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SELF-SYNCHRONIZATION OF DRIVE VIBRATORS OF AN ANTIRESONANCE VIBRATORY CONVEYOR

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A theoretical analysis of synchronization of inertial vibrators of a vibratory conveyor with a dynamic damper is presented in this paper. It is shown that for the over-resonance regime and counter-running drive vibrators, there is only one stable state of the system warranting formation of necessary sectional vibrations of the trough. The analytical form of the moment-synchronizing vibrators is also determined, and on the basis of this, the influence of angular vibrations of the body on the synchronizing process of the drive vibrators is determined. Due to the differences in the participation of angular vibrations in the self-synchronizing process in relation to classical solutions, the presented results fundamentally influence the design of long antiresonance conveyors.

Keywords: stability, vibratory conveyor, antiresonance, self-synchronizstion

1. Introduction

Among vibratory conveyors, antiresonance conveyors have become increasingly popular in recent years (Surówka and Czubak, 2021; Czubak and Gajowy, 2022; Gajowy, 2019). Such conveyors use dynamic dampers (Asami, 2019; Ascari, 1980; Fasana and Giorcelli, 2010) based on Frahm's patent from 1911 (Frahm, 1911) to decrease vibrations of drive frames as well as related harmful effects on the surrounding environment. The solution shown in Fig. 1, which is presently under prototype investigation, is one of the newest solutions of this type.

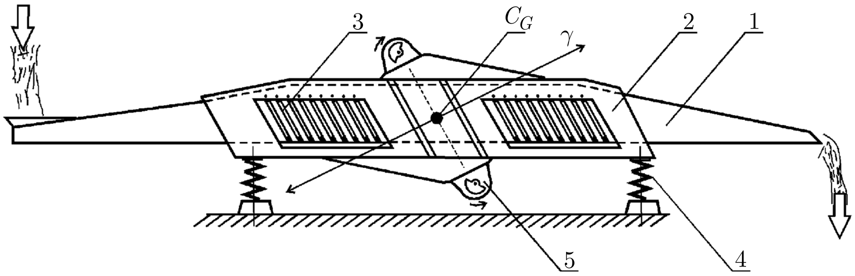

Fig. 1. Patent application No P.434041. Antiresonance vibratory conveyor (application drawing Fig. 2 in P. 434041 document). KMC Global Europe (2020), Opole, PL. Author: Jerzy Michalczyk, Cracow

It is written in the patent description that "This antiresonance vibratory conveyor is characterized by the fact that its body (2) is of a stiff structure, while trough (1) (functioning as a mass of the dynamic eliminator) is of a self-supporting structure and in its central segment is suspended inside body (2) with a clearance allowing for its effect on vibratory motion. The mass centres of trough (1) and of body (2) with vibrators (5) and leaf springs (3) overlap each other and – in addition – trough (1) is suspended inside body (2) on leaf springs (3), which are mounted in pairs symmetrically relative to the conveyor axial plane of symmetry, near the nodal points of the first form of the natural trough vibrations (1)".

The author of this solution aimed to build a long conveyor (several metres), which due to construction difficulties (mainly related to maintaining sufficient stiffness of the frame and trough) is not an easy task.

In addition to structural problems, issues arise related to the vibrator drive and self--synchronization processes of vibrators. When length of the conveyor is increased, its mass moment of inertia is also increased, which in turn leads to decreasing angular vibrations of the body and decreasing the synchronizing moment of the vibrators.

Knowledge concerning self-synchronization of unbalanced masses is very wide and includes problems of deterministic (Paz and Cole, 1992) and chaos theory (Chedjou *et al.*, 2008). Fundamental works in this area were published by Blekhman (1971, 2000). He formulated, among other concepts, the stability criterion for synchronized motion of unbalanced masses and, on its basis, developed detailed rules for constructing drive structures of inertial vibrators, which were cited later in papers by several authors (Michalczyk and Cieplok, 2014; Hou *et al.*, 2017; Li *et al.*, 2020).

Analyses concerning nonlinear systems are especially interesting. The conditions of synchronizing systems of two vibrators installed on a common platform and the criterion of global stability of the solutions obtained on the basis of analysing nonlinear equations were presented in (Smirnova and Proskurnikov, 2021). The synchronization of two exciters under the nonlinear influence of elastic elements of sectional-linear characteristics was investigated in (Zhang *et al.*, 2016). A system with a tri-motor was analysed in (Zou *et al.*, 2020), where the conditions of achieving the synchronous state were determined by means of the small-parameters method, while the stability criterion of synchronous motion was determined by means of the Poincaré--Lyapunov method. In (Zhao *et al.*, 2011), the problem of synchronizing two pairs of vibrators elastically placed on a common frame was reduced to the stability problem of two generalized systems. One of them was the generalized system of angular velocity disturbance parameters for four unbalanced rotors, and the other was the generalized system of three-phase disturbance parameters. Researchers have also obtained satisfactory results regarding the synchronization of unbalanced masses in spatial motion (Zhao, 2010; Cieplok and Wójcik, 2020; Fang *et al.*, 2019). Because of strong nonlinear connections between unbalanced masses and the body of the device (Dimentberg *et al.*, 1997), some analyses concerning transient processes are still based on numerical investigation (Zhang *et al.*, 2019; Shokhin *et al.*, 2021).

