

ON THE USE OF VISCOELASTIC MATERIALS CHARACTERIZED BY BAYESIAN INFERENCE IN VIBRATION CONTROL

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Viscoelastic materials are used to reduce vibrations in mechanical systems due to their control efficacy. Considering that the dynamic behavior of those materials may be described by means of complex moduli, and experimental data may present uncertainties, an alternative is to use probabilistic methods, especially the Bayesian inference approach. By that approach, probability distribution functions are obtained for parameters of a model which describes the behavior of a given material. The present work employs a viscoelastic material modeled by the Bayesian approach in two vibration control actions, namely: a) use of vibration isolators; b) use of dynamic neutralizers. Transmissibility and receptance curves are displayed as well as dimensions of the control devices. Performance predictions are carried out in both cases. It is shown that the Bayesian approach can favourably reflect the presence of the uncertainties and advance their effects. Thus, more information can be provided for the designer of viscoelastic vibration control devices to anticipate eventual corrective measures.

Keywords: viscoelastic material, Bayesian inference, vibration isolator, viscoelastic dynamic neutralizer

1. Introduction

Most mechanical systems generate vibration and noise. Vibration can act in a favorable way, such as in washing machines, electric toothbrushes, conveyor belts, dental drills, pile drivers and equipment for vibratory tests of materials. However, excessive vibration or noise may manifest in an unfavorable way, resulting in auditory annoyance to people, discomfort, loss of system efficiency, mechanical failures, structural fatigue, wear of components, engine unbalance, and bridge collapse, among other damages (Teng and Hu, 2001). Therefore, vibration and noise control consists of measures to reduce high levels of those phenomena (Felhi *et al.*, 2018).

For reducing excessive vibration and noise, the use of viscoelastic materials (VEMs) is an efficacious alternative. According to Nashif *et al.* (1985), viscoelastic materials display frequency and temperature dependent mechanical properties. A well established and widely used way of describing the dynamic behavior of viscoelastic materials – in wide ranges of frequency and temperature – has been the representation by complex moduli.

In the frequency domain, viscoelastic materials (VEMs) are successfully described in the above representation by the four-parameter fractional derivative model (Pritz, 1996). The temperature dependence can be added via either the Williams-Landel-Ferry (WLF) equation or the

Arrhenius equation, when the frequency-temperature superposition principle applies (Nashif *et al.*, 1985; Mead, 1999), which holds for typical VEMs.

Regarding the dynamic characterization of viscoelastic materials using probabilistic methods, it is possible to arrive at estimates for unknown material parameters. In this characterization, one seeks to describe the dynamic properties of a viscoelastic material from models that contain such parameters and their associated uncertainties. According to Trabelsi *et al.* (2019), when the uncertainties associated with some design parameters are inserted in the model, the designer has an idea of the effects of the variation of these parameters, in addition to assisting her/him in decision making about the most appropriate values in a preliminary concept of the product.

The Bayesian inference is a very powerful and convenient approach to properly deal with uncertainties on a coherent framework. It incorporates prior knowledge about a phenomenon before data collection using prior probability distribution functions for the parameters in the data model. Available information from the data is then gathered in the likelihood function to model the quantities of concern in a probabilistic way. The inference combines both prior and likelihood information on resulting probability density functions, called posterior distributions (Kruschke, 2015; Balbino *et al.*, 2021).

From a detailed knowledge of the dynamic behavior of viscoelastic materials, the implementation of effective control designs becomes possible. In view of the absence or presence of an external power source, vibration control systems can be divided into passive and active control techniques (Marra *et al.*, 2016; Inman, 2017). Among the passive control techniques using viscoelastic materials, one can mention the introduction of damping, the addition of viscoelastic links, the insertion of auxiliary devices known as viscoelastic dynamic neutralizers, and the implementation of isolation systems. The latter two are within the scope of the present work.

In a vibration isolation project, the resilient elements are isolators which connect the equipment to its supporting structure and may be made of a viscoelastic material. The transmissibility function is, as a general rule, used to describe the efficacy of the designed isolation system (Snowdon, 1968).

With regard to dynamic neutralizers, their aim is to reduce the response of a system submitted to excitations in a frequency region in which that system has one or more natural frequencies. A simple neutralizer is composed of a certain mass attached to a resilient element (made of a viscoelastic material or composed of a spring/damper, among others), which, in turn, is coupled to the primary system (the system to be controlled). The optimal parameters of the neutralizer, which must lead to a minimum system response, are determined by means of an adequate choice out of possible physical/dimensional parameters, as observed in Kitis *et al.* (1983).

In this work, vibration control actions in single degree-of-freedom (SDOF) systems by means of viscoelastic isolators and neutralizers are considered. A distinguished feature is the adoption of Bayesian inference in the dynamic characterization of the viscoelastic material. To the authors knowledge, focused efforts have not been previously reported in the literature.

2. Theoretical background

2.1. Viscoelastic materials

For viscoelastic materials, the complex elastic modulus approach is used to describe the stress and strain relationships resulting from the behavior observed in those materials (Nashif *et al.*, 1985). Thus, in general, the complex dynamics stiffness \bar{k} (in N/m) – frequency and temperature dependent – of a viscoelastic material (VEM) is given by (Espíndola *et al.*, 2008)

$$\bar{k}(\omega, T) = L\bar{E}(\omega, T) \quad (2.1)$$

where ω is frequency (in rad/s), T is temperature (in K, or °C), L is the so-called design factor (in m), and \bar{E} is the complex modulus of elasticity (in Pa), such that $\bar{E} = E_R + iE_I$. The quantities E_R and E_I are, respectively, the real and the imaginary moduli (in Pa). The way of representing the complex stiffness of a viscoelastic material element given by Eq. (2.1) is used because of its convenience and effectiveness in vibration and noise control designs. As to the design factor L , it relates a device property, the stiffness, to a material property, the modulus of elasticity, and contains geometrical characteristics of the device, as it will be shown below.

