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BIOMECHANICAL CRITERION OF DYNAMIC STABILITY BASED ON ZMP FORMULA AND FLASH-HOGAN PRINCIPLE OF MINIMUM JERK

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The main aim of the article is to define a criterion of dynamic stability based on the Flash--Hogan principle and the ZMP method. The gait researches were focused on analysis and observation of the human biomechanism with the optical system Optitrack. The smooth reference trajectory is defined forming a stability pattern. The optimal, due to the minimum jerk criterion, ZMP trajectory is illustrated in the results Section in order to demonstrate the dynamic stability pattern for the needs of rehabilitation in cases of neuromuscular damage or injuries affecting gait stability.

Keywords: biomechanical stability, jerk, Flash-Hogan principle, ZMP, Optitrack

1. Introduction

Stability is the ability to retrieve the state of equilibrium lost as a result of existing destabilizing factors. Loss of stability may be caused by incorrect setting of the foot, impaired control of movements in joints of the lower limb, wrong construction of lower limbs, but also the appearance of a slip phenomenon when foot touches the ground. Maintaining stability requires active involvement of biological motors (actin-myosin systems). The effect of the neuromuscular process is a human gait. An important biomechanical criterion for a human gait is dynamic stability. In biomechanical studies of the equilibrium state, cases of postural stability and locomotion of a human motor system are analyzed. Postural stability is related to a static issue. A biomechanism, by definition, is posturally stable when the projection of its mass center on the support surface is within the designated polygon. The gait, being the subject of work deliberations, is a two-legged and behavioral motor activity occurring with the interaction of human nervous and muscular systems.

Locomotion studies are conducted after adoption of various definitions of stability due to the diverse interpretation of the equilibrium state. A number of methods have also been defined to formulate the criterion of dynamic stability. The most important methods are: ZMP (zero moment point), FRI (foot rotation indicator) and CMP (centroidal moment pivot). In some work, the author assumes that dynamic stability can be determined on the basis of the ZMP method, in a combination with the minimal jerking theory formulated by Flash-Hogan (1985). According to Flash and Hogan, the optimal trajectory characterized by smoothness can be obtained by minimizing the integral criterion, which includes in itself a function of the square of the jerk, which is the third derivative of the displacement. Obtaining such a result for the ZMP trajectory makes it possible to achieve its smoothness.

The biomechanical criterion of dynamic stability in the work is formulated as: a human biomechanical system or biomechatronic human with an exoskeleton is dynamically stable while walking if the time integral has been minimized from the square of the jerk function point of the ZMP, specified by Flash and Hogan, and the ZMP point is inside the support polygon. Acceptance of the above criterion ensures no jerks in the trajectory of the ZMP, which means lack of the jerk in the biokinematical chain during the walk. The method for solving the case when the minimum ZMP trajectory is not inside the support polygon is to use an algorithm for controlling the dynamics of the biomechanical system with an exoskeleton, which will prevent such a situation or will cause the ZMP trajectory to return inside the support polygon.

The presented work is associated with a significant number of important scientific views. Researchers at various scientific centers have appointed multi-aspect methods for determining the dynamic stability of a gait to find out the properties of the biomechanical system precipitated from the equilibrium. In 1968, Miomir Vukobratović proposed a dynamic stability criterion for bipedal robots, which was applied sixteen years later to construct a WL-10RD biped robot in order to control its dynamics (Vukobratović and Borovac, 2004). The article gives various interpretations of the ZMP method. An elementary work in determining the stability for bipedal robots using the ZMP method is in the work (Vukobratović and Juričić, 1968). The examination method of static (postural) stability, defined as GCOM criterion in reference to the ZMP method, was analyzed in the work (Mrozowski *et al.*, 2008) by using an optical system consisting of two perpendicularly arranged cameras. The graphs show the course of ZMP and GCOM trajectories. In the following work, it is proposed to increase the accuracy of measuring the trajectory of the spatial model using six digital cameras and the Optitrack system.

