

VIBRATION TRANSIENTS OF RESERVOIR-PIPE-VALVE SYSTEM CAUSED BY WATER HAMMER

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A severe oscillation, accompanied with an abnormal “click” sound, of a fuel feeding pipe system during valve closing, when the feeding flowrate reaches a certain value, is observed experimentally. A fluctuation model in which stiffness and damping coefficients of the vibration system are time varying is proposed. Each coefficient is composed of two parts, one of which is constant and the other is time varying. Based on this model, simulation transients of the vibration displacement, velocity and pressure in the pipe are presented. Simulations of the pressure transients are compared with experimental data detected by pressure transducer, which shows that both have fluctuations in the transient process at a large flowrate.

Keywords: reservoir-pipe-valve system (RPV system), water hammer, vibration, transients

1. Introduction

In a reservoir-pipe-valve (RPV) system, water hammer (WH) will be generated during valve closing, which may affect the system stability or even produce undesired damage. The essence of this phenomenon is the conversion of liquid kinetic energy into its elastic potential energy which, for an elastic or weakly compressible liquid, may produce significant pressure variation. This variation propagates in the pipeline as an elastic wave. The simplest model is that the system structure is a rigid body and the fluid in the pipeline is an elastomer, and WH effect produces an elastic wave in the pipeline (Wylie *et al.*, 1993). Theoretically, partial differential equations (Wang and Eat Tan, 1997; Tijsseling, 2003; Yang *et al.*, 2004) are used to describe the pipe flow in which friction (Zarzycki *et al.*, 2011), Poisson’s (Wiggert and Tijsseling, 2001) and water hammer effects (Henclik, 2015) are generally included. Ferras *et al.* (2018) investigated the fluid-structure interaction in a pressurized fluid-filled pipe. Yang *et al.* (2017) developed a model resolving the pressure response of a pneumatic brake circuit with the effect of a transmission pipe. Meng *et al.* (2019) tried to apply graphic processing unit parallel computing to simulate hydraulic dynamics in large-scale water supply systems. Jiang *et al.* (2018), considering both the

normal and cavitation condition, developed the finite difference method to handle pressure transients. For unsteady pipe flow simulation, especially for fast transients, time varying wall shear stress have to be taken into account. Zarzycki and Urbanowicz reported a series of important works about the unsteady pipe flow. They proposed methods to confirm coefficients of weighting functions in a simple way (Urbanowicz, 2017), approximate weighting functions as sums of exponential components (Urbanowicz and Zarzycki, 2012) and improved the lumping friction model estimating the basic parameters (Urbanowicz and Zarzycki, 2015). They also simulated transients of a turbulent pipe flow (Zarzycki *et al.*, 2007) and a cavitating liquid flow (Urbanowicz *et al.*, 2012; Urbanowicz and Zarzycki, 2008; Zarzycki and Urbanowicz, 2008). In their works, the model of time-varying resistance was adopted for simulation of transient phenomena of unsteady flow. The instantaneous wall shear stress is a sum of two parts, one of which is related with quasi-steady and the other with unsteady stresses. The time varying component in the stress is derived as the convolution of liquid local velocity changes and a weighting function. WH may cause vibration of the RPV system during the valve closing, although the pipeline system is fixed by supports. The supports may be elastic (Adamkowski *et al.*, 2017) or viscoelastic (Henclik, 2018b). These supports have different characteristics and affect system vibrations (Covas *et al.*, 2005; Keramat *et al.*, 2012; Zanganeh *et al.*, 2015). In a majority of studies on vibrations of RPV systems, the pipe is modeled as a straight line and the supports distributed regularly. Bettaieb *et al.* (2019) developed a numerical model which combined the Kelvin-Voigt model and Vitkovsky formulation to obtain the response of the pipeline under transient events. Recently, a new model (Henclik, 2018a) was proposed, where the supports were replaced by a spring and a damper connected with a valve. Dong *et al.* (2019) analyzed the overall strain behavior of a buried pipeline subjected to an impact load.

In the present experiment, the RPV system is designed as an aircraft fuel feeding. The pipeline supports distribute irregularly owing to the limitation of space. A severe oscillation of the system while valve closing is observed. In the case of a lower feeding flowrate, the intensity of vibration is relatively small and its attenuation is regular, almost similar to that of a harmonic oscillator. However, when the feeding flowrate reaches a certain value, the system vibration is violent and, meanwhile, an abnormal “click” sound is accompanied. The transient process is no longer regular and fluctuations appear. In order to investigate this abnormal vibration, we propose a fluctuation model in which the stiffness and damping coefficients of the vibration system are not constants. But each of them is composed of two terms, one of which is constant and the other is time varying. The time varying coefficient is used to describe the fluctuation in the transient process. Based on the model, we present simulation transients of the vibration displacement and velocity. The pressure transient in the pipe is also presented. The simulation of the pressure transients are compared with the experimental data detected by a pressure transducer.