The four-parameter fractional derivative model, along with the WLF equation or the Arrhenius equation, can be employed to describe the frequency and temperature dependence required of the complex elastic modulus (Bagley and Torvik, 1986; Pritz, 1996; Mead, 1999). It is then expressed as

$$\bar{E}(\omega, T) = \frac{E_L + E_H b [i\alpha_T(T)\omega]^\beta}{1 + b [i\alpha_T(T)\omega]^\beta} \quad (2.2)$$

where E_L , E_H , b and β are material parameters (for the corresponding units, please see Table 1); $\omega_R = \alpha_T(T)\omega$ is the reduced frequency (in rad/s); and $\alpha_T(T)$ is the shift factor (dimensionless). This factor can be concisely determined using the Arrhenius equation (Jones, 1990; Mead, 1999), so that

$$\log_{10} \alpha_T(T) = T_A \left(\frac{1}{T} - \frac{1}{T_0} \right) \quad (2.3)$$

where T_A is the activation temperature (in K), and T_0 is the working temperature (also in K).

2.2. Vibration control

2.2.1. Vibration isolator

Vibration isolation encompasses procedures aimed at reducing a vibratory phenomenon when it is transmitted from a mechanical system to the medium in which it is located (force isolation) or vice-versa (motion isolation). A vibration isolator is a resilient element – usually made of a metal or elastomer, that connects a given machine to its base and promotes, by its characteristics, the desired reduction in transmission of the force, or of motion, according to the case.

The effectiveness of isolation can be determined through the concept of transmissibility, which is a complex frequency function when viscoelastic materials are modelled by complex moduli. For a single degree-of-freedom system, the transmissibility modulus (dimensionless) is (Snowdon, 1968)

$$T_M(\omega, T) = \sqrt{\frac{1 + [\eta_E(\omega, T)]^2}{[1 - r(\omega, T)\varepsilon^2 t]^2 + [\eta_E(\omega, T)]^2}} \quad (2.4)$$

where $\eta_E(\omega, T)$ is the (dimensionless) ratio between the imaginary $E_I(\omega, T)$ and the real $E_R(\omega, T)$ elastic moduli of the material (now expressed with the frequency and temperature dependence), $r(\omega, T) = E_R(\omega_n, T)/E_R(\omega, T)$, ω_n is the natural (or characteristic) frequency of the system (in rad/s), and $\varepsilon = \omega/\omega_n$ is the (dimensionless) ratio between the excitation and natural frequencies.

The transmissibility modulus is defined, for force isolation, as the ratio of the amplitude of the transmitted force (to the medium) to the amplitude of the excitation force. For motion isolation, it is the ratio of the amplitude of the transmitted vibration (to the equipment) to the amplitude of the base vibration. Reduction in the transmissibility modulus means, in amplitude, reduction either in the transmitted force for a given excitation force, or in the transmitted vibration, for a given base vibration. The transmissibility modulus can also be denoted by $T_M(f, T)$, where f is the frequency in Hz.

Based on Eq. (2.4) and on the desired transmissibility modulus T_{Md} for the passively controlled system, it is possible to determine the natural frequency ω_n of the isolated system, solving the following non-linear equation

$$T_M(\omega_e, T) - T_{Md} = 0 \quad (2.5)$$

where ω_e is the dominant excitation frequency (in rad/s). The natural frequency ω_n follows from the solution to Eq. (2.5) given the fact it is in Eq. (2.4) for the transmissibility modulus T_M .

Since the design factor L can be related, in general, to the real part of the complex stiffness by $k(\omega, T) = nLE_R(\omega, T)$ (see Eq. (2.1)) and the natural frequency $\omega_n = \sqrt{[k(\omega_n, T)/m]}$, it follows that

$$L = \frac{\omega_n^2 m}{nE_R(\omega_n, T)} \quad (2.6)$$

where m is the system mass (in kg), and n is the number of in-parallel isolators (usually, there are n isolators acting in parallel; in that case, the equivalent stiffness is the sum of the individual stiffnesses).

For cylindrical isolators under compression, the design factor may be expressed by (Rivin, 2003)

$$L = \frac{\pi d^2}{4h} \left[1 + \gamma \left(\frac{d}{4h} \right)^2 \right] \quad (2.7)$$

where d is diameter (in m), and h is height of the isolator (in m), whereas γ is a dimensionless constant, which is approximately equal to 2.0 for unfilled elastomers, and 1.5 for filled elastomers (case of the considered VEM). As noted in Eq. (2.7), and anticipated above, the design factor L contains geometrical characteristics of the cylindrical isolator, that is, its diameter and its height.

Equation (2.7) can be solved for d provided that the value of h is known. There are two alternatives: the first one is to make the static deflection of the isolated system equal to 10% of h , a limit for the load-deflection ratio to remain in the linear range (Rivin, 2003). Thus, for the isolated system, one has

$$h = \frac{mg}{0.1LE} \quad (2.8)$$

where g is the gravitational acceleration (in m/s^2) and E is the static modulus of elasticity (regarded as the real elastic modulus (in Pa) at a very low frequency, which is the meaning of the parameter E_L in Eq. (2.2)). The second one is to establish an arbitrary ratio between d and h , such that $d/h < 10$ (in fact, even this upper limit is arbitrary and is based on constraints imposed on the whole system under control, see Rivin (2003)). For $d/h = a$

$$h = \frac{d}{a} \quad (2.9)$$

where a is the aspect ratio. In this case, it can be shown from Eq. (2.7) that

$$d = \frac{64L}{\pi a(\beta a^2 + 16)} \quad (2.10)$$

Both the alternatives above allow the dimensions of concern for the vibration isolator to be obtained, and those dimensions will follow from the design factor previously determined. Given its dimensions, the viscoelastic device can be actually manufactured.

2.2.2. Vibration neutralizer

A dynamic vibration neutralizer (also called a dynamic vibration absorber) is a resonant system that, when properly designed and mounted on a mechanical system, usually known as the primary system, reduces the vibration level of that system. Its action can be suitably assessed in terms of the original and modified frequency response functions (FRFs) of the primary system. In structural dynamics, the display of an FRF is the most common method of visualizing the relationship between input (excitation) and output (response) of a linear mechanical system showing information such as resonance, anti-resonance, modal density and damping.

