RESEARCH ON CREEP-FATIGUE MODEL OF ANCHORED JOINTED ROCK MASS

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To explore the mechanical response of anchored rock mass with an interaction of fatigue and creep, this paper selects the fatigue amplitude and fatigue frequency as influencing factors to carry out a fatigue-creep experiment of the anchored rock mass. Based on fractional order theory and binary perturbation theory, a new fractional order creep model is constructed. The results show that compared with the Burgers model, the newly proposed creep model can better describe the whole process of creep deformation of the anchored jointed rock mass under fatigue loading.

Keywords: shear creep, anchored jointed rock mass, fractional order, fatigue load, damage

1. Introduction

With in large-scale strategic engineering of deep rock mass in China, ensuring stability of a deep rock mass structure and preventing catastrophes have become the first priority. The existing research shows: Deep rock mass has obvious rheological characteristics, and it is very easy to aggravate its rheological state due to blasting, earthquake and other disturbances, thus causing instability and failure of deep rock mass (Feng et al., 2022; Wiatowski et al., 2021). Therefore, the influence of load on deep rock masses cannot be ignored.

In the recent years, many scholars have conducted a series of studies on rheological properties of rock mass: Chen et al. (2014) used acoustic emission testing (AE) to record creep behavior of granite at different temperatures, and proposed a creep model based on a damage mechanism. Fu et al. (2008) used CT testing technology to study microdamage evolution characteristics of mudstone under an impact disturbance load. Hu et al. (2019) carried out long-term compressive creep tests and numerical simulation tests on single-fractured sandstone by means of a single-stage loading, and established a creep model considering rock damage. Huang et al. (2017) established a disturbance factor function based on plastic deformation, and further established the evolution equation of the creep disturbance factor through change characteristics of plastic deformation with time. Liu and Zhang (2020) established a creep equation, considering dual effects of stress and time, by analyzing the deterioration law of creep parameters under the action of time. Nadimi and Shahriar (2014) attempted to predict a long-term creep parameter using triaxial creep tests and to define time-dependent characteristics of the bounding material. It was observed that the creep rate of a grouting material specimen directly depends on the
deviator of stress. Wu et al. (2020) proposed a variable-parameter fractional derivative element based on fractional theory, and developed a mathematical model describing rock shear creep.

The above research mainly analyzed the damage evolution law of rock under creep load (Chen et al., 2013; Güneyisi et al., 2016; Luo et al., 2020). However, in the deep rock mass, creep deformation of rock will be aggravated under a fatigue load disturbance such as blasting and earthquake. At present, there are few reports on the research of this engineering phenomenon. Therefore, this paper studies the mechanical response of anchored jointed rock mass under creep-fatigue interaction, analyzes shear creep characteristics, and establishes a shear creep model by introducing fractional order theory and a binary disturbance principle.

2. Test overview

2.1. Experimental design

Marble is selected as a raw material for this test. First, marble is cut into a 100 mm × 100 mm × 50 mm rectangular block as the hanging wall and footwall of the joint test piece. Secondly, cement mortar (cement: river sand: water = 1:1.5:0.8) is selected for the joint part; it is used to bond the hanging wall and footwall. Third, the bolt adopts HRB335 steel with a yield strength of 335 MPa, and the grouting part adopts cement mortar. The test piece is shown in Fig. 1a.

Fig. 1. Specimen and loading path: (a) specimen and (b) loading path

This experiment adopts the method of controlling variables. Creep fatigue tests were carried out for four fatigue frequencies of 0.02 Hz, 0.04 Hz, 0.06 Hz and 0.08 Hz and four fatigue amplitudes of 2 kN, 4 kN, 6 kN and 8 kN. The test grouping is shown in Table 1.