2. Experiments

2.1. Description of the experimental setup

A diagram of the RPV system used in an experiment is illustrated in Fig. 1. It can be classified as two parts. The first part consists of a fuel delivery pump (1), filter (2), throttle valve (3), safety valve (4), pressure transducer (5), switch valve (6) and a flowmeter (7). The second component includes a part of the fuel transfer pipeline, directional valve (8) and a fuel consumption unit (9). The aim of the RVP system is originally designed for feeding the fuel to the consumption unit. In order to save the experimental cost, we turn off the fuel transmission to the consumption unit (9) with the valve (8). The pipeline of the central part is made of aluminum alloy and its length, diameter, and thickness are 8.7 m, 30 mm, and 3 mm, respectively. The

pipeline is fixed on the base with irregularly spaced supports. The fluid used in the experiment is aviation kerosene RP-5. The air content and the viscosity coefficient of the fuel in 20°C are 0.011 g/mol and 1.31 mPa·s, respectively. The delivery rate can be adjusted by the gear fuel pump (1) (KCB) and the throttle valve 3, and measured by the flowmeter (7) (model: LWGY-25A, range: 0.5-5 m³/h, deviation: ±1%). The delivery pump is not stopped during the valve closing. The average pressure in the pipe after the valve closed is preset. The increase of the average pressure is drained by the adjustable throttle valve (3). A pressure transducer (5) (model: MEAS U5300, range: 0-70 bar, deviation: ±0.5%) near the valve is used to monitor the fuel pressure in the pipe which connects with a data acquisition card (NIUSB-4431). The data collected by the card, with *A/D* conversion, is input to the computer.

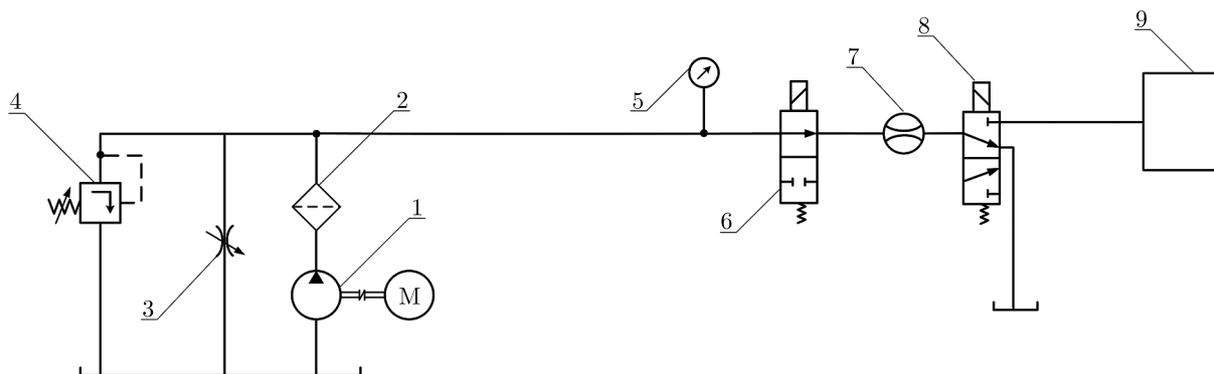


Fig. 1. A scheme of the RPV system used in the experiment: 1 – gear fuel pump, 2 – filter, 3 – throttle valve, 4 – safety valve, 5 – pressure transducer, 6 – switch valve, 7 – flowmeter, 8 – directional valve, 9 – fuel consumption unit

2.2. Experimental results

The fuel starts to flow through the pipeline when the delivery pump turns on. The flowrate is controlled by the pump. In the experiment, the flowrates are set at 0.3 m/s (12 L/min) and 0.8 m/s (56 L/min), respectively. In each case the delivery is constant. After the delivery pump runs smoothly, the computer executes the valve closing command. The closing command has a 5 seconds delay time, i.e. after 5 seconds of the command, the valve starts closing. The valve can be closed within 10 ms. In both cases, as the valve starts closing, the pipeline and valve begin to oscillate. After a short period of time of the oscillation, the system is stable again. In the case of a flowrate at 0.3 m/s, the vibration intensity is relatively small. However, in the case of a flowrate at 0.8 m/s, the oscillation appears violently and randomly. Meanwhile, an abnormal “click” sound is accompanied. The vibration continues for a period of time which is within 20 seconds and the system restores the stable state at 0.7 MPa of the preset pressure.

The transducer installed near the valve reads the pressure transient in the pipe. Figure 2 depicts the pressure transients recorded by the transducer in which curves (a) and (b) are corresponding to the flowrates 0.3 m/s and 0.8 m/s. Both curves in Fig. 2 show that the pressure starts oscillation during the valve closing and, at the beginning, the pressure rises rapidly for a very short period of time. But their increase ranges are completely different. The pressure increases from 0.25 MPa to 0.9 MPa in the first case, and the range is from 0.3 MPa to 2.6 MPa in the second. The pressure vibration attenuates from their maximum to the preset stable state 0.7 MPa in both cases. The decay time in the first case is shorter than that in the second case. The most important difference is the feature of the vibration decay process. In the first case, the decay process is similar as the classic exponential type. However, fluctuations appear in the second case, which are random, flickering and with a large variable change range.