The use of three principles of determining the dynamic stability, i.e. ZMP (zero moment point), FRI (foot rotation indicator), CMP (centroidal moment pivot) in robotics and biomechanics of gait is described in a complex way in the work (Popovic *et al.*, 2005).

The works (Bruijn *et al.*, 2012; Gouwanda and Muraledharan, 2012) describe the dynamic stability of a gait based on analysis of the maximum of Lapunov's exponent. The normal gait and a gait with an attached knee orthosis joint were examined. Gyroscopic sensors connected to s computer using a wireless network were used to measure the trajectory. These sensors are an alternative to optical systems because during a gait test, however, it is compulsory to make numeric integration of the measurement signal.

Analytical formulas defining dynamic stability based on the correlation of the trajectory of the generalized center of gravity and the trajectory of the assumed points of the feet during normal walking were also introduced (Antipov *et al.*, 2018).

The application of the finite difference method for approximation of time derivatives of the displacement during gait is described in the work (Ilewicz and Wojnarowski, 2012). Papers (Zatziorsky, 1998; Zatziorsky and Seluyanov, 1983) present some ways of obtaining the mass parameters of the human body.

Modeling of human walking patterns has been described in work (Kazemi and Ozgoli, 2019). In this report, Pontryagin's minimum principle was applied in order to obtain a smoother trajectory.

The key studies in the area of minimum jerk are the papers (Flash and Hogan, 1995; Flash *et al.*, 2003).

Engelbrecht (2001) describes the use of mechanical minimum principles in a motor control. It describes the principle of Flash-Hogan's minimum jerk and the principle of Uno's minimal change of the moment.

The application of the minimum jerk principle and the minimum moment principle are shown in the article (Fligge *et al.*, 2015). The methods discussed were used to find a way to generate a trajectory for a biped robot.

The principle of minimum jerk was used, stating that the trajectory of minimum jerk movement on the surface of a sphere should be geodetic. A mechatronic robot for studying the interaction of a human-machine system was used in the study (Sha *et al.*, 2006).

The simultaneous use of the ZMP criterion and the minimum jerk to generate a smooth trajectory of a bipedal robot is described in (Suleiman, 2016).

Arisumi *et al.* (2008) dealt with problems of dynamic stability related to transport of objects with the use of robotic systems. In the article (Xiang and Arefeen, 2020) an optimization problem of the dynamic human system and the load being lifted is described.

The aim of this work is to achieve s criterion of dynamic stability based on the Flash-Hogan principle and the ZMP method. The accepted criterion is universal for various gait cases, however, its usefulness in the case of a gait with the assistance of a supporting exoskeleton is particularly evident. A premise for the adopted goal is to say that the optimization of human body movements is a difficult issue that can be accomplished by using a supporting exoskeleton with an optimal ZMP trajectory generator. It is also stated that the movement along a smoother ZMP trajectory reduces wear of the musculoskeletal system, thus the exoskeleton with the applied trajectory of the optimal ZMP can be used for early rehabilitation of injured patients. Scientific researches (experimental and numerical) were carried out to seek for the criterion of dynamic stability.

2. Materials and methods

The article focuses on obtaining a new method for determining dynamic stability. It was assumed that the criterion sought would be based on the ZMP method. Coordinates of the ZMP point on the surface were obtained on the basis of an experimental study using an optical system. It is then demanded from the trajectory generator that the generated trajectory of the ZMP point would be as smooth as possible. The above assumption will be fulfilled by using one of the principles of the minimum mechanics of the so-called Flash-Hogan's jerks theory (Flash and Hogan, 1985). The principle resulting from this theory is called the principle of the minimum jerk. A series of experimental tests were carried out with the usage of the Optitrack research equipment consisting of six digital optical sensors in order to register the motion. The image from optical sensors was recorded at 120 Hz. Despite the right preparation of the experimental environment, it was necessary to correct the obtained trajectories by interpolating the appearing discontinuities (their cause was a momentary lack of visibility of the marker by the optical system) in positions by using parabolic polynomials. To determine the trajectory of the ZMP point, Ha and Choi's model (Ha and Choi, 2007), consisting of the lower limbs and torso was used