For a single degree-of-freedom (SDOF) primary system containing a simple (SDOF) viscoelastic dynamic neutralizer, the FRF of the compound (or composite) system (primary system plus neutralizer) is given by Bronkhorst *et al.* (2018)

$$\overline{H}(\omega, T) = \frac{1}{-\omega^2 m_1 + i\omega c_1 + (k_1 + \overline{k}_{eq})} \quad (2.11)$$

where m_1 is the primary system mass (in kg), c_1 is the primary system damping constant (in kg/s), k_1 is the primary system stiffness (in N/m), and $\overline{H}(\omega, T)$ is the compound system receptance (in m/N). The receptance is a complex FRF which relates the vibration (given in terms of displacement) of the compound system to the applied force. Its modulus is the ratio between the modulus (amplitude) of the vibration and the modulus (amplitude) of the applied force.

Yet in Eq. (2.11), \overline{k}_{eq} , which incorporates the effects of the inserted neutralizer, is the equivalent complex stiffness (in N/m) such that its real and imaginary parts are given (in N/m) by the following equations (Lopes, 1998)

$$\begin{aligned} (k_{eq})_r(\omega, T) &= \frac{m_a r(\omega, T) \omega_a^2 \{\varepsilon^2 - r_a(\omega, T) [1 + \eta_E(\omega, T)^2]\} \varepsilon^2}{[\varepsilon^2 - r(\omega, T)]^2 + [r(\omega, T) \eta_E(\omega, T)]^2} \\ (k_{eq})_i(\omega, T) &= \frac{m_a r(\omega, T) \omega_a^2 \eta_E(\omega, T) \varepsilon^4}{[\varepsilon^2 - r(\omega, T)]^2 + [r(\omega, T) \eta_E(\omega, T)]^2} \end{aligned} \quad (2.12)$$

where m_a is the neutralizer mass (in kg) and ω_a is the neutralizer natural frequency (in rad/s).

Actually, the compound system, comprising the primary system and the viscoelastic neutralizer, is a two degree-of-freedom system. However, the above expressions reflect the fact that the effect of the viscoelastic neutralizer can be represented by that of the equivalent and complex viscoelastic spring connected to the primary system and to the “earth” (that is, for the current case, a more convenient alternative to that of the equivalent mass and dashpot shown in Bronkhorst *et al.*, 2018). Then, the compound system can still be regarded as a SDOF system under the action of the equivalent spring, as seen in Section 3.3.

When controlling the SDOF primary system by using a viscoelastic neutralizer with the aid of a non-linear optimization technique, a desirable aim is to obtain the value of the neutralizer natural (characteristic) frequency that leads to the lowest amplitude of the response of the compound system. The general standard optimization problem for controlling the system may then consist of

$$\min f_{obj}(x) \quad f_{obj}(x) : \mathbb{R}^n \rightarrow \mathbb{R} = \|\overline{H}(\omega, x)_{\omega_i \leq \omega \leq \omega_f}\|_F \quad (2.13)$$

where $\|\cdot\|_F$ represents the Frobenius norm; ω_i and ω_f are the lowest and highest limits of the frequency range of interest, respectively; and x is the design vector comprising, in this case, the frequency ω_a .

That means to minimize as much as possible the amplitude of the receptance (FRF) of the compound system. The procedure consists of searching for an optimal neutralizer natural frequency that may match the lowest values of the compound system FRF (receptance) amplitude

in the frequency range $[\omega_i, \omega_f]$. As the receptance relates, in the frequency, vibration (displacement) and excitation, by reducing the receptance amplitude the vibration amplitude will be also reduced to a given excitation.

After determining the optimal frequency of the neutralizer, it is possible to calculate the corresponding design factor L and, consequently, the dimensions of the resilient element of the device. Based on that optimal frequency $\omega_{a_{op}}$, the design factor (in m) is obtained by

$$L = \frac{m_a \omega_{a_{op}}^2}{E(\omega_{a_{op}}, T)} \quad (2.14)$$

an equation similar to Eq. (2.6) since the viscoelastic neutralizer is a single degree-of-freedom system. It is noted that the optimal frequency is $\omega_{a_{op}} = \sqrt{[k(\omega_{a_{op}}, T)/m_a]}$ and that the design factor L relates the stiffness and the modulus of elasticity as mentioned in Section 2.2.1 for the vibration isolator.

If the resilient element of the neutralizer is considered cylindrical, the determination of its dimensions follows the exposed methodology based on Eq. (2.7). It is observed that the neutralizer mass is, as a rule, predefined between 2.5% and 10% of the primary system mass, the vibration of which one intends to control (Den Hartog, 1956; Espíndola *et al.*, 2008).

3. Methodology of design of control devices

In this Section, the methodology employed for designing the viscoelastic vibration control devices is detailed, after specifying the viscoelastic material data considered in the designs. Based on the data, the isolator design was initially carried out, followed by the neutralizer design.

3.1. Viscoelastic material data

The viscoelastic material (VEM) used in the present work was studied by Jones (1992) and Lopes (1998), and is a well-known and typical material. It is manufactured and marketed by the EA-RTM Aearo Technologies LLC (3M Group) and commercially labeled as ISODAMP C-1002.

In a companion paper, Balbino *et al.* (2021) put forward a Bayesian inference approach to characterize the dynamic properties of a typical VEM modeled by means of the four-parameter fractional derivative model associated to the Arrhenius equation. The dynamic properties were represented by the real and imaginary moduli of elasticity and the corresponding loss factor. The proposed approach, which was applied to the aforementioned VEM, provides as output probability distribution for each considered material parameter, taking the frequency and temperature dependence into account.

The Bayesian inference approach described in Balbino *et al.* (2021) was followed herein, however, in a fresh implementation, for sake of completeness. From the posteriori distribution functions resulting from the Bayesian approach, 2000 sets of parameters which characterized the VEM were randomly selected (that is, the entire inference process was run with fresh simulations of the Bayesian algorithm of inference). From each one of those sets, 1750 pairs of values of real and imaginary moduli were generated, corresponding to the number of samples, frequencies and temperatures of Balbino *et al.* (2021).