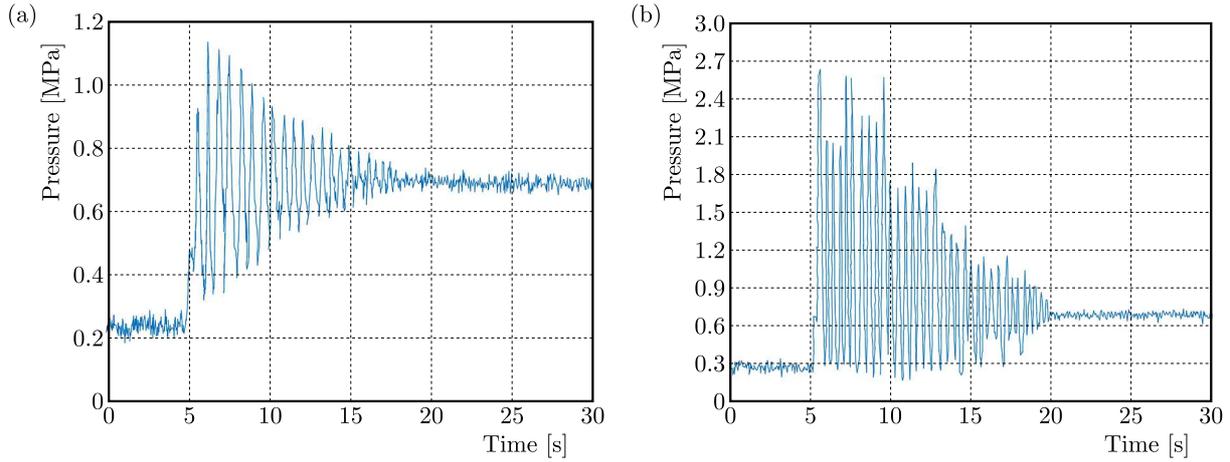


Fig. 2. Pressure transients in the pipe during the valve closing: (a) $v_0 = 0.3\text{ m/s}$, (b) $v_0 = 0.8\text{ m/s}$

3. Theoretical model

3.1. Physical model of RPV system

The physical model of the RPV system is illustrated in Fig. 3. The output of the fuel pump is defined as the starting point, i.e. $x = 0$, the position of the valve is at $x = L$, and velocity of the fuel in the pipeline is $v(x, t)$. In order to investigate the water hammer for the PRV system, the support is simplified as a spring and damper which are connected to the valve node. Suppose α and b represent the coefficients of stiffness and damping, respectively. When the flowrate in the pipe is low, both coefficients can be considered as constants. However, when the flowrate is large, the water hammer effect during valve closing may be strong and, in this case, the coefficients of stiffness and damping cannot be considered as constants. Because the pipeline supports distribute irregularly, the fluctuation easily appears in the system at a large flowrate. Under this circumstance, the coefficients of stiffness and damping are supposed to be as $\alpha_0 + \Delta\alpha(t)$ and $b_0 + \Delta b(t)$ where α_0 and b_0 are constants as in classic theory, and $\Delta\alpha(t)$, $\Delta b(t)$ are time-varying. The time variables $\Delta\alpha(t)$ and $\Delta b(t)$ are introduced to describe fluctuations in the system.

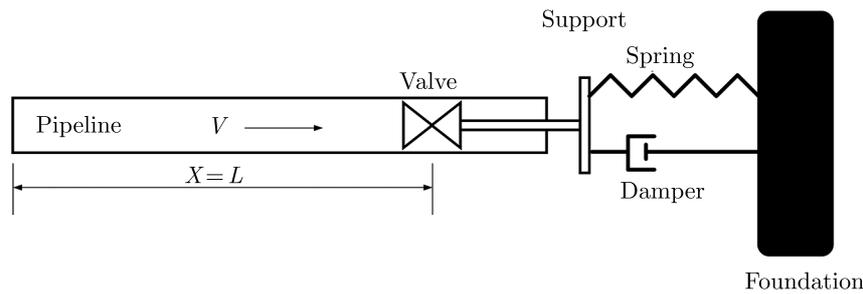


Fig. 3. Physical model of the RPV system

3.2. Mathematical model of the pipeline flow

The basic fluid-solid coupling equations are shown as below (Wiggert and Tijsseling, 2001)

$$\begin{aligned}
 \frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial x} &= -g \sin \gamma - \frac{4\tau_s}{\rho D} & \frac{\partial v}{\partial x} + \frac{1}{\rho c^2} \frac{\partial P}{\partial t} &= 2\mu \frac{\partial w}{\partial x} \\
 \frac{\partial w}{\partial t} + \frac{1}{\rho_s} \frac{\partial \sigma}{\partial x} &= -g \sin \gamma + \frac{\tau_s}{e\rho_s} & \frac{\partial w}{\partial x} + \frac{1}{\rho_s c_s^2} \frac{\partial P}{\partial x} &= -\frac{\mu D}{2Ee} \frac{\partial P}{\partial t}
 \end{aligned}
 \tag{3.1}$$