Table 1. Trial grouping

<table>
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<tr>
<th>Influencing factors</th>
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<th>Fatigue frequency [Hz]</th>
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<td>0.02</td>
</tr>
<tr>
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<td>2</td>
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</tr>
<tr>
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<tr>
<td></td>
<td>2-4</td>
<td>2</td>
<td>0.08</td>
</tr>
</tbody>
</table>
2.2. Loading scheme

The shear device of a TAW2000 triaxial tester is utilized in this test. As a step-by-step loading can avoid the discrete phenomenon of the test results, the shear creep test of the anchored jointed rock mass is carried out by a step loading in this test. The loading path is shown in Fig. 1b. The specific loading steps are listed as follows:

(1) Normal load: Apply the normal stress to the test piece at a loading rate of 0.05 MPa/s and keep the normal load unchanged when the normal stress reaches the established stress level of 0.2 MPa.

(2) Creep load: Based on the direct shear test, the primary shear stress is determined to be 0.1 MPa. When the creep load reaches the predetermined value of 0.1 MPa, the load remains constant for a certain period. Afterward, the creep shear stress of each stage increases by 0.1 MPa.

(3) Fatigue load: According to the experimental groups shown in Table 1, the fatigue load is applied after the creep of each load level in the test piece. The loading path is shown in Fig. 1b.

3. Test overview

3.1. Test curve analysis

Through sorting and analyzing the test data, the data with typical curve characteristics are selected and plotted as shown in Figs. 2 and 3. In this study, the stress threshold of creep failure refers to the stress value at the time of failure in the process of creep. The following results can be obtained from the figures.

When the fatigue amplitude is increased from 2 kN to 8 kN, the creep failure stress thresholds of the specimen are 0.56 MPa, 0.50 MPa, 0.45 MPa and 0.40 MPa in turn, and the degrees of decline were 9.28%, 18.73% and 28.19%. When the fatigue frequency is increased from 0.02 Hz to 0.08 Hz, the creep failure stress thresholds of the specimen are 0.56 MPa, 0.50 MPa, 0.46 MPa and 0.40 MPa in turn, and the degrees of decline of the creep failure stress threshold are 9.60%, 18.17% and 28.14%, respectively, as shown in Fig. 2. The creep failure stress thresholds for the two groups of specimens show a linear decreasing trend, and the degree of decline is similar. This finding shows that the sensitivities of the fatigue amplitude and fatigue frequency to the creep failure stress threshold of the anchored jointed rock mass are equivalent.
According to Fig. 3, the change trend of the shear creep fatigue curve with different fatigue factors, which is similar, can be divided into four stages: attenuation creep stage, stable creep stage, fatigue creep stage and accelerated creep stage. When the fatigue amplitude is increased from 2 kN to 8 kN, and the fatigue frequency is increased from 0.02 Hz to 0.08 Hz, the speed of the specimen entering the accelerated creep stage is obviously accelerated. Specifically, when the influence factor is the fatigue amplitude, the failure time of the specimen decreases from 4.83 d to 2.98 d, with a decrease of 38.30%. When the influence factor is the fatigue frequency, the failure time of the specimen decreases from 5.30 d to 3.47 d, with a decrease of 32.22%.

3.2. Isochronal shear stress-strain curves

According to the stress-strain curve, the strain-time history curve and the Boltzmann superposition principle, isochronous curve clusters of shear stress-strain of the specimen are drawn. The time intervals of the curves are 0.0 d, 0.2 d, 0.4 d and 0.6 d. Thus, the long-term strength for accelerated creep deformation of the specimen with different fatigue amplitudes is determined, where 0 d is the relative starting time when each level of shear stress is applied. Due to the limited scope of the article, only isochronous curve clusters with a fatigue amplitude of 4 kN and a fatigue frequency of 0.04 Hz are given, as shown in Fig. 4.

![Fig. 4. Stress-strain isochronous curve cluster: (a) 4 kN and (b) 0.04 Hz](image-url)
Research on creep-fatigue model of anchored jointed rock mass

Fatigue frequency is increased from 0.02 Hz to 0.08 Hz, the corresponding long-term strengths of the specimen are 0.403 MPa, 0.302 MPa, 0.298 MPa and 0.300 MPa. These results conclude that the long-term strength of the specimen shows an initially decreasing and then stabilizing trend. The main reason is that when the fatigue load is at a low level, it can promote rapid development of internal cracks of the specimen, thereby reducing the long-term strength of the specimen. When the fatigue amplitude is large, the specimen can quickly form through cracks and, simultaneously, the strength of the joint part is weakened to a great extent so that the shear strength of the specimen is mainly provided by the anchor bolt. Therefore, the long-term strength of the specimen will be stabilized.