Then, the mean values and the limits for the 2.5th percentile (2.5th per.) and the 97.5th percentile (97.5th per.) were computed. Those values were submitted to a curve fitting process (Medeiros Jr *et al.*, 2019) so that the associated material parameters could be extracted, as shown in Table 1. Thus, it was possible to estimate the dynamic properties for those sets of parameters (mean, 2.5th percentile and 97.5th percentile) at the frequencies and temperatures of concern. More details can be found in Préve (2019) and Balbino *et al.* (2021).

Table 1. Parameters of the viscoelastic material characterized via Bayesian inference

Parameters	E_L [Pa]	E_H [Pa]	b [s $^\beta$]	β	T_A [K]
mean	$4.77 \cdot 10^6$	$2.58 \cdot 10^9$	0.0042	0.39	9495.77
2.5th per.	$3.68 \cdot 10^6$	$1.99 \cdot 10^9$	0.0043	0.33	10986.08
97.5th per.	$6.06 \cdot 10^6$	$3.54 \cdot 10^9$	0.0040	0.48	7458.06

3.2. Isolator design

Viscoelastic isolators were then designed for the case of a 4.9 kg bench grinder. Its operating angular speed was 3500 rpm, which corresponded to a rotating unbalance frequency of approximately 58.3 Hz (366.3 rad/s). The bench grinder on cylindrical isolators, schematically shown in Fig. 1a, was modelled as a SDOF system in vertical motion, as shown in Fig. 1b.

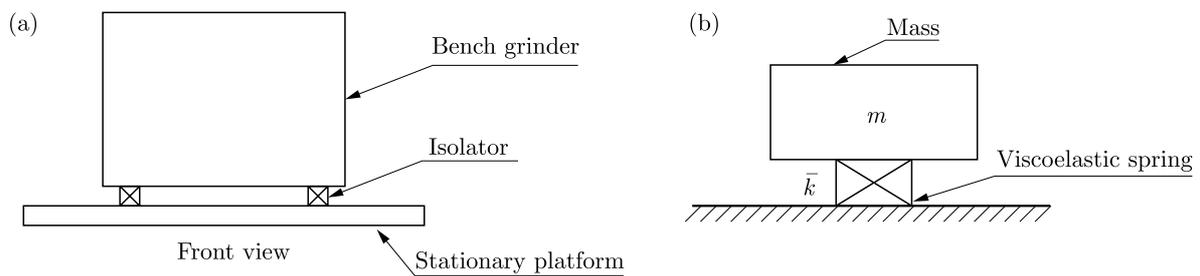


Fig. 1. (a) Schematic diagram of a bench grinder on cylindrical isolators; (b) mechanical model

The aim was to design an isolation system that could provide an 80% reduction in terms of transmissibility modulus which would be thus equal to 0.2. In other words, the amplitude of the transmitted harmonic force would be 20% of the amplitude of the harmonic force generated by the equipment, in the frequency of interest. In dB, such a transmissibility modulus would be nearly -14 dB (ref. 1 N/N).

The working temperatures considered were -2.1°C , 17.4°C and 33.5°C (270.9 K, 290.4 K and 306.5 K, respectively). They were the minimum, mean and maximum temperatures, respectively, registered in the city of Curitiba, southern Brazil, in 2013 by the National Institute of Meteorology.

With the value of the desired transmissibility modulus, the natural frequency of the isolated system was obtained. That was achieved by solving the non-linear equation given in Eq. (2.5) in MATLAB with the aid of function *fzero*. This provided the characteristic frequency of the system for each working temperature and for each set of the parameters: mean, 2.5th percentile, and 97.5th percentile.

From the determined characteristic frequencies, the corresponding real stiffness values were obtained and, consequently, the design factors, given by Eq. (2.6). As the design factor was directly linked to the dimensions of the resilient element chosen in the proposed design, and the isolators were chosen to have the convenient cylindrical form, the two previously exposed alternative ways of obtaining the associated dimensions were employed (see Section 2.2.1).

3.3. Neutralizer design

For designing the viscoelastic neutralizer, it was considered the same bench grinder now mounted in such a way that the resonance frequency of concern was 45 Hz (282.7 rad/s). That was then the primary system, modeled as a SDOF system. In the face of operating conditions, the control frequency range was established in an interval between 30 Hz and 60 Hz (188.5 rad/s and 377.0 rad/s, respectively).

As before, the working temperatures were -2.1°C , 17.4°C and 33.5°C . The mass of the primary system was 4.9 kg and the parameters of the viscoelastic material were those already presented in Table 1. The neutralizer mass was arbitrarily chosen as 5% of the primary system mass, and it was understood that one single device could carry out the passive control action, as shown in Fig. 2a. As shown in Fig. 2b, the compound system, comprising primary system (bench grinder) and viscoelastic neutralizer, can be modeled as a two degree-of-freedom system in vertical motion. However, as mentioned in Section 2.2.2, the effect of the viscoelastic neutralizer can be represented by that of an equivalent viscoelastic spring connected to the primary system and to the ‘‘earth’’. Then, the compound system can be regarded as a SDOF system under the action of that equivalent spring, as shown in Fig. 2c.