The left hand side of equations (3.1)_{1,2} represents the fluid parameters of the system. The left hand side of equations (3.1)_{3,4} represents the pipeline. The right hand side of these equations describe the fluid-structure interaction (FSI), where x is the positional variable in the pipeline, c is the speed of sound in the fluid, t is time, v is fuel velocity, P is fluid pressure, w is pipe section velocity, σ is longitudinal stress, ρ is fuel density, ρ_s is density of the pipeline, E is the Young modulus of the pipeline, μ is the Poisson coefficient, τ_s is frictional stress of the pipeline wall, g is the gravitational acceleration, γ is the angle between the pipeline axis and the horizontal plane, e is thickness of the pipeline, and D is inner diameter of the pipeline.

3.3. Vibration model of RPV system

As the valve is closing, pressure of the water hammer acts on the pipeline system. Energy exchanges between the fuel and the structure, which causes vibrations. According to the physical model illustrated in Fig. 3, the system can be simplified as a harmonic oscillator. The oscillator is located at the node of the valve. In essence, vibration of the pipeline system is caused by pressure variation of the fluid acting on the valve node. The pressure variation of the fluid results from its energy conversion between the kinetic and potential form. Assuming that, the water hammer makes the valve node move with a distance y . When the node moves to the maximum distance y_0 , the valve velocity is zero, and the elastic potential of the RPV system reaches its maximum. Thereafter, the pipeline system generates vibrations under the action of elastic restoring, damping, and water hammer forces. The resilience of the system is defined as follows

$$F_r = -\alpha y \quad (3.2)$$

where α is the stiffness coefficient, and y is expansion length of the spring, which is equal to the displacement of the valve.

Assuming that the fuel initial flowrate is v_0 , then, the kinetic energy of the water hammer is $m_w v_0^2/2$, where m_w is mass of the fuel in the pipeline. When the valve moves to the maximum distance y_0 , the potential energy of the system is $-\alpha y_0^2/2$. According to the law of energy conservation, we obtain

$$\alpha y_0^2 = m_w v_0^2 \quad (3.3)$$

Thus, the vibration amplitude y_0 can be obtained according to this equation (energy dissipation in motion is neglected here).

The damping force is

$$F_d = -b \dot{y} \quad (3.4)$$

where b is the damping coefficient, and \dot{y} is the valve velocity.

Considering the restoring force, damping force and the excitation force caused by WH, the dynamic equation of the pipeline system is

$$m \ddot{y} + b \dot{y} + \alpha y = (P - P_0) S \quad (3.5)$$

where m is mass of the RPV system, S is the cross-sectional area of the pipeline, P_0 is the fuel pressure before the valve closing, P is the fuel pressure when the valve starts to close.

For the pipeline with a given section, the larger the initial speed of the fluid, the larger its kinetic energy, and the stronger the water hammer. In the case of a large flowrate or a strong water hammer, according to the fluctuation model established in this paper, equation (3.5) becomes

$$m \ddot{y} + [b_0 + \Delta b(t)] \dot{y} + (\alpha_0 + \Delta \alpha) y = (P - P_0) S \quad (3.6)$$

From the physical point of view, this is a forced and damped vibration equation. It can also be considered as a force balance equation in which each term (from left to right) represents inertia, damping, restoring and excitation forces. It is different from the classic vibration equation in which coefficient fluctuations of stiffness and damping are considered. The fluctuation reveals randomness of the RPV system with irregular distribution of the supports.

4. Analysis of fluctuation parameters of the system

4.1. Velocity and pressure of fluid in the pipeline

In our experiment, the viscosity coefficient of kerosene RP-5 is 1.31 mPa·s in 20°C, thus, the frictional stress in equation (3.1)₁ is small. The action of the gravitational force is zero in this equation because it is assumed that the pipeline is laid in the horizontal direction. Meanwhile, we mainly pay attention here to the fluid velocity and pressure without considering deformation of the pipeline. So, as in the reference (Henlik, 2018a), we neglect friction and Poisson effects in equations (3.1)₁ and (3.1)₂. Under these conditions and based on equations (3.1)_{1,2}, the equation of fluid velocity in the pipeline can be obtained as follows

$$\frac{\partial^2 v}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 v}{\partial t^2} = 0 \quad (4.1)$$

where c is the wave speed, and

$$c = \sqrt{\frac{K}{\rho} \frac{1}{\sqrt{1 + (1 - \mu^2) \frac{KD}{Ee}}}} \quad (4.2)$$

where K is the bulk modulus of the fluid.

Because the fluid pressure at the output of the pump is constant, the boundary condition of the left hand side of the pipeline is $x = 0$, $\partial v(x, t)/\partial x = 0$. When the valve moves to the maximum position y_0 , the velocity of vibration is zero. At the right end of the pipe, the fluid velocity is as same as that of the valve node. Therefore, the boundary condition at right is $x = L + y_0$, $\partial v(x, t)/\partial x = 0$.

