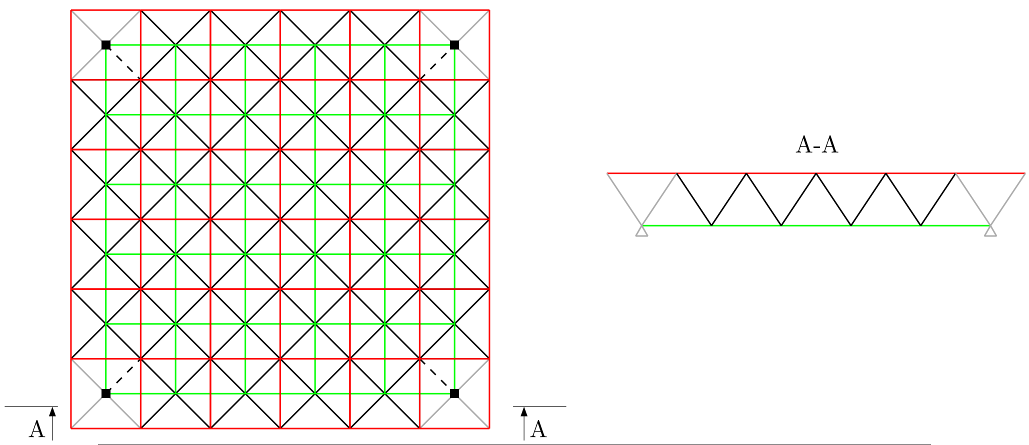
Random variable	Coefficient of variation [%]
Yield strength f_y	6
Cross-sectional area A	8
Modulus of elasticity E	5
Moment of inertia J	8
Effect of action Eff	6

Table 5. Reliability analysis results for the plane truss

No. of attempt	1	2		3
		Method of moments	Collocation point method	
R_{sw}	1.0	1.0	1.0	$0.(9)^{12}88$
R_c	$0.(9)^{8}829535$	$0.(9)^{8}829535$	$0.(9)^{8}829535$	$0.(9)^{5}8634554069$
R_s	$0.(9)^{7}84158383$	$0.(9)^{7}71059492$	$0.(9)^{7}83384258$	$0.(9)^{5}8634554069$
R_w	1.0	1.0	1.0	$0.(9)^{10}83968$
R_{fin}	$0.(9)^{7}82387918$	$0.(9)^{7}69289027$	$0.(9)^{7}69289027$	$0.(9)^{5}76909$
β	5.513	5.415	5.506	4.546

3.2. Spatial truss

The next example concerns the spatial truss presented in Fig. 4. Similarly, like in the case of the plane truss, some elements are drawn with dashed lines, what means they were excluded from the system reliability analysis. The reason was analogous as in the previous example, the bearing capacity of this elements was exceeded according to FEM analysis, but the system reliability analysis indicated that the structure is safe as a whole, despite of the fact that some of elements are unreliable as individuals.



Element	Bottom chord (green)	Top chord (red)	Cross-braces	
			grey	black
Profile	IPE 100	IPE 200	CHS 70x5	CHS 51x4

Fig. 4. The analysed spatial truss

Identification of cut-sets was realized again with the assumption that the cross-braces are most likely elements to fail. Creating the elements, 1-,2-, 3- and 4-element cut-sets are presented in Fig. 5. In Table 6, the identified cut-sets are presented. The interpretation is the same as in the case of the plane truss analysed in Section 3.1.