2. Theoretical analysis

The conveyor of a structure corresponding to the one presented in Fig. 1 was analyzed in this study. The analyses dealt with three problems: dynamic equations of motion of the system, analysis of motion stability of drive vibrators, and determination of an analytical formula for the synchronizing moment of vibrators. Realization of the last two problems required knowing the analytical solutions of equations of motion of the system. They were determined for the steady state, for which it was possible to assume a constant rotational speed of vibrators. This assumption allowed one to obtain linear equations and to lower the number of degrees of freedom by two. The dynamic equations of motion became also the basis for verifying analytical results.

2.1. Dynamic equations of the conveyor

Let us discuss the system presented in Fig. 2. It consists of four solid bodies representing: drive frame of a conveyor m_k with mass inertia J_{Ck}, transporting trough m_r with mass inertia J_{Cr} and two inertial vibrators each of them having mass m_w and radius of unbalance e_w. The conveyor is placed on a viscoelastic suspension described by parameters k_x, k_y, b_x, b_y; while the trough is connected with the body by means of a leaf springs system described by parameters k_{si}, b_{si}.

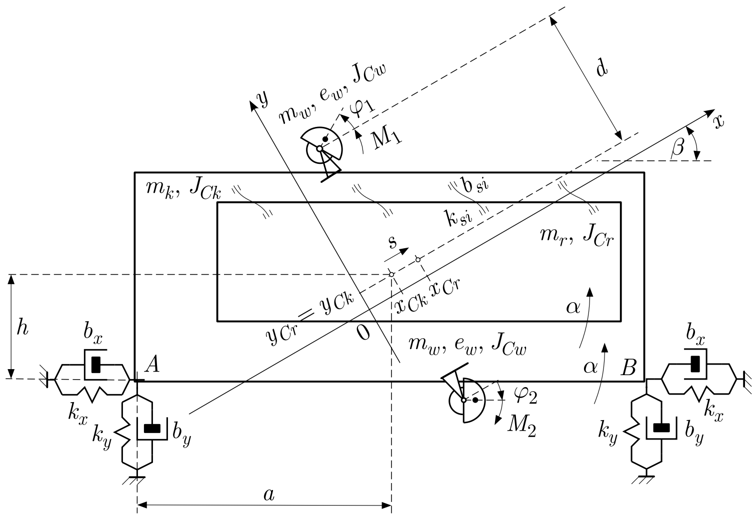

Fig. 2. Schematic presentation of the system.

Inertial vibrators are driven counter running by means of two asynchronous motors of identical drive characteristics M_1 and M_2. The total value of the axial moment of inertia of the motor and the central moment of inertia of the vibrator mass is marked as J_w. Vibrators are placed in the line perpendicular to the direction of motion of the transporting trough. They are placed symmetrically at a distance d, in relation to the mass centre of the drive frame. 6 generalized coordinates were used to describe the system location. Coordinates x_{Ck}, y_{Ck} represent the mass centre of the drive frame, α – angle of rotation of the drive frame and transporting trough, s – displacement of the trough in relation to the frame, and angles φ_1, φ_2 describe angular positions of the vibrators. On the basis of Fig. 2 the kinematic dependencies imposed on the positions of mass centres of the trough and drive vibrators were determined

$$\begin{aligned} &x_{Cr} = x_{Ck} + s \qquad && y_{Cr} = y_{Ck} \\ &x_{C1} = x_{Ck} + e_w \cos\varphi_1 + d\alpha && y_{C1} = y_{Ck} + e_w \sin\varphi_1 \\ &x_{C2} = x_{Ck} + e_w \cos\varphi_2 - d\alpha && y_{C2} = y_{Ck} - e_w \sin\varphi_2 \end{aligned} \tag{2.1}$$

Geometrical dependencies imposed on shifting of springs of the suspension system were also determined

$$\begin{aligned} &\Delta x_A = h\alpha + x_{Ck}\cos\beta - y_{Ck}\sin\beta \qquad && \Delta y_A = -a\alpha + y_{Ck}\cos\beta + x_{Ck}\sin\beta \\ &\Delta x_B = h\alpha + x_{Ck}\cos\beta - y_{Ck}\sin\beta && \Delta y_B = a\alpha + y_{Ck}\cos\beta + x_{Ck}\sin\beta \end{aligned} \tag{2.2}$$