Using the separation variable method to solve equation (4.1), the solution can be expressed as

$$v(x, t) = V(x)T(t) \quad (4.3)$$

where $V(x)$ and $T(t)$ are all univariate functions.

4.2. Excitation force

When the valve starts closing, the relative velocity of the fluid is $v_0 - v(x, t)$. According to the Joukovsky formula, the excitation force acting on the valve caused by WH is

$$(P - P_0)S = \rho c(v_0 - v)S \quad (4.4)$$

The function $v(x, t)$ in equation (4.3) can be expressed as a Fourier series (Dong *et al.*, 2019), where the n -th order component is

$$v_n(x, t) = v_{0n} \cos(k_n x) \cos(\omega_n t) \quad (4.5)$$

and $\omega_n = k_n c$. For simplicity, we only consider the fundamental component in calculation.

Substituting the fundamental component in equation (4.5) into (4.4) and then into equation (3.6), we obtain

$$m\ddot{y} + [b_0 + \Delta b(t)]\dot{y} + (\alpha_0 + \Delta\alpha)y = \rho v_0 c S [1 - \cos(k_0 L) \cos(\omega_0 t)] \quad (4.6)$$

The initial conditions of the equation are $t = 0$, $y = y_0$, $\dot{y} = 0$.

4.3. Pressure in the pipeline

Ignoring the friction and Poisson effects of the system, according to equation (3.1)₂, the relation between the velocity and pressure in the pipeline is

$$\frac{\partial v(x, t)}{\partial x} + \frac{1}{\rho c^2} \frac{\partial P(x, t)}{\partial t} = 0 \quad (4.7)$$

The fluid pressure in the pipeline can be obtained by integrating equation (4.7), and we have

$$P(x, t) = P(x, 0) - \rho c^2 \int_0^t \frac{\partial v(x, t)}{\partial x} dt \quad (4.8)$$

where $P(x, 0)$ is the initial pressure of the fluid in the pipeline. Assuming that, the fluid velocity $v(x, t)$ at the valve input is equal to the valve node vibration velocity \dot{y} . Based on equation (4.5), we have

$$\frac{\partial v(x, t)}{\partial x} = k \tan(kL) \dot{y}(t) \quad (4.9)$$

where k is the wave vector and $k = \omega/c$, c is the wave speed. Suppose the liquid and the valve vibrate at the valve node synchronously. Thus, in the case of non-fluctuation, $\omega = \sqrt{\alpha_0/m}$ and then

$$k = \frac{1}{c} \sqrt{\frac{\alpha_0}{m}} \quad (4.10)$$

Substituting equations (4.9) and (4.10) into equation (4.8), we obtain

$$P(t) = P(0) + \rho c \sqrt{\frac{\alpha_0}{m}} \tan(kL) [y_0 - y(t)] \quad (4.11)$$

5. Results and discussion

In the numerical calculation, the parameters of the line were as follows: pipe length $L = 8.7$ m, inner diameter $D = 30$ mm, thickness $e = 3$ mm, bulk modulus $E = 69$ GPa, density $\rho_s = 2.73 \cdot 10^3$ kg/m³. The working liquid is kerosene RP-5 with density $\rho = 0.78 \cdot 10^3$ kg/m³ and the bulk modulus $K = 1.36$ GPa. The wave speed is $c = 1324$ m/s, mass $m = 9.3$ kg and is estimated as the sum of masses of the fluid, pipe and the valve. The stiffness and damping coefficients are $\alpha_0 = 274$ N/mm and $b_0 = 54$ Ns/m. The values of stiffness and damping coefficients are estimated with the formula $\alpha_0 = 4\pi^2 [c/(4L)]^2 m_w$ and $b_0 = 2\xi \sqrt{K m_w}$, where m_w is mass and ξ is a non-dimensional damping degree ($0 < \xi < 1$) (Henclik, 2018a). There are two cases in the numerical calculation. In the case of a flowrate at 0.3 m/s, the stiffness and damping coefficients are constants respectively, i.e. $\Delta\alpha(t)$ and $\Delta b(t)$ in equation (4.6) are zero. Using the Runge-Kutta method to calculate equation (4.6), solutions $y(t)$, $\dot{y}(t)$ can be obtained. Then, substituting the solution $y(t)$ into equation (4.11), the pressure $P(t)$ on the valve is obtained.

In the second case of a flowrate at 0.8 m/s, fluctuations of the stiffness and damping coefficients $\Delta\alpha(t)$ and $\Delta b(t)$ should be considered. Usually, the fluctuation can be mathematically described as a pulse function which is expressed as a Fourier series, e.g. the pulse delivery of the pump is taken as a sum of Fourier components (Zarzycki *et al.*, 2017). Our aim is mainly to explain the fluctuation and, in simple calculations, we only consider the fundamental frequency in the pulsation. So, we assume the time varying stiffness and damping $\Delta\alpha(t)$ and $\Delta b(t)$ as

$$\Delta\alpha = \alpha_0 \cos(\omega_f t) \quad \Delta b = b_0 \cos(\omega_f t) \quad (5.1)$$

where ω_f is the frequency related with fluctuation. In calculations, we suppose that $\omega_f = \omega_0/5$. This assumption means that one fluctuation appears for five WH oscillations.

