

TIMOSHENKO BEAM MODEL FOR VIBRATION ANALYSIS OF COMPOSITE STEEL-POLYMER CONCRETE BOX BEAMS

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The free vibration model of a steel-polymer concrete beam based on Timoshenko beam theory is presented in this paper. The results obtained on the basis of the model analysis, describing values of the natural frequencies of the beam, were compared with the results obtained by the solution of the model formulated on the basis of the classical Euler-Bernoulli beam theory, the finite element model and the results of experimental studies. The developed model is characterized by high compliance with the experimental data: the relative error in the case of natural vibration frequencies does not exceed 0.4%, on average 0.2%.

Keywords: Timoshenko beam, Euler-Bernoulli beam, free vibration, composite beam, finite element model

List of used symbols

E_i, G_i – Young’s and shear modulus [Pa]
 k – shear correction factor
 I_i, I_{pi} – area and polar moment of inertia [m⁴]
 A_i – cross section area [m²]
 w – transverse deflection [m]
 l – length of beam [m]
 K_i – torsional parameter [m²]
 H, h – outer and inner height of beam cross section [m]
 ρ – mass density [kg/m³]
 i – material: steel (1), polymer concrete (2)

1. Introduction

Dynamic properties constitute one of the fundamental aspects considered during design of mechanical structures. Their evaluation is particularly important when a structure is exposed to time-varying loads that give rise to vibrations of different nature. In the vast majority of mechanical structures, generation of excessive vibration is undesirable and adversely affects their operating properties (Jasiewicz and Miądlicki, 2019; Marchelek *et al.*, 2002; Pajor *et al.*, 1999; Wojciechowski *et al.*, 2019). Therefore, at the design stage of mechanical structures, the aim is to shape their structure in such a way as to ensure the best possible resistance to vibrations.

Mathematical models on the basis of which dynamic properties of mechanical structures can be evaluated are a tool supporting designers in conscious shaping of mechanical structures. The latest scientific research is aimed at preparing new and developing existing modeling methods to reliably assess static and dynamic properties of composite structural elements at the design stage (Rakowski and Guminiak 2015; Roque *et al.*, 2013; Batra and Xiao 2013; Marczak and

Jędrysiak, 2015; Grygorowicz *et al.*, 2015; Domagalski and Jędrysiak, 2016; Magnucka-Blandzi *et al.*, 2017; Magnucka-Blandzi, 2018; Kurzawa *et al.*, 2018; Mackiewicz *et al.*, 2020).

Berczyński and Wróblewski (2005) present models enabling description of free vibrations of composite steel and concrete beams, consisting of a steel I-section combined with a concrete slab by means of special stud connectors. The paper presents two models based on the classical Euler-Bernoulli beam theory (taking into account deformability of the stud connection and treating it as rigid) and a model built according to Timoshenko beam theory taking into account stiffness of the junction. The developed models were then subjected to experimental verification on the basis of which it was concluded that the model based on the Timoshenko beam theory best described dynamic properties of the considered steel and concrete beams. In the case of the Timoshenko model, the relative error determined for values of natural vibration frequencies did not exceed 5.4%, on average 3.3%. In the case of models based on the classical Euler-Bernoulli beam theory, the relative errors for the natural vibration frequencies corresponding to higher mode shapes reached even 50%.

Chakrabarti *et al.* (2012) presents a model of a composite beam built using the finite element method and using higher order beam theory considering the shear of beams. The proposed model took into account the influence of partial shear interaction between the adjacent layers of the composite, as well as transverse shear deformations of the beams. The values of natural vibration frequencies obtained based on the analysis of the developed model were compared with the results available in literature, obtaining a high compatibility (relative error not exceeding 2.82%) with the models presented in (Huang and Su, 2008; Xu and Wu, 2008).

Huang *et al.* (2019) presents a finite element model for the analysis of layered beam vibrations with a core of a viscoelastic material placed between two elastic layers. The construction of the model was carried out using the finite element method. A seven-parameter Biot model was used to describe viscoelastic properties depending on the frequency. After the model parameters were identified, calculations were carried out to determine natural vibration frequencies and corresponding loss factors. Comparing the calculation results with the results of experimental studies, a relative error for natural vibration frequencies not exceeding 3.97%, on average 3.62%, and in the case of the loss factor, a relative error not exceeding 4.53%, on average 3.66%, were obtained.