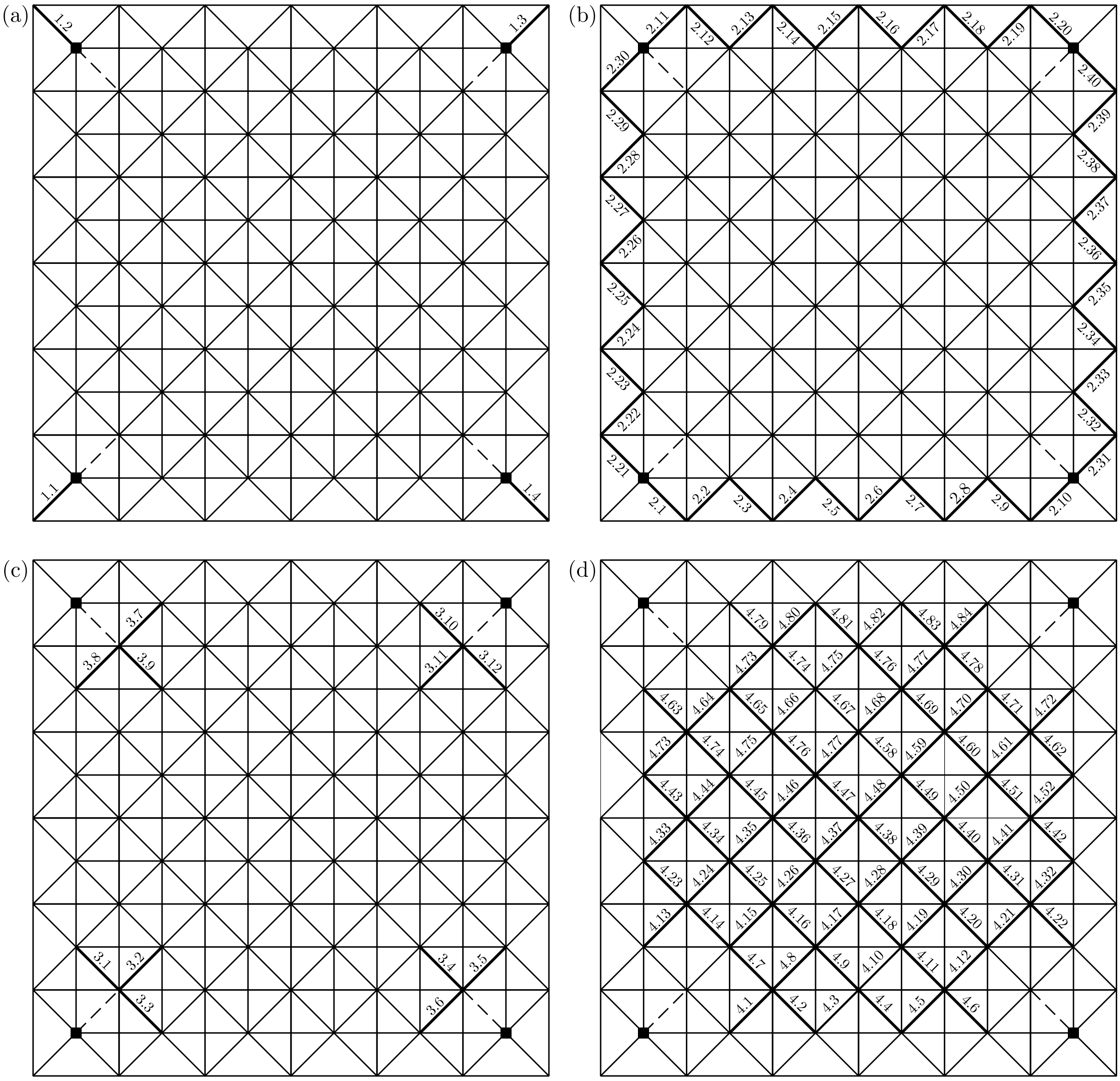


Fig. 5. Creating the elements (a) 1-, (b)2-, (c) 3-, (d) 4-element cut-sets for the analysed spatial truss

For the analysed spatial truss, the final reliability computed analogously as in the case of the plane truss was equal to 1.0, what means that the probability of failure of the structure is practically equal to 0. This is the result of the fact that the analysed structure is highly statistically indeterminate, so it is redundant what means that the failure of a single member merely changes the system behaviour and does not result in the collapse of the whole structure. That is why the system reliability analysis is a useful tool during designing of the structure. It allows the designer to follow possible failure path and decide which element may be designed without satisfying ULS.

Table 6. The cut sets identified for the analysed spatial truss

1-element cut-sets	2-element cut-sets	3-element cut-sets	4-element cut-sets
{1.1}	{2.1, 2.2}	{3.1, 3.2, 3.3}	{4.1, 4.2, 4.7, 4.8}
{1.2}	{2.3, 2.4}	{3.4, 3.5, 3.6}	{4.3, 4.4, 4.9, 4.10}
{1.3}	{2.5, 2.6}	{3.7, 3.8, 3.9}	{4.5, 4.6, 4.11, 4.12}
{1.4}	{2.7, 2.8}	{3.10, 3.11, 3.12}	{4.13, 4.14, 4.23, 4.24}
	{2.9, 2.10}		{4.15, 4.16, 4.25, 4.26}
	{2.11, 2.12}		{4.17, 4.18, 4.27, 4.28}
	{2.13, 2.14}		{4.19, 4.20, 4.29, 4.30}
	{2.15, 2.16}		{4.21, 4.22, 4.31, 4.32}
	{2.17, 2.18}		{4.33, 4.34, 4.43, 4.24}
	{2.19, 2.20}		{4.35, 4.36, 4.45, 4.46}
	{2.21, 2.22}		{4.37, 4.38, 4.47, 4.48}
	{2.23, 2.24}		{4.39, 4.40, 4.49, 4.50}
	{2.25, 2.26}		{4.41, 4.42, 4.51, 4.52}
	{2.27, 2.28}		{4.53, 4.54, 4.63, 4.64}
	{2.29, 2.30}		{4.55, 4.56, 4.65, 4.66}
	{2.31, 2.32}		{4.57, 4.58, 4.67, 4.68}
	{2.33, 2.34}		{4.59, 4.60, 4.69, 4.70}
	{2.35, 2.36}		{4.61, 4.62, 4.71, 4.72}
	{2.37, 2.38}		{4.73, 4.74, 4.79, 4.80}
	{2.39, 2.40}		{4.75, 4.76, 4.81, 4.82}
			{4.77, 4.78, 4.83, 4.84}

4. Conclusions

The presented analysis undoubtedly indicated that the system reliability analysis is an appropriate tool to estimate the reliability of both plane and spatial trusses. What is more, the proposed method is elastic, so the user can define characteristics of random variables. The method have to be developed especially by taking into account the load combination in the probabilistic point of view. What is more, in the initial research all processes were considered as time-independent. In fact, wind should be considered as time-dependent, what can be realized by using stochastic dynamics (Śniady, 2000). That is what the author is going to do in the nearest future. After this, the proposed probabilistic method can be considered as a complement to the traditional design method based on the partial safety factor. It seems that such an approach could result in limitation of the structure cost. Because of the redundancy of highly statically indeterminate structures it is not always necessary to select profiles for some group of elements according to "the weakest" element, what may lead to the situation that some of the elements are almost not stressed. Thanks to the system reliability analysis, it is possible to choose profiles with smaller dimensions, which reduces the volume of steel used for the structure and directly translates into not only lower costs, but also lower self-weight of the structure.

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