3.3. Equivalent Shear Modulus

To further evaluate creep mechanical properties of the anchored jointed rock mass under fatigue loading, the concept of Equivalent Shear Modulus is proposed in this paper. Namely, the equivalent shear modulus is the ratio of shear stress to shear strain corresponding to the long-term strength of the specimen. Equivalent Shear Modulus can be obtained by sorting and analysing the stress-strain isochronous cluster curve data. To quantitatively evaluate the change law of Equivalent Shear Modulus, the author uses the Origin software to fit the data; the fitting curve is shown in Fig. 5. Under the action of different fatigue amplitudes and fatigue frequencies, the change law of Equivalent Shear Modulus of the specimen shows a downward quadratic curve. When the influence factor is the fatigue amplitude, Equivalent Shear Modulus of the specimen decreases from 0.827 MPa to 0.321 MPa, with a decrease of 61.18%. When the influence factor is the fatigue frequency, the long-term shear modulus of the specimen decreased from 0.827 MPa to 0.519 MPa, with a decrease of 37.24%. Under the effect of fatigue amplitude, Equivalent Shear Modulus decreases more sharply. Therefore, the fatigue amplitude is more sensitive to Equivalent Shear Modulus of the specimen.

3.4. Failure mode of joint surface

Figure 6 is the typical failure mode of the joint surface during the test. It can be seen from the figure that, with an increase of fatigue load, the influence of shear stress on the joint surface gradually increases, which makes the crack gradually expand from one end to both ends, specifically, the crack shape of the joint surface changes from $V$ to $X$. 
4. Establishment of the shear creep model

4.1. Abel element

As fractional order theory can solve complex mechanical models, an increasing number of scholars worldwide have tried to apply fractional calculus theory to research of rock rheological models in recent years. The most commonly utilized definition of fractional calculus is the definition method of Riemann-Liouville (R-L) fractional calculus (Chen et al., 2010), which is defined as follows: the function \( f \) is piecewise continuous in the interval \((0, +\infty)\), and it is integrable in any subinterval of \((0, +\infty)\)

\[
\frac{d^{-\gamma}f(t)}{dt^{-\gamma}} = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t - \xi)^{\gamma - 1} f(\xi) \, d\xi
\]  

(4.1)

It is known that the ideal elastic body satisfies Hooke’s law, while the Newtonian fluid satisfies Newton’s law. Based on the above theory, we can estimate the rheological element

![Abel element](image)

(Abel element) between the ideal elastomer and the Newtonian fluid, as shown in Fig. 7a. The constitutive relation may satisfy

\[
\tau(t) = \eta \frac{d^\alpha \gamma(t)}{dt^\alpha} \quad \gamma(t) = \frac{\tau}{\eta t^{\alpha}} \quad 0 < \alpha < 1
\]

(4.2)
where $\gamma(t)$ is the shear strain, $\tau(t)$ is the shear stress, $\eta$ is the viscosity coefficient, and $\alpha$ is the derivative order.

As is apparent from Fig. 7b, when the derivative order decreases, the slope of the curve gradually increases. This phenomenon is consistent with the characteristics of the accelerated creep stage of soft rock, which shows that the smaller the derivative order is, the easier it is for the Abel element to characterize the accelerated creep stage of soft rock.