To sum up, the development and related increase in the number of industrial implementations using new composite construction materials, which for years have been used mainly in scientific works and prototype machine elements, forces the improvement of calculation models enabling the assessment of dynamic properties of composite structural elements at the stage of their design.

In response to the presented problem, this paper presents a free vibration model of a steel-polymer concrete beam based on the Timoshenko beam theory. The natural vibration frequencies obtained based on the model were compared with the results obtained in the course of experimental studies, obtaining a high compatibility. Additionally, the developed model was compared with the model formulated on the basis of Euler-Bernoulli's classical beam theory and the model built on the finite element method convention, demonstrating superiority of the model over the others under analysis. The paper also proved that it is possible to build a model of a steel-polymer concrete beam using equivalent flexural stiffness.

2. Steel-polymer concrete beam concept

The subject matter of the study presented in this paper is a steel-polymer concrete beam being the basic structural component of, among others, structures presented in (Dunaj *et al.*, 2019, 2020). The structure of the beam is based on synergic use of properties of two materials: steel –

ensuring the assumed stiffness of the structure and polymer concrete – increasing its capacity to dissipate vibration energy. By properly selecting the arrangement and the degree of filling of the profiles, it is possible to intentionally influence the resulting dynamic properties of the structure composed of such beams. Beams with appropriate cross-sections are selected depending on the requirements for structures consisting of steel-polymer concrete beams. The paper contains an analysis of a beam consisting of a steel profile with length of 1000 mm, square cross-section with dimensions of 70×70 mm and wall thickness of 3 mm, which was filled with polymer concrete.

The filling polymer concrete consists of epoxy resin and mineral filling of various sizes: ash, fine fraction – consisting mainly of sand of 0.25-2 mm grain size, medium fraction of 2-10 mm grain size and coarse fraction of 8-16 mm grain size (Dunaj *et al.*, 2020). The coarse and medium fraction was mainly gravel with irregular grain shape. The percentage mass share of individual fractions in the polymer concrete used is presented in Table 1. The concept of a steel-polymer concrete beam is presented in Fig. 1.

Table 1. Composition of the applied filling polymer concrete

	Epoxy resin	Ash	Fine fraction (0.25-2 mm)	Medium fraction (2-10 mm)	Coarse fraction (8-16 mm)
Percentage share of component weight	15%	1%	19%	15%	50%

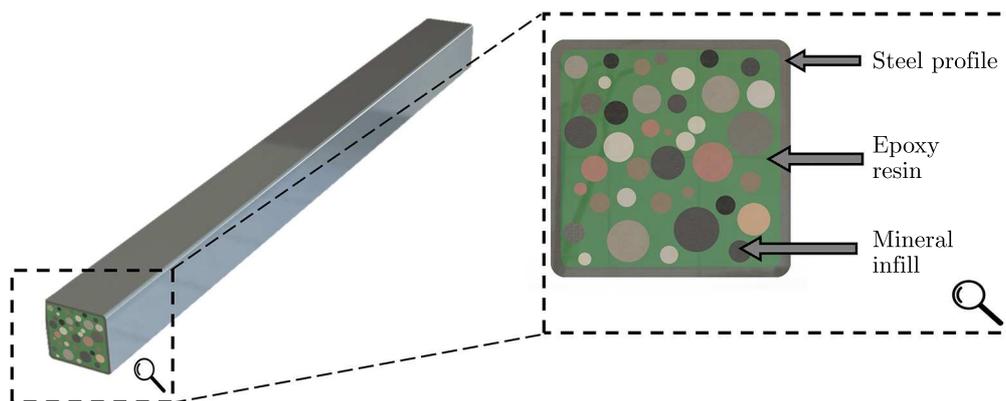


Fig. 1. Steel-polymer concrete beam concept

3. Modeling of steel-polymer concrete beams

On the basis of observations resulting from experimental studies, simplifications were adopted, which made it possible to build a mathematical model of the beam. It was assumed that in order to describe the dynamic properties of polymer concrete, despite its heterogeneous structure, a linear elastic model of the material could be used. The basis for this assumption were the results of experimental studies: fulfilment of the Maxwell principle of reciprocity, linear rigidity characteristics and symmetry of mode shapes determined based on an impulse test. The experimental studies are presented in detail in (Dunaj *et al.*, 2020).