These dependencies allowed one to formulate the Lagrange kinetic potential of the system in the form

$$\begin{aligned} L &= \frac{1}{2}m_k\Big(\frac{d}{dt}x_{Ck}\Big)^2 + \frac{1}{2}(m_k + m_r)\Big(\frac{d}{dt}y_{Ck}\Big)^2 + \frac{1}{2}(J_{Ck} + J_{Cw})\Big(\frac{d}{dt}\alpha\Big)^2 + \frac{1}{2}m_r\Big(\frac{d}{dt}x_{Cr}\Big)^2 \\ &+ \frac{1}{2}m_w\Big(\frac{d}{dt}x_{C1}\Big)^2 + \frac{1}{2}m_w\Big(\frac{d}{dt}y_{C1}\Big)^2 + \frac{1}{2}J_{Cw}\Big(\frac{d}{dt}\varphi_1\Big)^2 + \frac{1}{2}m_w\Big(\frac{d}{dt}x_{C2}\Big)^2 + \frac{1}{2}m_w\Big(\frac{d}{dt}y_{C2}\Big)^2 \\ &+ \frac{1}{2}J_{Cw}\Big(\frac{d}{dt}\varphi_2\Big)^2 - \Big(\frac{1}{2}k_x\Delta x_A^2 + \frac{1}{2}k_y\Delta y_A^2 + \frac{1}{2}k_x\Delta x_B^2 + \frac{1}{2}k_y\Delta x_B^2 + \frac{1}{2}k_s s^2\Big) \end{aligned} \tag{2.3}$$

and on its basis and by making use of the Lagrange formula

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \frac{1}{2}\frac{\partial N}{\partial \dot{q}_i} = Q_i \tag{2.4}$$

to determine the dynamic equation of motion. In Eq. (2.4) marked are: by q_i the i-th generalized coordinate, by Q_i – i-th generalized force and by N – value of linear loses. The last quantity (2.5) was determined on account of a damping present in elements of the system suspension and in leaf springs of the trough

$$N = b_x\Big(\frac{d\Delta x_A}{dt}\Big)^2 + b_y\Big(\frac{d\Delta y_A}{dt}\Big)^2 + b_x\Big(\frac{d\Delta x_B}{dt}\Big)^2 + b_y\Big(\frac{d\Delta y_B}{dt}\Big)^2 + b_s\Big(\frac{ds}{dt}\Big)^2 \tag{2.5}$$

Dependencies from (2.6) to (2.11) present dynamic equations of motion determined one after another in the coordinates: x_{Ck}, y_{Ck}, s, α, φ_1 and φ_2

$$\begin{aligned}
&[\sin(2\beta)b_y - \sin(2\beta)b_x]\Big(\frac{d}{dt}y_{Ck}\Big) + [\sin(2\beta)k_y - \sin(2\beta)k_x]y_{Ck} \\
&\quad + (2m_w + m_r + m_k)\Big(\frac{d^2}{dt^2}x_{Ck}\Big) + [(1-\cos(2\beta))b_y + (\cos(2\beta)+1)b_x]\Big(\frac{d}{dt}x_{Ck}\Big) \\
&\quad + [(1-\cos(2\beta))k_y + (\cos(2\beta)+1)k_x]x_{Ck} + m_r\Big(\frac{d^2}{dt^2}s\Big) \\
&\quad - e_w m_w \sin(\varphi_2)\Big(\frac{d^2}{dt^2}\varphi_2\Big) - e_w m_w \cos(\varphi_2)\Big(\frac{d}{dt}\varphi_2\Big)^2 - e_w m_w \sin(\varphi_1)\Big(\frac{d^2}{dt^2}\varphi_1\Big) \\
&\quad - e_w m_w \cos(\varphi_1)\Big(\frac{d}{dt}\varphi_1\Big)^2 + 2\alpha\cos(\beta)hk_x + 2\Big(\frac{d}{dt}\alpha\Big)\cos(\beta)b_x h = 0
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
&(2m_w + m_r + m_k)\Big(\frac{d^2}{dt^2}y_{Ck}\Big) + [(\cos(2\beta)+1)b_y + (1-\cos(2\beta))b_x]\Big(\frac{d}{dt}y_{Ck}\Big) \\
&\quad + [(\cos(2\beta)+1)k_y + (1-\cos(2\beta))k_x]y_{Ck} + [\sin(2\beta)b_y - \sin(2\beta)b_x]\Big(\frac{d}{dt}x_{Ck}\Big) \\
&\quad + [\sin(2\beta)k_y - \sin(2\beta)k_x]x_{Ck} - e_w m_w \cos(\varphi_2)\Big(\frac{d^2}{dt^2}\varphi_2\Big) + e_w m_w \sin(\varphi_2)\Big(\frac{d}{dt}\varphi_2\Big)^2 \\
&\quad + e_w m_w \cos(\varphi_1)\Big(\frac{d^2}{dt^2}\varphi_1\Big) - e_w m_w \sin(\varphi_1)\Big(\frac{d}{dt}\varphi_1\Big)^2 - 2\alpha\sin(\beta)hk_x \\
&\quad - 2\Big(\frac{d}{dt}\alpha\Big)\sin(\beta)b_x h = 0
\end{aligned} \tag{2.7}$$