### 4.2. Damage function

Desai and Zhang (1998) proposed a concept of the relative disturbance state. This theory can provide a unified theoretical basis for elastic, plastic and creep deformation, microcracks, damage and softening, hardening and cyclic fatigue. Based on this theory, Desai proposed theory of duality disturbance (TDD). He suggests that engineering materials gradually change from a relatively undisturbed dynamic state to a relatively completely disturbed dynamic state through self-regulation of microcrack structures, producing a mixed response. This approach is not considered by classical damage theory. Therefore, the theory of duality disturbance can be applied to better describe the damage of rock mass under a complex stress state. The basic equation is

$$
\sigma^0 = (1 - D)\sigma^n + D\sigma^c
$$

$$
d\sigma^0 = (1 - D)d\sigma^n + dD\sigma^c - dD(\sigma^n - \sigma^n)
$$

(4.3)

where $0, n, and c$ represent the apparent state, relatively undamaged state and relatively completely damaged state, respectively. $D$ is the disturbance function and it is a measure of the degree of disturbance. Since the internal mechanism of the disturbance is not clear, for the sake of simplification and application, it is assumed that it is only related to a certain internal variable $f$. For typical elastoplastic materials, it is assumed to be related to the equivalent shear modulus. And it is usually represented by Weibull curves

$$
D = D(f) = D_u\sigma^c[1 - \exp(-Af^B)]
$$

(4.4)

where $D_u$ is the ultimate fatigue damage variable, usually taken as 1, $A$ and $B$ are material related constants, $f$ is the equivalent shear modulus.

From Section 3.3, under the action of different fatigue amplitudes and fatigue frequencies, the long-term shear modulus of the anchor body changes in a quadratic curve, as shown in Fig. 5. The specific expression of the equivalent shear modulus is

$$
f = ax^2 + bx + c
$$

(4.5)

where $a, b, c$ are correlation coefficients.

### 4.3. Fractional damage creep model

The anchored jointed rock mass is a composite material body, and its internal stress distribution is extremely complex and changeable. To simplify the stress calculation process, the bolt, joint and rock mass are regarded as a unified whole, and we only consider the macroscopic mechanical response of the jointed rock mass with anchors. As we know, the classical rheological model can better characterize decay creep and stable creep, but it lacks a quantitative description of the accelerated creep stage of soft rock. This problem is also an urgent problem to be solved.

According to the investigation, the generalized Kelvin model has characteristics of instantaneous deformation, attenuation creep and elastic aftereffect, and it can better characterize viscoelastic rock with attenuation creep and stable creep properties. As described in Section 3.1, the Abel element is a rheological element between the elastic element and the Newtonian element. The rheological rate of the Abel element gradually increases with the decreasing derivative...
order. Therefore, the Abel element can effectively represent the accelerated creep stage of the rock mass. To establish a creep model capable of characterizing the acceleration stage of soft rock, the author concatenates the generalized Kelvin style with Abel elements to establish a new fractional shear creep model. In this model, the long-term strength serves as the accelerated creep stress threshold. When the shear stress is less than the long-term strength, the new model degenerates into the generalized Kelvin one. A schematic of the model is shown in Fig. 8.

The creep equation of the fractional order creep model is expressed as follows

\[
\gamma = \begin{cases} 
\frac{\tau}{G_1} + \frac{\tau}{G_2} \left[ 1 - \exp \left( -\frac{G_2}{\eta_1} t \right) \right] & 0 < \tau < \tau_s \\
\frac{\tau}{G_1} + \frac{\tau}{G_2} \left[ 1 - \exp \left( -\frac{G_2}{\eta_1} t \right) \right] + \frac{\tau - \tau_s}{\eta_2} \frac{t^\alpha}{\Gamma(\alpha + 1)} & \tau \geq \tau_s 
\end{cases}
\] (4.6)