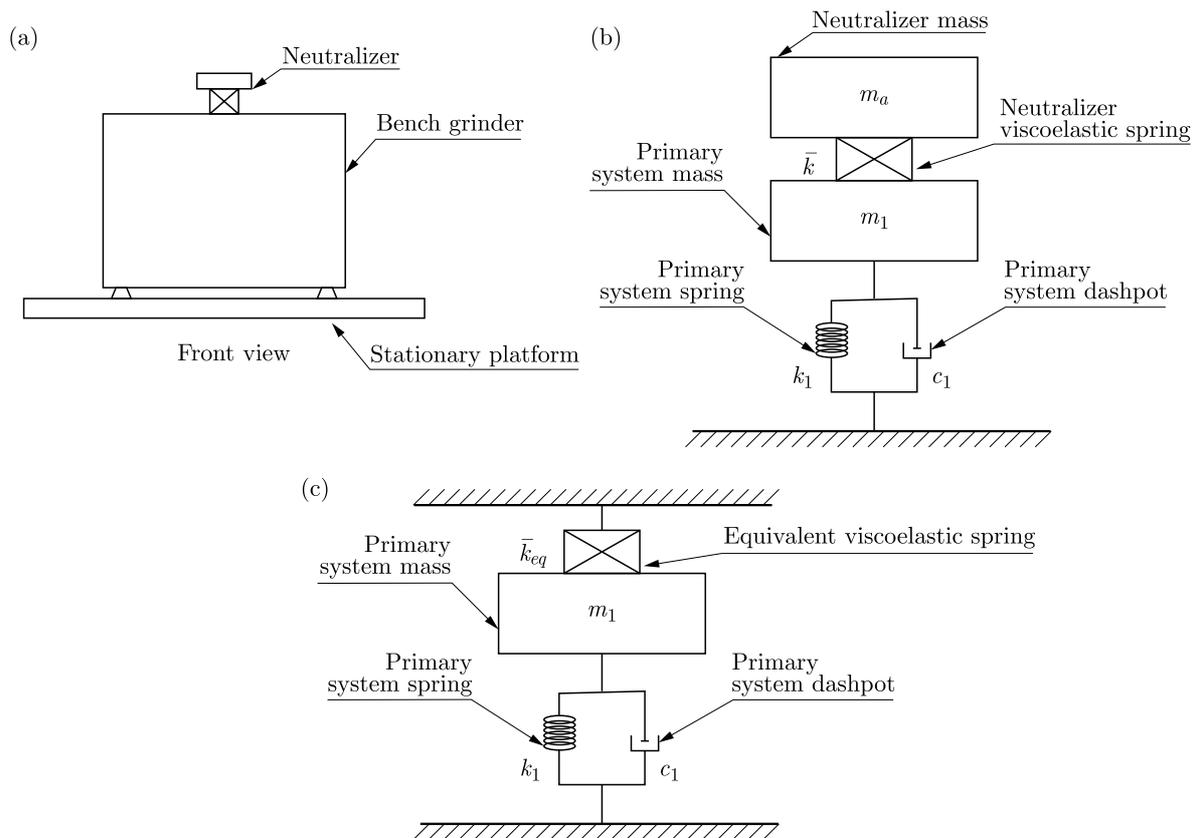


Fig. 2. (a) Schematic diagram of the bench grinder on the cylindrical neutralizer; (b) mechanical model; (c) equivalent mechanical model

The predicted compound system receptance was computed by Eq. (2.11), and the optimal neutralizer characteristic (natural) frequency was that which provided the composite vibrating system with the greatest possible reductions in terms of receptance within the specified conditions. From the initial estimate for each neutralizer characteristic frequency corresponding to each set of parameters, the non-linear optimization process was carried out in MATLAB with the aid of function *fminsearch*, leading to the respective optimal neutralizer characteristic frequency (see Section 2.2.2 for the definition of the neutralizer characteristic frequency).

Having determined the optimal frequency for each set of parameters and also having chosen the neutralizer mass, the design factors associated to each working temperature were obtained by Eq. (2.14). As in the previous case of isolator design, from the design factors, it was possible to determine, in each instance, the dimensions of the resilient element of the neutralizer.

4. Presentation and analysis of results

4.1. Results for isolator design

4.1.1. Computation of dimensions

Figure 3 shows the transmissibility curves of the three sets of parameters for each considered working temperature (the transmissibility modulus, as defined in Section 2.2.1, is expressed in dB and the frequency is in Hz, for sake of convenience). It is possible to observe that the curves of the transmissibility modulus intercept the straight line corresponding to the desired transmissibility of -14 dB in the approximate frequency of 58 Hz. That is exactly in the frequency of interest for the case.

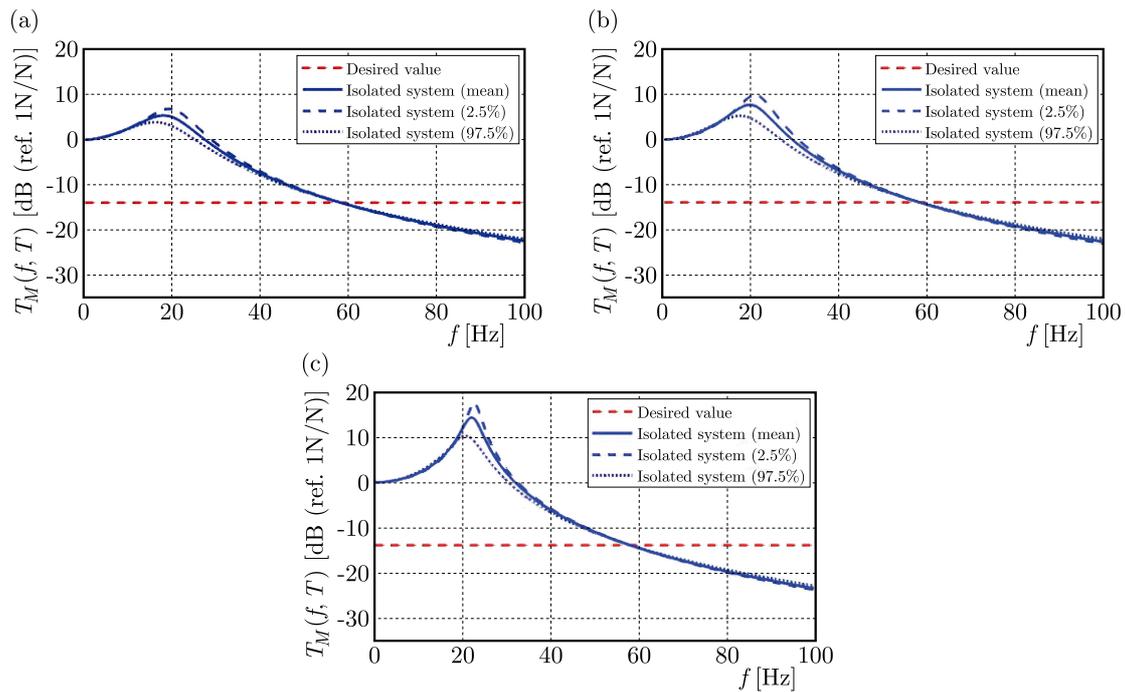


Fig. 3. Transmissibility modulus: (a) -2.1°C , (b) 17.4°C , (c) 33.5°C

It is also observed that, for each temperature, the transmissibility modulus behaves in a different way for the set of parameters of the mean, the 2.5th percentile and the 97.5th percentile. As the working temperature increased, the transmissibility curve – in the characteristic frequency – showed a more pronounced peak. This is due to the fact that, with the considered increase in temperature, the system is less damped as a result of the temperature dependence of the employed VEM.