Substituting equation (5.1) into equation (4.6), the vibration equation of the system under fluctuation conditions is obtained as

$$m\ddot{y} + b_0[1 + \cos(\omega_f t)]\dot{y} + \alpha_0[1 + \cos(\omega_f t)]y = \rho v_0 c S [1 - \cos(k_0 L) \cos(\omega_0 t)] \quad (5.2)$$

According to equations (5.2) and (4.6), the displacement $y(t)$, velocity $\dot{y}(t)$, and pressure $P(t)$ of the system under the fluctuation conditions are calculated.

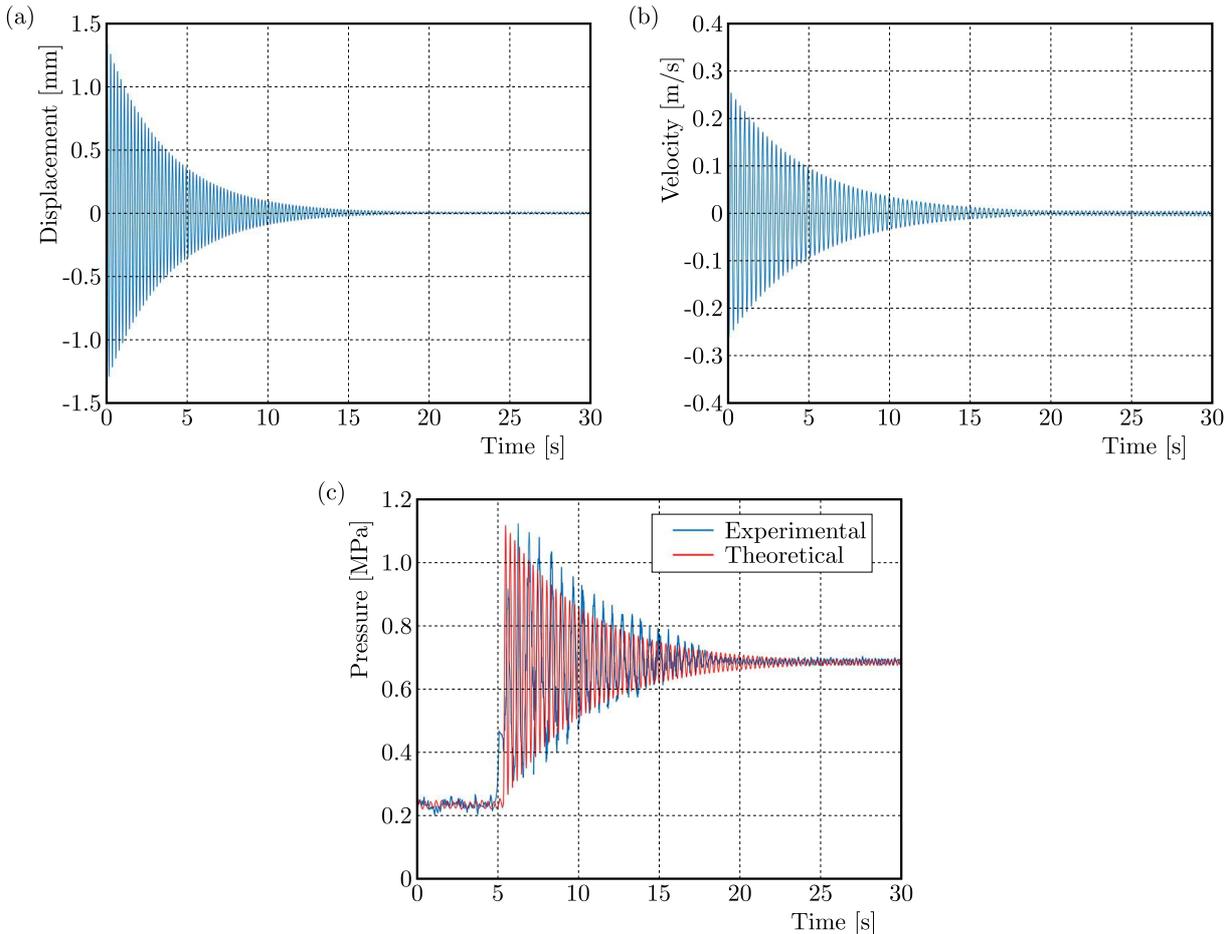


Fig. 4. Vibration of the system and the pressure transient during the valve closing $v_0 = 0.3$ m/s