$$m_r\Big(\frac{d^2}{dt^2}x_{Ck}\Big) + m_r\Big(\frac{d^2}{dt^2}s\Big) + b_s\Big(\frac{d}{dt}s\Big) + k_s s = 0 \tag{2.8}$$

$$\begin{aligned}
&-2\sin(\beta)b_x h\Big(\frac{d}{dt}y_{Ck}\Big) - 2\sin(\beta)hk_x y_{Ck} + 2\cos(\beta)b_x h\Big(\frac{d}{dt}x_{Ck}\Big) + 2\cos(\beta)hk_x x_{Ck} \\
&\quad + de_w m_w \sin(\varphi_2)\Big(\frac{d^2}{dt^2}\varphi_2\Big) + de_w m_w \cos(\varphi_2)\Big(\frac{d}{dt}\varphi_2\Big)^2 - de_w m_w \sin(\varphi_1)\Big(\frac{d^2}{dt^2}\varphi_1\Big) \\
&\quad - de_w m_w \cos(\varphi_1)\Big(\frac{d}{dt}\varphi_1\Big)^2 + 2\Big(\frac{d^2}{dt^2}\alpha\Big)d^2 m_w + 2a^2\alpha k_y + 2\alpha h^2 k_x \\
&\quad + 2\Big(\frac{d}{dt}\alpha\Big)b_x h^2 + 2a^2\Big(\frac{d}{dt}\alpha\Big)b_y + (J_{Cr} + J_{Ck})\Big(\frac{d^2}{dt^2}\alpha\Big) = -M_1 + M_2
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
&e_w m_w \cos(\varphi_1)\Big(\frac{d^2}{dt^2}y_{Ck}\Big) - e_w m_w \sin(\varphi_1)\Big(\frac{d^2}{dt^2}x_{Ck}\Big) + (e_w^2 m_w + J_{Cw})\Big(\frac{d^2}{dt^2}\varphi_1\Big) \\
&\quad - \Big(\frac{d^2}{dt^2}\alpha\Big)de_w m_w \sin(\varphi_1) = M_1 - \frac{1}{2}M_{desync}
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
&-e_w m_w \cos(\varphi_2)\Big(\frac{d^2}{dt^2}y_{Ck}\Big) - e_w m_w \sin(\varphi_2)\Big(\frac{d^2}{dt^2}x_{Ck}\Big) + (e_w^2 m_w + J_{Cw})\Big(\frac{d^2}{dt^2}\varphi_2\Big) \\
&\quad + \Big(\frac{d^2}{dt^2}\alpha\Big)de_w m_w \sin(\varphi_2) = M_2 + \frac{1}{2}M_{desync}
\end{aligned} \tag{2.11}$$

where M_{desync} is the moment desynchronizing the vibrators.

2.2. Equations of the steady state above resonances of the suspension system

The analyzed machine is of the over-resonance type, for which the effect of the suspension system elements on the drive frame can be omitted in relation to influences of inertial vibrators. In turn, on account of the application – in the further part of this study – the self-synchronization criterion of vibrators, which is based only on the Lagrange kinematic potential, a component of damping forces between the drive frame and trough was also omitted in the equations.

Steady state equations were obtained assuming the constant value of angular velocities of vibrators $d\varphi_1/dt = d\varphi_2/dt = \omega$. Angular positions of the vibrators are then

$$\varphi_1 = \omega t + \varphi_{10} \qquad \varphi_2 = \omega t + \varphi_{20} \tag{2.12}$$

where φ_{10} and φ_{20} denote constants during the pathway phases.

In the equations of steady state, an expression for the adjusting frequency of the dynamic damper (in the analyzed case, the transporting trough) to the excitation frequency originating from forces of the drive vibrators, was also taken into account

$$k_s = m_r\omega^2 \tag{2.13}$$

In such a way, equations (2.14) were determined

$$\begin{aligned}
&(2m_w + m_r + m_k)\Big(\frac{d^2}{dt^2}x_{Ck}\Big) + m_r\Big(\frac{d^2}{dt^2}s\Big)\\
&\qquad = e_w m_w\omega^2\cos(\omega t + \varphi_{10}) + e_w m_w\omega^2\cos(\omega t + \varphi_{20})\\
&(2m_w + m_r + m_k)\Big(\frac{d^2}{dt^2}y_{Ck}\Big)\\
&\qquad = e_w m_w\omega^2\sin(\omega t + \varphi_{10}) - e_w m_w\omega^2\sin(\omega t + \varphi_{20})\\
&m_r\Big(\frac{d^2}{dt^2}x_{Ck}\Big) + m_r\Big(\frac{d^2}{dt^2}s\Big) + m_r\omega^2 s = 0\\
&(J_{Cr} + J_{Ck})\Big(\frac{d^2}{dt^2}\alpha\Big) + 2\Big(\frac{d^2}{dt^2}\alpha\Big)d^2 m_w\\
&\qquad = de_w m_w\omega^2\cos(\omega t + \varphi_{10}) - de_w m_w\omega^2\cos(\omega t + \varphi_{20})
\end{aligned} \tag{2.14}$$

It can be noticed that equations are of a linear form, heterogeneous, in which forcing elements are composed of harmonic expressions. Equations of this type can be solved by symbolic calculation, transforming the time depending differential equations into algebraic equations, also linear and depending on $\mathrm{i}\omega$, where i is the imaginary unit.