The procedure for obtaining the dimensions of the isolators followed the steps of the two already mentioned alternatives – hereafter called “Alternative 1” and “Alternative 2” – as can be seen in Tables 2 and 3 for each of the 3 working temperatures with dimensions expressed in cm for sake of understanding. In those tables, it is observed that, for “Alternative 1”, height and diameter values increase for the sequence 2.5th percentile, mean and 97.5th percentile, whereas, for “Alternative 2”, they decrease. That is related to the way those alternatives are built, as explained in Section 2.2.1.

When the results were analyzed in terms of eventual manufacturing, it was verified that it would not be viable to use the isolator with the dimensions of $h = 12.47$ cm and $d = 1.13$ cm, obtained by “Alternative 1”, with the set of parameters of the mean, at -2.1°C , due to its awkward geometrical characteristics. That is, a device like this at this temperature would not actually be used.

Table 2. Dimensions of isolators – Alternative 1

	Temperature	Mean	2.5th per.	97.5th per.
height [cm]	−2.1°C	12.47	6.97	25.70
diameter [cm]		1.1345	1.1335	1.1348
height [cm]	17.4°C	1.79	1.10	3.46
diameter [cm]		1.11	1.09	1.13
height [cm]	33.5°C	0.70	0.49	1.11
diameter [cm]		1.03	0.97	1.09

Table 3. Dimensions of isolators – Alternative 2

	Temperature	Mean	2.5th per.	97.5th per.
height [cm]	−2.1°C	12.75	22.79	6.18
diameter [cm]		1.15	2.05	0.56
height [cm]	17.4°C	1.81	2.95	0.90
diameter [cm]		1.12	1.83	0.58
height [cm]	33.5°C	0.70	1.00	0.44
diameter [cm]		1.03	1.48	0.65

However, it would be viable to use the isolator with the dimensions of $h = 1.00$ cm and $d = 1.48$ cm, obtained by “Alternative 2”, with the set of parameters of the 2.5th percentile, at 33.5°C. It was noticed that those dimensions were compatible with the focused system in question and, therefore, this could be a suitable device for eventual manufacturing.

Although some of the above isolators do not present viable dimensions for the application under study (that is, those dimensions are not adequate for manufacturing of viscoelastic isolators and/or for use under the bench grinder, given its purpose), it is still worth to note the wide variation between the dimensions. And regardless of the approach chosen for calculating the dimensions of the isolators (the dimensions may result from the approaches called herein alternative 1 and alternative 2), clear variations in height and diameter can be observed, reflecting the variations found in the dynamic properties of the viscoelastic material.

4.1.2. Evaluation of performance

As observed in the results obtained for the isolator design, the temperature effect already known in the literature is pronounced. Anyway, by checking the transmissibility curves it is possible to observe that in all temperatures and for all sets of parameters the desired transmissibility is achieved. Meanwhile, what is the performance variation among the designed isolators?

Now suppose one desires to build real isolators for an effective application and, for that, the set of mean parameters at 33.5°C is chosen. What happens if, by any chance, due to uncertainties, the resulting isolators present the parameters of the set of 2.5th or 97.5th percentiles? The result of the performance benchmarking in those conditions can be visualized in Fig. 4c (again, transmissibility modulus expressed in dB and frequency in Hz for sake of convenience).

The benchmarking idea is, therefore, to illustrate that if variation in the dynamic properties exists then the isolating system must be such that it becomes possible to have an adjustment mechanism after the isolators are manufactured. One could think, for example, that instead of using four simple in-parallel isolators, one could use 4 sets of two in-series isolators which would reduce the system stiffness by half. Another adjustment alternative would be to alter the base of the bench grinder by inserting a metallic base, thus increasing the system mass and decreasing the natural frequency.