Figure 4 depicts numerical calculation results for vibrations of the pipeline under a weak water hammer effect. Figures 4a and 4b are corresponding to the transients of vibration displacement and velocity, respectively. It can be seen from them that the pipeline system generates vibrations when the valve is closing. The initial amplitudes are determined by strength of the water hammer effect. Both vibration amplitudes of displacement and velocity attenuate. And the attenuation tendency is approximately exponential. The attenuation speed is related with the damping coefficient. After a period of time, the system tends to a stable state. Figure 4c shows the liquid pressure transients at the valve in which the red curve is simulation and the blue one is experimental data detected by the pressure transducer. Both of them show that the pressure in the pipe exhibits vibration when the valve is closing, and the vibration attenuates nearly exponentially and finally tends to the stable state. The fluctuations among the vibration transient processes are completely not observed in theoretical curves and almost not presented in the experimental data. The experimental data shows that the pressure vibration amplitude

increases rapidly during a short period of time at the beginning, and reaches maximum and then attenuates. However, this rising edge does not appear in the theoretical curve. It is also observed that pressure in the experimental data is increasing from 0.25 MPa at the beginning, but it starts to decrease from the central value 0.7 MPa. In addition, the theoretical vibration transient is completely symmetrical. These differences between the theoretical and experimental result from theoretical assumptions: (i) the valve is closed instantaneously, (ii) the initial value of pressure is supposed at 0.7 MPa and (iii) the expression of vibration equation (4.6) has a symmetrical feature.

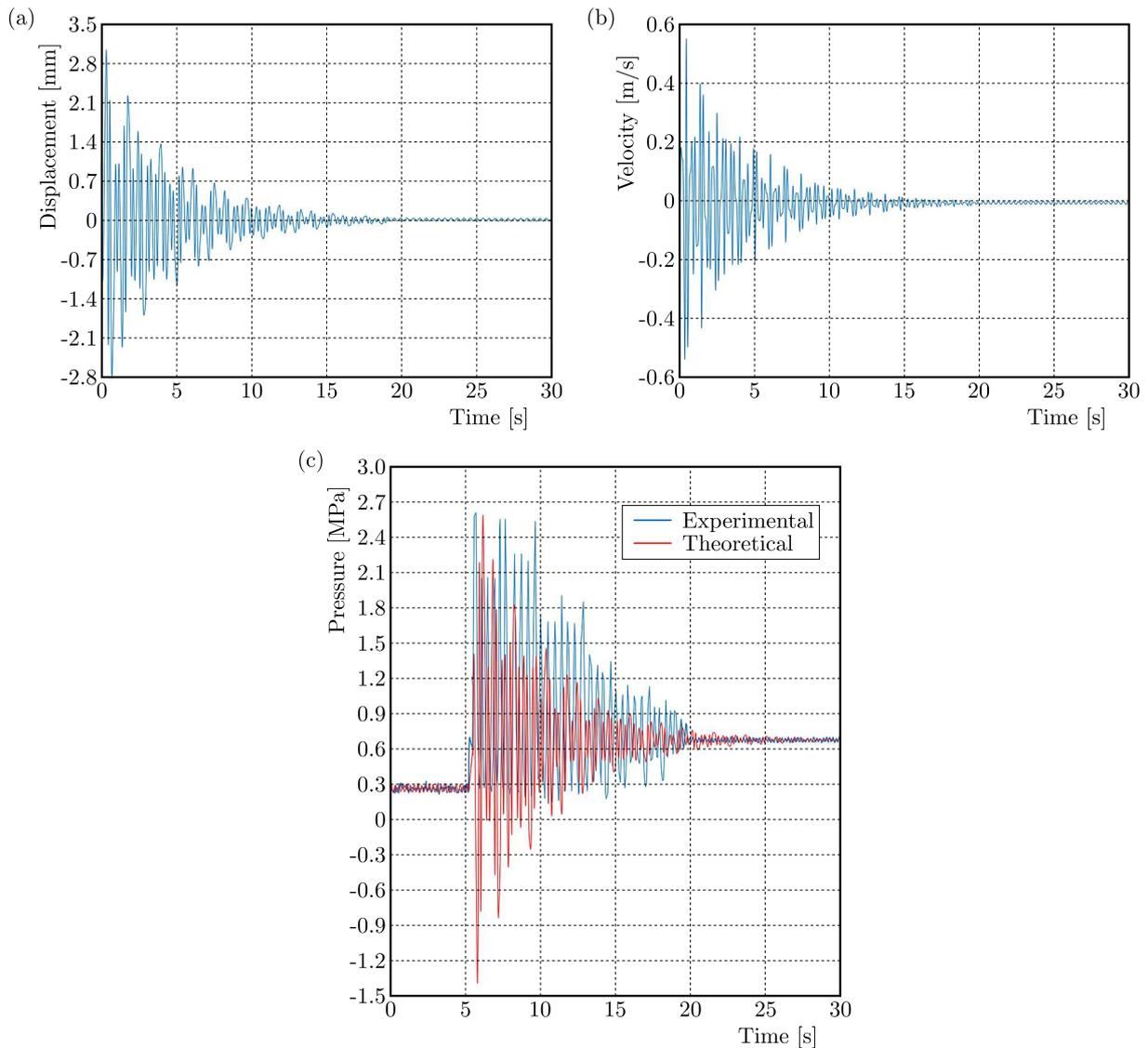


Fig. 5. Vibration of the system and pressure transient in the pipe during the valve closing $v_0 = 0.8 \text{ m/s}$