Defining representations of coordinates

$$x_{Ck}(t) \rightleftharpoons \underline{X}(\mathrm{i}\omega) \qquad y_{Ck}(t) \rightleftharpoons \underline{Y}(\mathrm{i}\omega) \qquad s(t) \rightleftharpoons \underline{S}(\mathrm{i}\omega) \qquad \alpha(t) \rightleftharpoons \underline{\alpha}(\mathrm{i}\omega) \tag{2.15}$$

excitations

$$\sin(\omega t + \varphi_{10}) \rightleftharpoons \mathrm{e}^{\mathrm{i}\varphi_{10}} \qquad \cos(\omega t + \varphi_{20}) \rightleftharpoons \mathrm{i}\mathrm{e}^{\mathrm{i}\varphi_{20}} \tag{2.16}$$

operators

$$\frac{d}{dt} \rightleftharpoons \mathrm{i}\omega \qquad \frac{d^2}{dt^2} \rightleftharpoons -\omega^2 \tag{2.17}$$

solutions of the system motion can be obtained in a complex form

$$\begin{aligned}
&\underline{X} = 0 \qquad \underline{Y} = \frac{e_w m_w}{2m_w + m_r + m_k}\Big(\mathrm{e}^{\mathrm{i}\varphi_{20}} - \mathrm{e}^{\mathrm{i}\varphi_{10}}\Big)\\
&\underline{S} = -\frac{\mathrm{i}e_w m_w}{m_r}\Big(\mathrm{e}^{\mathrm{i}\varphi_{20}} + \mathrm{e}^{\mathrm{i}\varphi_{10}}\Big) \qquad \underline{\alpha} = \frac{\mathrm{i}de_w m_w}{2d^2 m_w + J_{Cr} + J_{Ck}}\Big(\mathrm{e}^{\mathrm{i}\varphi_{20}} - \mathrm{e}^{\mathrm{i}\varphi_{10}}\Big)
\end{aligned} \tag{2.18}$$

Time waveforms corresponding the above numbers are obtained on the basis of the formula

$$\begin{aligned} &y_{Ck}(t) = \Re(\underline{Y})\sin(\omega t) + \Im(\underline{Y})\cos(\omega t) \\ &s(t) = \Re(\underline{S})\sin(\omega t) + \Im(\underline{S})\cos(\omega t) \qquad \alpha(t) = \Re(\underline{\alpha})\sin(\omega t) + \Im(\underline{\alpha})\cos(\omega t) \end{aligned} \tag{2.19}$$

where $\Re$ and $\Im$ denote the real and imaginary parts of complex numbers.

Coordinates determined in such a way were used in the analysis of the kinematic potential and in calculating the moment synchronizing the drive vibrators in the steady state operations of the machine.

2.3. Problem of the stability of the system motion

The position of the stable equilibrium of vibrators was determined on the basis of the criterion

$$D(\varphi_{10},\varphi_{20}) = \frac{1}{T}\int\limits_0^T L\,dt \to \min \tag{2.20}$$

formulated in the studies by Blekhman (2000). According to these findings, vibrators obtain stable positions when the average value of the Lagrange function L, determined for the vibration period T of the system, obtains its minimal value. On the basis of (2.3) and solutions (2.19), the function D in the explicit analytical form was derived

$$\begin{aligned} D(\varphi_{10},\varphi_{20}) &= m_w^2 e_w^2\omega^2[\cos(\varphi_{20}-\varphi_{10})-1](4d^4m_km_w^2 + 8J_{Ck}d^2m_w^2 + 8J_{Ck}d^2m_rm_w \\ &+ 4J_{Cr}d^2m_km_w + 12J_{Ck}d^2m_km_w + 2J_{Ck}d^2m_r^2 + 4J_{Ck}d^2m_km_r + 2J_{Ck}d^2m_k^2 \\ &+ J_{Cr}^2m_k + 2J_{Ck}J_{Cr}m_k + J_{Ck}^2m_k)/[2(2m_w+m_r+m_k)^2(2d^2m_w+J_{Cr}+J_{Ck})^2] \end{aligned} \tag{2.21}$$

Its cosinusoidal dependence on the difference of angles φ_{20} and φ_{10} means, that there is only one minimum of the function and it occurs for the angle

$$\gamma = \varphi_{20} - \varphi_{10} = \pi \tag{2.22}$$

The obtained result allows one to state that in the case of the counter running drive the system has only one state of stable work, in which vibrators generate a sinusoidally variable force of a rectilinear direction, perpendicular to the segment connecting the points of its fastening to the machine body, Fig. 3.