$$\begin{aligned} X_{ZMP}(t) &= \frac{\sum_{i=1}^{5} m_i(\ddot{y}_i + g)x_i - \sum_{i=1}^{5} m_i\ddot{x}_i y_i}{\sum_{i=1}^{5} m_i(\ddot{y}_i + g)} \\ Z_{ZMP}(t) &= \frac{\sum_{i=1}^{5} m_i(\ddot{y}_i + g)z_i - \sum_{i=1}^{5} m_i\ddot{z}_i y_i}{\sum_{i=1}^{5} m_i(\ddot{y}_i + g)} \end{aligned} \tag{2.1}$$

where: m_i is mass of the link i, x, y, z are coordinates of the center of the mass of the link i.

A criterion of dynamic stability was proposed, based on finding the minimum of the function of the square of jerk for the trajectory of the ZMP. Such an assumption gives an opportunity to state that the optimal trajectory can be found, therefore, there is no unexpected displacement of the ZMP point on the walking surface. This corresponds to minimal jerks of the human biokinematical chain and minimization of the usage of the musculoskeletal system. The jerk is referred to as the third displacement derivative. It informs how fast the force will change over time according to the equation

$$\frac{\partial F(\dot{x}, x, t)}{\partial t} = \frac{\ddot{x}F}{\partial \dot{x}} + \frac{\dot{x}\partial F}{\partial x} + \frac{\partial F}{\partial t} = mj \tag{2.2}$$

where x is displacement, $\dot{x}$ – velocity, $\ddot{x}$ – acceleration, $\dddot{x} = j$ – jerk, m – mass.

To determine the trajectory of the minimum ZMP jerk, one of the minimum mechanical principles developed by Flash and Hogan (1985) is used. It postulates that the integral

$$\int_0^t \dddot{x}^2\, dt \to \min \tag{2.3}$$

where $\dddot{x}$ is jerk of ZMP point, reaches the minimum value. Then, the optimal trajectory of the ZMP point is achieved. Most researchers, dealing with motor control, believe that this principle reflects biological utility of the biomechanism. After solving Euler-Lagrange's equation, the smooth trajectory equation of the ZMP point is obtained

$$x(t)_{ZMP} = c_5 t^5 + c_4 t^4 + c_3 t^3 + c_2 t^2 + c_1 t + t_0 \tag{2.4}$$

where c_i are polynomial coefficients.

Polynomial coefficients (2.4) are gained from the equation

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ t^5 & t^4 & t^3 & t^2 & t & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ t^4 & t^3 & t^2 & t & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ t^3 & t^2 & t & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_5 \\ c_4 \\ c_3 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix} = \begin{bmatrix} a \\ b \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.5}$$

where a, b are boundary conditions.

Todorov-Jordan's algorithm is a numerical modification of the theory given by Flash-Hogan and describes the problem of optimal control assuming various cost functions such as maximizing smoothness by minimizing jerks. Todorov and Jordan used a numerical approach to obtain velocity profiles in perfect harmony with the experiment. Their numerical algorithm is based on minimizing the jerk along the trajectory. The algorithm assumes that the trajectory of the ZMP point is given, and only space of velocity profiles is minimized. The Flash-Hogan algorithm requires both trajectory and velocity to be calculated. Details of Todorov-Jordan's algorithm are given in (Todorov and Jordan, 1998).

3. Results

Based on Todorov-Jordan's algorithm, the important trajectories were obtained in the Matlab environment. The trajectories of the ZMP point during normal gait of various qualities obtained by the optical system are illustrated in Figs. 1 to 3. The trajectories of the minimum jerk shown with the red line are the trajectories that give the result of minimal jerks of the biokinematical chain during the gait. During the gait with an exoskeleton, some jerks may appear during realization of the rehabilitation algorithm (braking, acceleration), whose influence on the ZMP trajectory can be leveled based on the Flash-Hogan principle and the used algorithm. This gives a positive effect of the smooth and rhythmic movement of the patient in the exoskeleton.