With the interaction of fatigue and creep, the shear modulus of the anchored jointed rock mass is bound to decay with the continuous expansion of cracks. As previously mentioned, the long-term shear modulus of the specimen shows a quadratic curve decay trend with the increasing fatigue load. Therefore, the author establishes a damage factor by combining binary disturbance theory and damage theory to characterize the process of the specimen gradually changing from a relatively undisturbed dynamic state to a relatively completely disturbed dynamic state. The fractional creep constitutive equation considering damage is expressed as follows

\[
\gamma = \begin{cases} 
(1 - D) \left\{ \frac{\tau}{G_1} + \frac{\tau}{G_2} \left[ 1 - \exp \left( -\frac{G_2}{\eta_1} t \right) \right] \right\} & 0 < \tau < \tau_s \\
(1 - D) \left\{ \frac{\tau}{G_1} + \frac{\tau}{G_2} \left[ 1 - \exp \left( -\frac{G_2}{\eta_1} t \right) \right] \right\} + D \frac{\tau - \tau_s}{\eta_2} \frac{t^\alpha}{\Gamma(\alpha + 1)} & \tau \geq \tau_s 
\end{cases}
\] (4.7)

where there are shear moduli of the elastic body and viscosity coefficients.

4.4. Parameter identification

To solve the relevant parameters of the fractional damage creep model, this paper mainly adopts the segmented solution method to process the experimental data. Now, the solution method of the model at different shear stress levels is given:

(1) The first step is to judge the long-term strength of the specimen under different fatigue loads. The long-term strength can be determined according to the isochronous stress-strain curve cluster, as shown in Section 3.2.

(2) The second step is to determine the damage function. First, we can fit and determine the value of damage functions \( f = ax^2 + bx + c \), as shown in Section 3.3. Then, according to the research results of Deng et al. (2017), we determine the damage variable parameters \( A \) and \( B \).

(3) The mathematical expression of the nonlinear fractional model, Eqs. (4.7), is programmed into the Origin software, and the Levenberg-Marquardt method is used to fit the rheological parameters of the nonlinear fractional model for the test data of each stress level, so as to determine values of \( G_1, G_2, \eta_1, \eta_2, \alpha \).
To verify the accuracy of the model, the author selects a group of experimental data from the two variables of the fatigue amplitude and fatigue frequency and solves the relevant parameters according to the above steps. The fitting results are shown in Fig. 9 and Table 2. The results show that the fractional damage model proposed in this paper can fit the accelerated creep stage of the anchored jointed rock mass better than the Burgers model, which indicates that the model can better reflect the overall shear creep process of the anchored jointed rock mass under different fatigue loads.

![Fig. 9. Schematic of the fitting curve: (a) 4kN and (b) 0.04Hz](image)

**Table 2. Summary of model parameters**

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<tr>
<th>Grading</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$\eta_1$</th>
<th>$A$</th>
<th>$B$</th>
<th>$a$</th>
<th>$b$</th>
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5. Conclusion

To reveal the accelerating effect of low cycle fatigue loads such as earthquakes and blasting on the shear creep instability of deep rock mass, we adopt the control variable method to investigate the effect of different fatigue amplitudes and fatigue frequencies on the creep characteristics of the rock mass with anchored joints. According to the experimental results, we use the Abel element to characterize the accelerated creep stage and simulate the transition process of the specimen from a relatively undisturbed dynamic state to a relatively fully disturbed dynamic state with the theory of duality disturbance. The conclusions are summarized as follows:

- A comparison of the shear creep stress-strain curves of the anchored jointed rock mass with different influence factors reveals that the creep failure stress threshold of the specimen linearly decreases with the increasing fatigue amplitude and fatigue frequency. The
sensitivity of the two to the creep failure stress threshold is equivalent, which indicates that the fatigue load has an obvious accelerating effect on the anchored jointed rock mass.

- To evaluate the macro performance of the anchored jointed rock mass, we propose a concept of the long-term shear modulus. Through the Origin software fitting, it is determined that the long-term shear modulus of the specimen decreases in a quadratic curve with the increasing fatigue amplitude and fatigue frequency. The difference between them is that the fatigue amplitude is more sensitive to the long-term shear modulus of the specimen, and the decrease is larger.

- In this paper, we connect the generalized Kelvin model and the Abel element in series. Combined with the theory of duality disturbance, a nonlinear fractional order creep model is constructed, which can describe the transition process of the anchored jointed rock mass from a relatively undisturbed to relatively fully disturbed state. Compared with the Burgers model, this model can well simulate the whole process of three stages of rock specimens: attenuation creep, stable creep and accelerated creep.

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