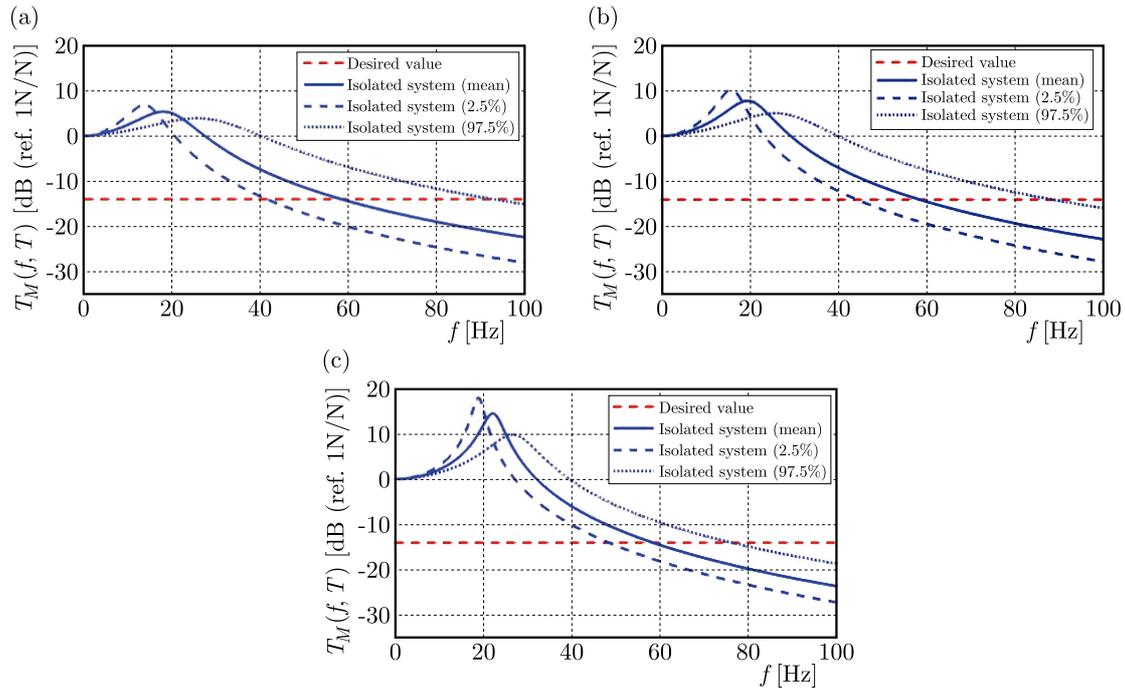


Fig. 4. Performance in vibration isolation. Parameters of the mean: (a) -2.1°C , (b) 17.4°C , (c) 33.5°C

4.2. Results for neutralizer design

4.2.1. Computation of dimensions

Regarding the neutralizer design, Fig. 5 shows the curves of the receptance modulus for each temperature and each set of material (model) parameters (the receptance modulus, as defined in Section 2.2.2 is expressed in dB, for sake of convenience). The red line refers to the primary system behavior, while the other lines refer to the compound system (primary system plus neutralizer). The latter show more attenuated curves due to the vibration control performed with the viscoelastic neutralizer.

From the receptance curves, little difference was observed among the curves of each set of parameters (mean and percentiles), except for the curves at 33.5°C , in which the difference was somehow more appreciable. At that temperature, the difference was approximately 8 dB between 2.5th and 97.5th percentiles. In Fig. 5c, the receptance curves clearly show two peaks, evidencing that the composite system behaves like a two degree-of-freedom system. Regardless of the set of parameters used in the neutralizer design, there was a vibration reduction at those three working temperatures. This can be seen in the curves of the receptance modulus of the composite system.

The same procedure adopted for calculating the dimensions of the isolators was adopted for calculating the dimensions of the resilient element of the neutralizer. Tables 4 and 5 show the results for the two used alternatives (dimensions expressed in cm for sake of understanding).

Again, as noted for the dimensions of the isolators, some of the dimensions of the neutralizer resilient element were not compatible for the bench grinder vibration control design. Anyway, it was also noted that the variation in height and diameter values persisted due to the variation in the dynamic properties of the viscoelastic material.

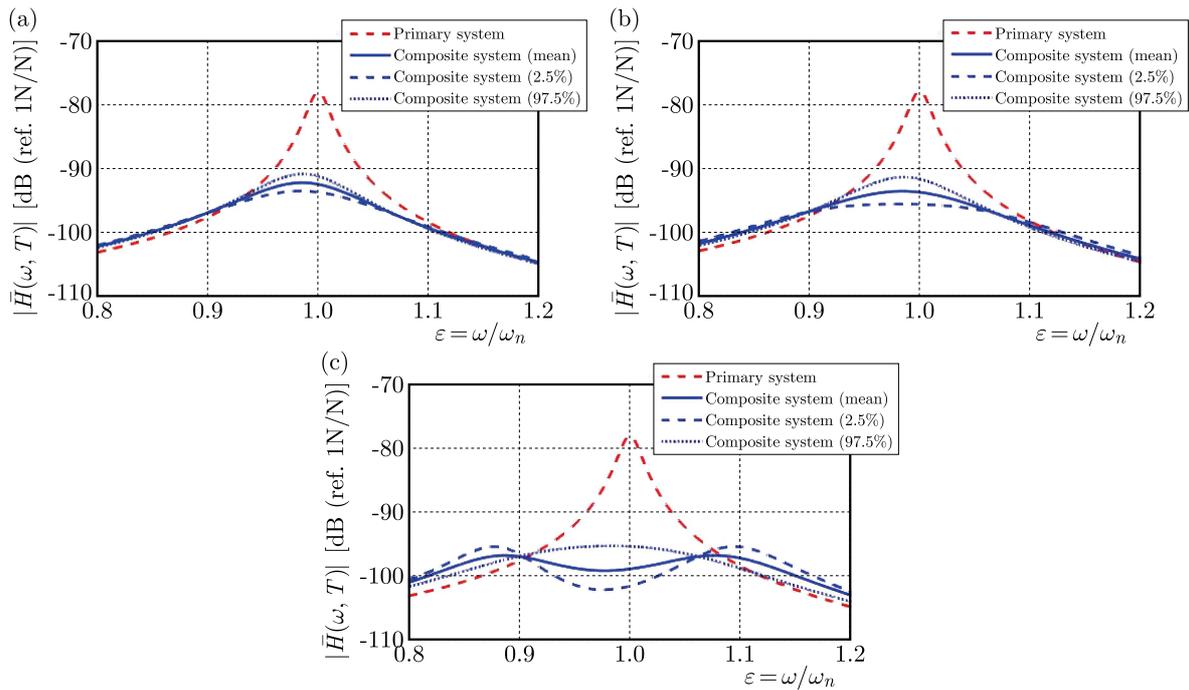


Fig. 5. Receptance modulus: (a) -2.1°C , (b) 17.4°C , (c) 33.5°C

Table 4. Dimensions of resilient element – Alternative 1

	Temperature	Mean	2.5th per.	97.5th per.
height [cm]	-2.1°C	69.11	38.93	141.05
diameter [cm]		1.134936	1.134905	1.134946
height [cm]	17.4°C	9.99	6.20	19.02
diameter [cm]		1.1343	1.1332	1.1348
height [cm]	33.5°C	4.04	2.85	6.23
diameter [cm]		1.1308	1.1267	1.1332

Table 5. Dimensions of resilient element – Alternative 2

	Temperature	Mean	2.5th per.	97.5th per.
height [cm]	-2.1°C	0.16	0.30	0.08
diameter [cm]		0.05	0.09	0.02
height [cm]	17.4°C	1.17	1.89	0.61
diameter [cm]		0.38	0.62	0.20
height [cm]	33.5°C	2.90	4.10	1.88
diameter [cm]		0.96	1.36	0.62

4.2.2. Evaluation of performance

Performance benchmarking was also carried out for the vibration neutralization. Again, the set of parameters of the mean at 33.5°C was selected. The result can be observed in Fig. 6c (again, the receptance modulus expressed in dB for sake of convenience).