The used algorithm also gives the possibility of global smoothing of the ZMP trajectory for various smoothing filter coefficients while minimizing the jerk phenomenon (Todorov and Jordan, 1998; Meirovitch *et al.*, 2016). Figures 1 to 3 illustrate the smoothing for a variety of smoothing coefficients λ. For $\lambda = 1$, the trajectory is a straight line and it strongly deviates from the trajectory of the minimum jerk. By increasing the value of the smoothing factor, it increases the accuracy of matching the optimal trajectory to the ZMP trajectory points. Figure 2 illustrates the ZMP trajectory for the coefficient λ equal to 12. There is an improvement in the smoothness of the ZMP trajectory.

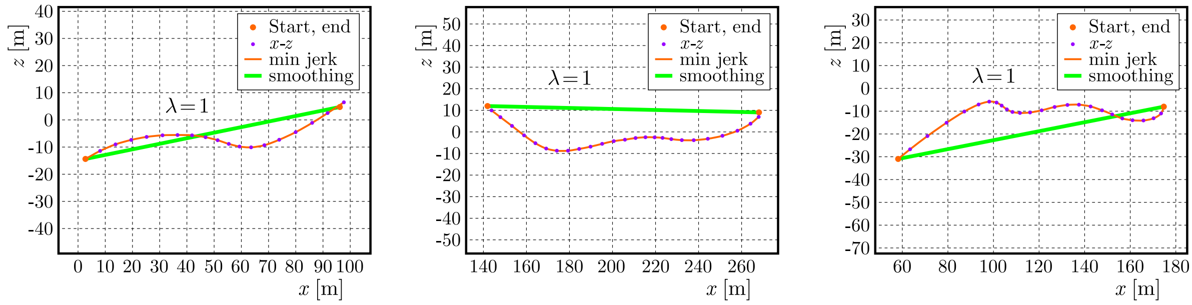

Fig. 1. Optimal ZMP trajectory for the coefficient $\lambda = 1$

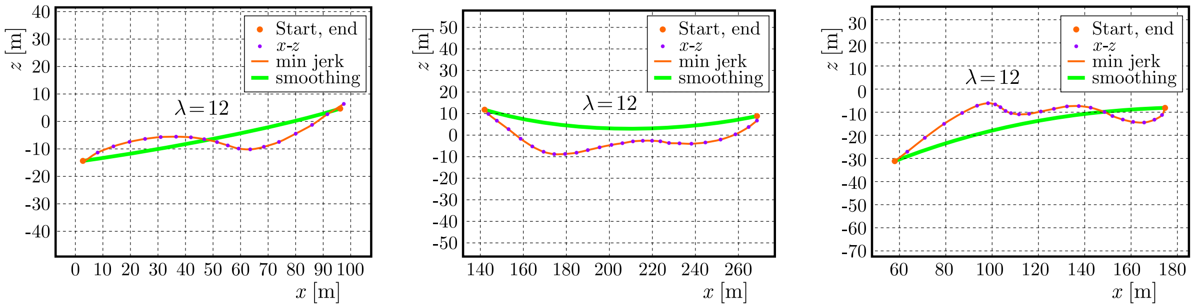

Fig. 2. Optimal ZMP trajectory for the coefficient $\lambda = 12$

For the value of the lambda smoothing filter equal to 36 (Fig. 3), a simultaneous effect of minimizing the emerging or might occurring jerk and matching the smoothed trajectory to the trajectory of the ZMP is obtained, whereas the trajectory becomes smooth.