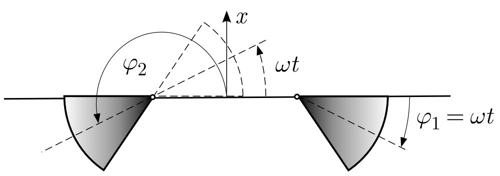

Fig. 3. System of phase angles of vibrators in the synchronous running state

2.4. Moment synchronizing the drive vibrators

Left sides of equations (2.10) and (2.11), apart from the components $(J_{Cr} + m_r e^2)(d^2/dt^2)$, present the effect of the drive frame on drive vibrators. When transferred them into the right hand side of equations, they represent vibration moments as (Dimentberg *et al.*, 1997)

$$\begin{aligned} M_{vibr1}(t) &= \Big(\frac{d^2}{dt^2}\alpha\Big)de_wm_w\sin\varphi_1 - e_wm_w\cos\varphi_1\Big(\frac{d^2}{dt^2}y_{Ck}\Big) + e_wm_w\sin\varphi_1\Big(\frac{d^2}{dt^2}x_{Ck}\Big) \\ M_{vibr2}(t) &= -\Big(\frac{d^2}{dt^2}\alpha\Big)de_wm_w\sin\varphi_2 + e_wm_w\cos\varphi_2\Big(\frac{d^2}{dt^2}y_{Ck}\Big) + e_wm_w\sin\varphi_2\Big(\frac{d^2}{dt^2}x_{Ck}\Big) \end{aligned} \tag{2.23}$$

Depending on the mutual positions of the vibrators these moments may obtain different values. However, from the practical point of view, the most essential value is the average difference of moments (2.24). This value decides on the size of disphasing of the vibrators and on the reserve of stability related to the stable equilibrium of vibrators

$$M_{sync} = \frac{1}{T}\int_0^T [M_{vibr2}(t) - M_{vibr1}(t)]\,dt \tag{2.24}$$

After calculating two times the derivative of (2.19) versus time and inserting it into equation (2.24), the synchronizing moment was determined in the form

$$M_{sync} = m_w^2 e^2 \omega^2 \frac{(4m_w + m_r + m_k)d^2 + J_{Cr} + J_{Ck}}{(2m_w + m_r + m_k)(2m_w d^2 + J_{Cr} + J_{Ck})} \sin\gamma \tag{2.25}$$

Transferring to the boundary $(J_{Ck} + J_{Cr}) \to \infty$ or inserting into equation $d = 0$, provides an interesting result. In such a situation, we obtain the formula

$$M_{sync}^0 = \frac{m_w^2 e^2 \omega^2}{2m_w + m_r + m_k} \sin \tag{2.26}$$

which allows one to state that when there is a lack of angular vibrations, the synchronizing moment will still exist and will have a high enough value to maintain synchronous motion of the drive vibrators. This property is quite different than that of the conveyors without dynamic damper systems, for which the coaxial fastening of vibrators does not generate any synchronizing moments.

3. Numerical analysis

The verification of analytical formulas derived in Section 2 is based on dynamic equations (2.6)-(2.11) and on parameters of the exemplary conveyor. Parameters presented in Table 1 are related to the long conveyor, approximately 5 m, for which – due to a high value of $J_{Ck} + J_{Cr}$ – there is a danger of a low level of body angular vibration and difficulties with self-synchronizing of vibrators.

Table 1. Physical parameters of the system

Parameter	Value	Unit	Parameter	Value	Unit
m_k	500.0	kg	k_x	65023.3	N/m
m_r	350.0	kg	k_y	1.30E+05	N/m
$m_w e_w$	0.9625	kg m	b_x	114.0	Ns/m
a	1.5	m	b_y	161.3	Ns/m
h	0.5	m	β	$\pi/6$	rad
d	0.5	m	k_s	3.84E+06	N/m
J_{Cw}	0.001	kg m^2	b_s	733.04	Ns/m
J_{Ck}	416.67	kg m^2	ω	$100\pi/3$	rad/s
J_{Cr}	729.17	kg m^2			

However, the main investigations should be veryfied because of errors resulting from simplifications made while formulating equations of steady state (2.14).

On the basis of analytical formulas (2.18), the following results were obtained

$$\begin{aligned} \underline{Y} &= -0.0011073 + 0.0011073\mathrm{i} = 0.0015659\mathrm{e}^{+\mathrm{i}135^0}\ \mathrm{m} \\ \underline{S} &= -0.0011073 + 0.0011073\mathrm{i} = 0.0038891\mathrm{e}^{-\mathrm{i}45^0}\ \mathrm{m} \\ \underline{\alpha} &= -0.0011073 + 0.0011073\mathrm{i} = 0.00059149\mathrm{e}^{-\mathrm{i}135^0}\ \mathrm{rad} \end{aligned} \tag{3.1}$$

and on the basis of equation (2.25) it was found

$$M_{sync} = 13.89\ \mathrm{Nm} \tag{3.2}$$

Analogous procedure was applied in the case of $d = 0$. Then, it was obtained

$$\begin{aligned} \underline{Y} &= -0.0011073 + 0.0011073\mathrm{i} = 0.0015659\mathrm{e}^{+\mathrm{i}135^0}\ \mathrm{m} \\ \underline{S} &= +0.0027500 - 0.0027500\mathrm{i} = 0.0038891\mathrm{e}^{-\mathrm{i}45^0}\ \mathrm{m} \\ \underline{\alpha} &= 0\ \mathrm{rad} \end{aligned} \tag{3.3}$$

and

$$M_{sync} = 11.68\ \mathrm{Nm} \tag{3.4}$$

The dynamic equations of motion enabled obtaining the pathways as presented in Figs. 4-6.