Vibration reduction for the set of parameters of the mean is the same as predicted in the design stage. However, for the sets of parameters of 2.5th and 97.5th percentiles, one loses approximately between 6 dB and 10 dB of the performance (when comparison is made at the peaks). A mechanism that can be adopted after manufacturing this device with one of those two

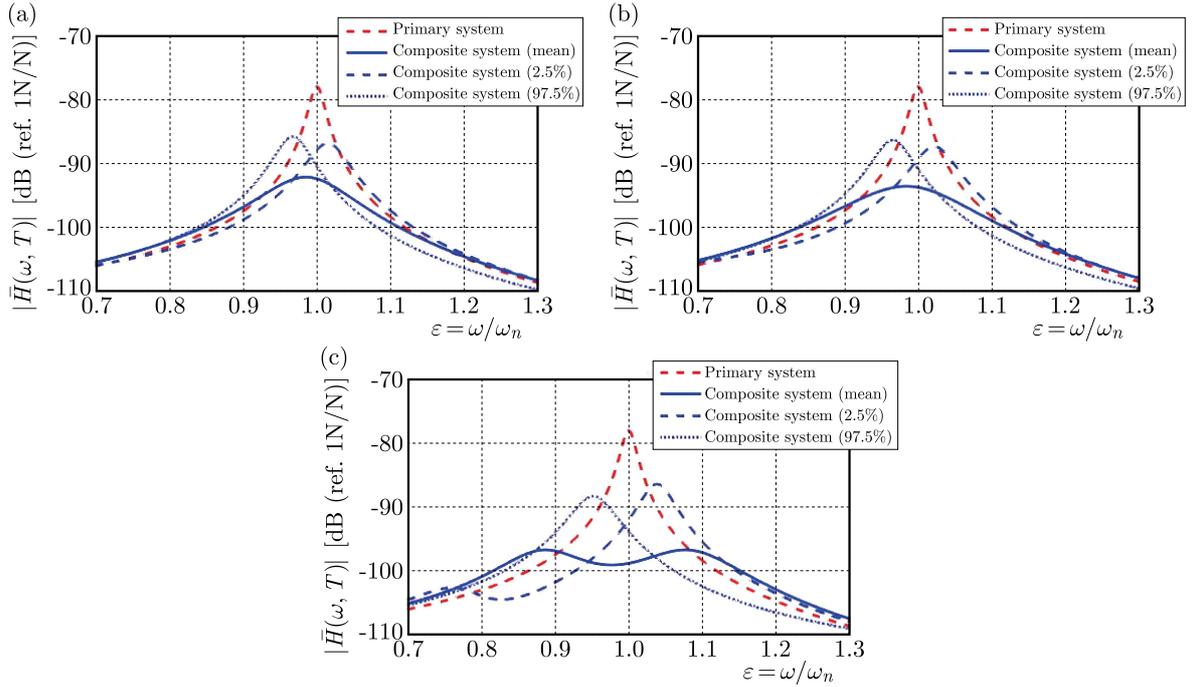


Fig. 6. Performance in neutralization. Parameters of the mean: (a) -2.1°C , (b) 17.4°C , (c) 33.5°C

sets of parameters is retuning, which could be performed in this case by a change in the neutralizer mass.

Benchmarking for temperatures from -2.1°C to 17.4°C is presented in Fig. 6a and Fig. 6b. In such cases, reductions between approximately 5 dB and 8 dB can be seen.

4.3. Supplemental remarks

It has already been shown that dynamic properties of viscoelastic materials (VEMs) are subject to uncertainty (Balbino *et al.*, 2021), and the above results suggest that uncertainty may significantly affect manufacturing and performance of viscoelastic vibration control devices. By applying the Bayesian inference in the dynamic characterization of VEMs, it is possible to propagate the uncertainty from the model parameters associated with the dynamic properties to the actual design of the control devices, for instance, vibration isolators and neutralizers. Thus, ranges of likely performances of those devices may be established and eventual corrective measures anticipated, if needed.

In this context of controlling vibrations by means of viscoelastic devices, it is of concern to know the probability distributions of material behavior model parameters in order to probabilistically evaluate the possible effects of those parameters over their entire domains. That cannot be accomplished, for instance, if the model parameters are obtained either as single values, as it follows by the use of the least-squares method, or by means of confidence intervals, as it usually happens in the frequentist inference (Wakefield, 2013). It is worth emphasizing that the approach taken implies no criticism to the least squares method, on the contrary, adds to it the accounting for extra uncertainties.

Besides, it is noted that the Bayesian inference is well regarded for incorporating available information on phenomena under investigation (Kruschke, 2015). That means that the outcomes of the Bayesian inference can be employed as valuable prior information for a future work and a useful knowledge base for investigations on viscoelastic materials and devices updated continuously.

5. Conclusion

The present work used a typical viscoelastic material characterized via the Bayesian inference in passive vibration control design actions. Two control designs were performed in a single degree-of-freedom system, aiming at vibration isolation and vibration neutralization, respectively. In both designs, the dimensions of the corresponding resilient elements were calculated.

It was observed that both in the isolator and neutralizer designs, the dimensions exhibited variation due to temperature and uncertainties. It was also shown how the behavior of chosen designs could be predicted in the face of temperature changes and uncertainties in the dynamic properties.

It can be then concluded that the uncertainties associated with the dynamic properties of a viscoelastic material studied in vibration control designs is better accounted for through the Bayesian paradigm of inference. Those uncertainties are properly propagated, so that a more realistic assessment can be performed when obtaining prediction curves. That provides more information for the designer to anticipate eventual corrective measures.

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