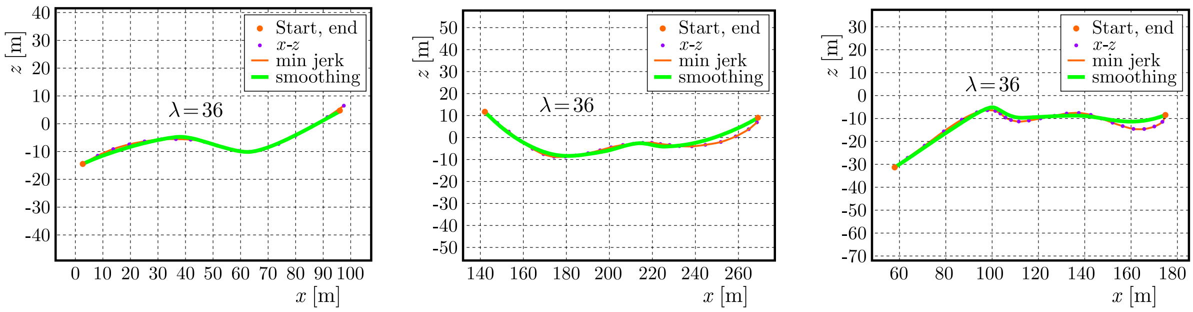

Fig. 3. Optimal ZMP trajectory for the coefficient $\lambda = 36$

The errand of the optimal ZMP trajectory to the exoskeleton control system will allow one to obtain better dynamic characteristics of the biokinematical chain during the gait. This will also result in increasing the coordination of the structure of human movements. From the point of biomechanics, it is stated that the movement on the optimal trajectory will minimize the usage of the musculoskeletal system. Undoubtedly also, for this reason, it can be applied to the exoskeleton control system for the early rehabilitation of injured patients.

4. Discussion

The obtained model determining the dynamic stability criterion can be added to the structure of the exoskeleton control system and enable smooth, rhythmic and coordinated gait by a smooth regulation of servomotors that drive individual degrees of freedom. Obtaining such an effect will give favorable effects of rehabilitation enabling the rehabilitated patient's gait to be adjusted with the help of an exoskeleton to the predetermined ZMP patterns, which can be described as optimal or, from a medical point of view, to be correct. The conducted research works made it possible to determine the innovative criterion of dynamic stability on the basis of Flash-Hogan's jerk principle. The adopted dynamic stability criterion specifies that human biomechanism during gait is dynamically stable in the case when the integral function, containing the square of the ZMP point jerk, is minimized and the ZMP point is within the designated support polygon. The trajectory of the ZMP point, after obtaining the minimum of integral criterion (2.3), becomes smooth and no jerks are observed in it. The obtained method of determining the dynamic stability gives the possibility to apply it to the exoskeleton control program, therefore, it is a significant innovation in relation to the exoskeleton control models created in research and development centers around the world. The obtained effect also gives the possibility of using it in the process of rehabilitation of patients, due to the fact that after applying the algorithm to the exoskeleton control system, the rehabilitated patient's movement can be forced in trajectories of various smoothness and thus, achieve the desired therapeutic effects. Not without significance is also the fact that the movement along the optimal trajectory will minimize wear of the musculoskeletal system, so that the exoskeleton could be used for early rehabilitation of the patient after the operation or injury.

5. Conclusions

The work proposes an innovative formulation of the criterion of dynamic stability. The outcome of applying Flash-Hogan's minimum principle is an optimal trajectory of the ZMP point, for the integral squared jerk criterion. ZMP trajectories that deviate from the trajectory of the minimum jerk and those which are not inside the support polygon are not being considered to be optimal dynamic, so that the criterion of dynamic stability defined in this work is not met for them. Optimality of the ZMP trajectory reflects the lack of jerks in the biokinematic chain during a human walk. Conducting rehabilitation activities can be used to move toward the trajectory of the ZMP of a patient with a damaged nervous system or trauma of the musculoskeletal system to the given optimal trajectory. Rehabilitation research, which may be conducted in the future using this criterion, rely on determining the optimal ZMP trajectory and the patterns that allow gradual approach to its quality. It is evident that this type of rehabilitation can be carried out with the help of an exoskeleton that emables sooner return to normal dynamic stability during the gait. The function of this interactive medical robot with a rehabilitated patient, after losing the ability to remain stable, will also take place with the lack of jerks resulting from actions of the mechatronic system of the exoskeleton during contact with the ground.

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