Figure 5 depicts simulation of the system vibration under a strong water hammer effect with the fluctuation model. Figures 5a and 5b correspond to the transients of vibration displacement and velocity, respectively. It is observed from them that the pipeline system generates vibrations when the valve is closing. The vibration amplitudes of displacement and velocity, on the overall trend, attenuate from large to small with time and, finally, tend to stable states. But the attenuation tendency is not exponential, and fluctuations do appear in the transient process. Figure 5c shows liquid pressure transients in the pipe at the valve, in which the red curve is simulation and the blue one is experimental data detected by the pressure transducer. Both

of them verify that the pressure vibrates during valve closing and the pressure change range is much larger, which is different from that in Fig. 4c. The experimental data shows that, at the beginning, the pressure increases rapidly from 0.3 MPa to 2.6 MPa during a short period of time, but this rising edge is not visible in the theoretical curve. The pressure oscillation attenuates with time and finally tends to a stable state at 0.7 MPa. However, the damping process is not completely exponential. The most important facts are that the fluctuations in the transient process are not only observed in experimental data, but also reflected in the simulation curve. Perhaps, the appearance of the abnormal click sound is related with these fluctuations. It should be pointed that the practical pressure fluctuations in the experiment are random, impulsive and the pressure oscillation is asymmetrical. But they are periodical, less violent and the vibration is symmetrical in the theoretical prediction curve. Especially, the minimum pressure of the simulation curve is in an unreasonable range. Maybe, these defects result from other physical factors existing in extreme cases, which affect pressure of the pipe flow. For example, when pressure falls to a very low value equal to vapor pressure, cavitations will occur in an unsteady pipe flow. By taking into account unsteady friction loss in a transient cavitating pipe flow, (Urbanowicz and Zarzycki, 2008; Zarzycki and Urbanowicz, 2008) showed that great pressure changes are accompanied with the cavitations. Another obvious reason is that the time varying coefficients of stiffness and damping are assumed to be harmonic oscillation functions in order to simplify calculation. We will try to use an impulsive random sampling function instead of the harmonic and flow cavitations in the low pressure range to modify the model in the future work.

In addition, comparison of double experimental pressure oscillations in Fig. 2 shows that there is some frequency difference. The frequency in Fig. 2a is slightly smaller than that in Fig. 2b. Based on the fluctuation model, the stiffness is not a constant and its value is related with the flowrate. The phenomenon of the system affected by the input is non-linear. Henclik investigated the relation between the frequency of pressure oscillation and stiffness of the spring attached to the valve. His result (Henclik, 2018a) shows that the frequency difference of pressure oscillations in the pipe is 0.8 Hz when the stiffness difference is 102 N/mm.

6. Conclusions

- Vibration transients of the RPV system are experimentally studied. A severe oscillation of the fuel feeding the RPV system with irregular spaced supports is observed and pressure transients in the pipe are detected experimentally at the moment of valve closing. In the case of a lower feeding flowrate, the range of pressure changes is relative small and the oscillation decay is relative regular. However, when the feeding flowrate reaches a certain value, the system vibration becomes violent. Meanwhile, a series of abnormal “click” sounds are accompanied. The fluctuation phenomena appear in the detected pressure data.
- A fluctuation model for simulation of the RPV system vibrations is proposed. In the model, irregular spaced supports are equivalent to a spring and a damper, and time varying terms are introduced into the coefficients of stiffness and damping. Each coefficient is composed of two terms in which one is constant as in the classic description and the other is time varying to describe the fluctuation in the vibration process.
- A new dynamic vibration equation is established based on the fluctuation model, the expressions for damping and restoring forces are modified. Using these modified damping and restoring forces as well as the excitation force caused by WH effect, a new vibration dynamic equation is arrived at. And then, an expression for pressure in the pipe is derived.
- Numerical results from the vibration equation of the RPV system are calculated by the Runge-Kutta method. Simulation transients of the vibration displacement and velocity as well as simulation of pressure transients at lower and larger flowrates are obtained, respectively.

- Simulations of pressure transients are compared with experimental data detected by a pressure transducer. Both results show that pressure in the pipe exhibits vibration during valve closing. The oscillation attenuates with time and finally tends to a stable state. However, pressure damping transients are completely different at lower and larger flowrates. Both experimental and theoretical results show that the pressure vibration attenuates near-exponentially in the former case. Yet in the latter case, both the experiment and simulation show that the fluctuations appear in decay processes although their features have some differences.

In this work, the time varying coefficients in the model are assumed as a simple harmonic function for simplicity of calculation. This assumption may result in defects of the simulation. We will take into account cavitation in the case of a low pressure unsteady flow and use a more reasonable function replacing the harmonic assumption to modify the model in the future work.

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