Figure 4 presents the pathways of coordinates of conveyor motion in the steady work state at the maximal desynchronizing moment. On the basis of the data read from the figure, it is possible to determine error values for individual pathways

$$\begin{aligned} \delta y_{Ck} &= \frac{0.00155 - 0.0015659}{0.0015659} 100\% = 1\% \qquad \delta s = \frac{0.00397 - 0.0038891}{0.00397} 100\% = 2\% \\ \delta\alpha &= \frac{0.00397 - 0.0038891}{0.00397} 100\% = 2\% \end{aligned} \tag{3.5}$$

The simulation results for the vibrators loaded by external desynchronizing moments are presented in Fig. 5 for three values: 3 Nm, 14.2 Nm and 14.3 Nm. The load was applied at 10 s, 20 s and 35 s of the simulation time. It can be noticed that breaking of the synchronizing bond occurred above 14.2 Nm. Thus, it is possible to determine the error of synchronizing moment to be

$$\delta M_{sync} = \frac{14.2 - 13.89}{14.2} 100\% = 2.1\% \tag{3.6}$$

The pathways of coordinates of the mass centre of the drive frame, corresponding with loads from Fig. 5, are presented in Fig. 6. From Figs. 4-6 a change in the direction and amplitude of vibrations of the conveyor trough can be pointed out. When the desynchronizing moment is increasing then vibrations of the trough are smaller and more bent in the direction of transport.

4. Summary

The self-synchronization process of drive vibrators (of the inertial type) in a vibratory conveyor with a dynamic damper was analyzed in this study. This type of solution allows us to use the antiresonance effect to achieve a significant decrease in the drive frame vibration amplitude and, consequently, a significant decrease in forces transmitted to the foundation. It was revealed that in this type of solution, the nonzero average value of the synchronizing moment allowed for mutual synchronization of the vibrators and that these vibrators could assume only one

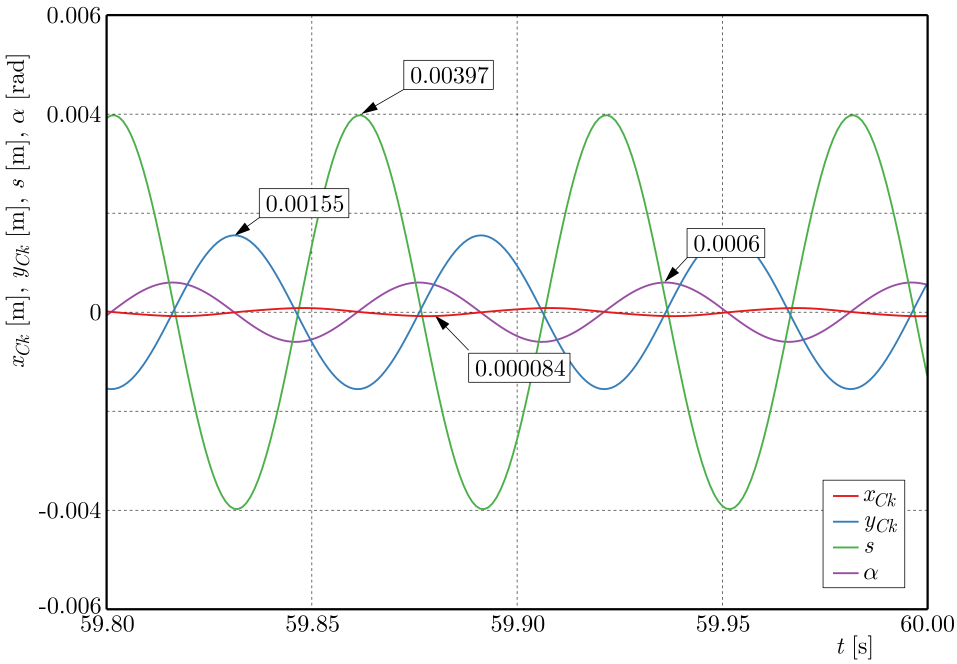

Fig. 4. Pathways of generalized coordinates in the steady state obtained on the basis of dynamic equations (2.6)-(2.11)

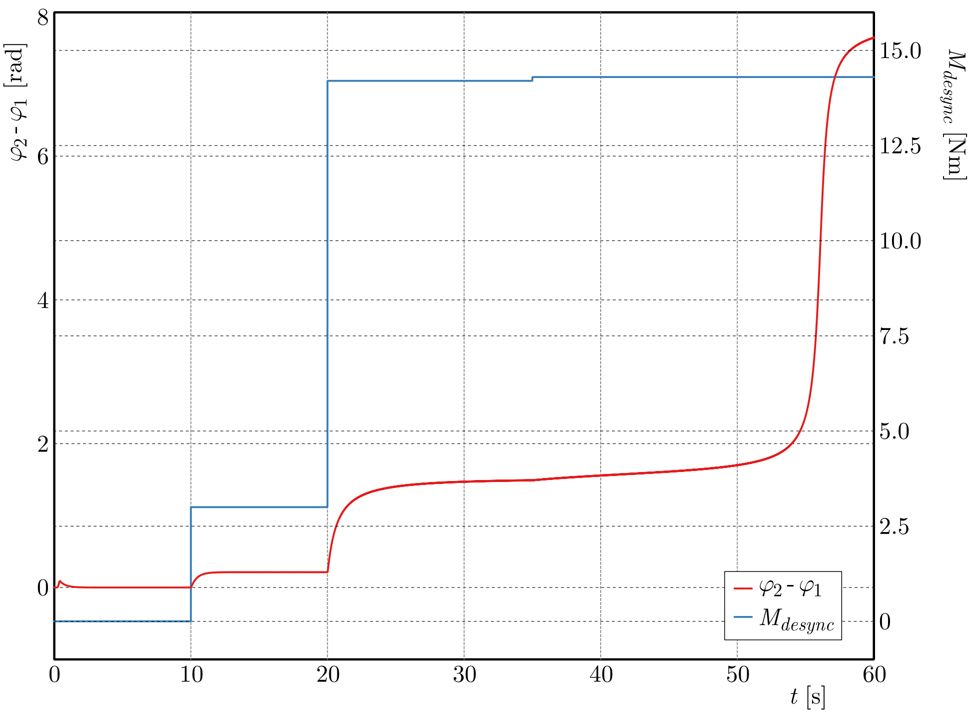

Fig. 5. Pathways of the disphasing angle of drive vibrators and desynchronizing moment

stable position with respect to each other. It was shown that in the case of counter running, the drive vibrators generated a force that was sinusoidally variable in the rectilinear direction and perpendicular to the segment connecting the mounting points to the machine body. The analytical formula for the average value of the synchronizing moment was determined, and on the basis of it, the meaning of the component originating from angular vibrations of the conveyor was determined. It was shown that in a system containing a dynamic damper, the synchronization of

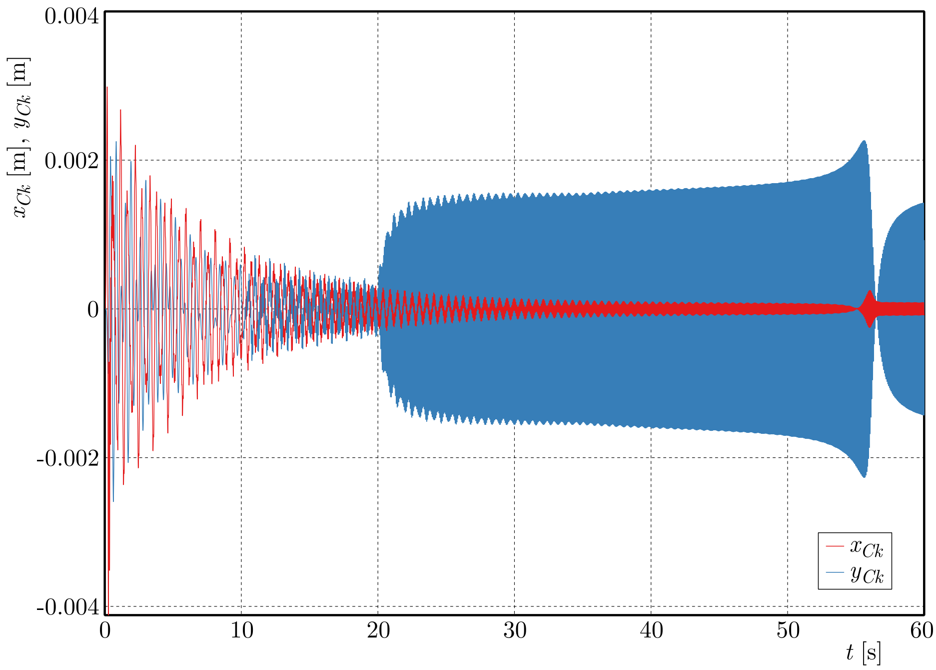

Fig. 6. Pathways of coordinates of the mass centre of the drive frame

vibrators was possible without angular vibrations of the machine, which provided the possibility of installing drive vibrators on the same axis. This is completely different than the situation for conveyors without a dynamic damper system, in which the coaxial mounting of vibrators does not generate a synchronizing moment.

The last conclusions have fundamental importance for building long conveyors, in which mass moments of inertia of the drive frame and trough take high values, which leads to low values of angular vibrations.

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