It was then assumed that contact of the steel profile with the polymer concrete filling occurs on the entire internal surface of the steel profile. In addition, adhesion forces prevent tangential displacement within the contact area of the materials. A schematic representation of the adopted model simplifications is presented in Fig. 2. The use of such a model was dictated by the analogy between the beam in question and the concrete filled steel tubes (CFST) used in the civil engineering. When analyzing the studies on the interaction between a concrete filling and a steel

coating (Duarte *et al.*, 2016; Li *et al.*, 2018), it was noticed that both the steel profile and the concrete filling transfer the load behaving as a structure made of a homogeneous material.

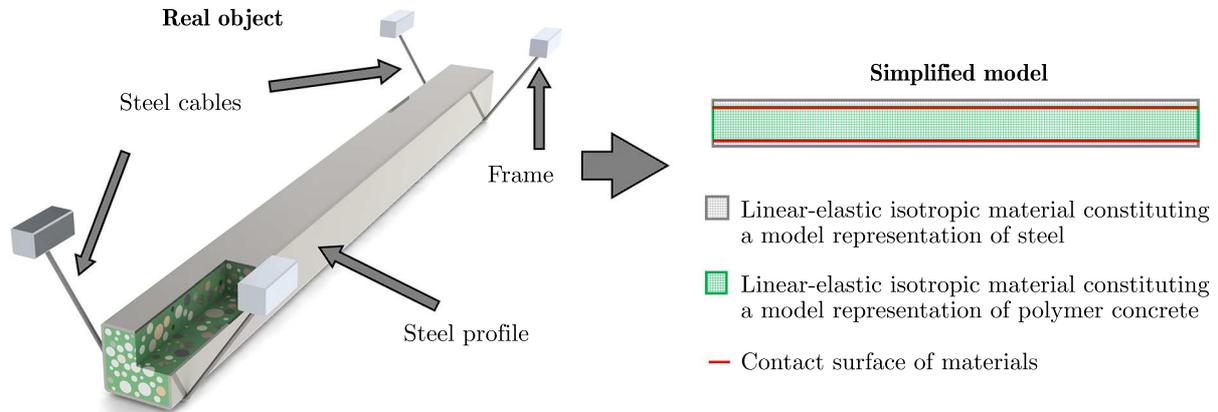


Fig. 2. Simplifications applied at the stage of mathematical modeling for the steel-polymer concrete beam model

The material properties describing steel and polymer concrete were defined based on experimental results – a static compression test carried out on Instron 8850 testing system and an impulse test. The results of these studies are presented in Table 2.

Table 2. Material properties of steel and polymer concrete determined based on experimental tests

Property	Steel	Polymer concrete
Young's modulus E	210 ± 5 GPa	17.2 ± 0.2 GPa
Poisson's ratio ν	0.28 ± 0.03	0.20 ± 0.05
Density ρ	7812 ± 35 kg/m ³	2200 ± 6 kg/m ³
Loss factor η	0.00220 ± 0.00005	
Equivalent loss factor	0.00480 ± 0.00024	

3.1. Timoshenko and Euler-Bernoulli beam model

In the first stage of modeling, it was assumed that the steel-polymer concrete beam could move in the XZ plane, Fig. 3.

In connection with the adopted assumption related to the impossibility of mutual movement of the steel profile in relation to the polymer concrete filling, it was assumed that the axes neutral to the steel and polymer concrete beams overlapped, which can be expressed by the following equation

$$u_1 = u_2 = 0 \quad v_1 = v_2 = 0 \quad w_1 = w_2 = w(x, t) \quad (3.1)$$

Therefore, non-zero stresses and deformations can be expressed in the following form

$$\varepsilon_{zx} = \frac{\partial w}{\partial x} \quad \sigma_{zx} = kG \frac{\partial w}{\partial x} \quad (3.2)$$

Next, a mathematical model of a steel-polymer concrete beam based on the Timoshenko beam theory was defined. The Hamilton principle was applied in order to determine the coefficients of the steel-polymer concrete beam equation. Considering the fact that the Timoshenko model

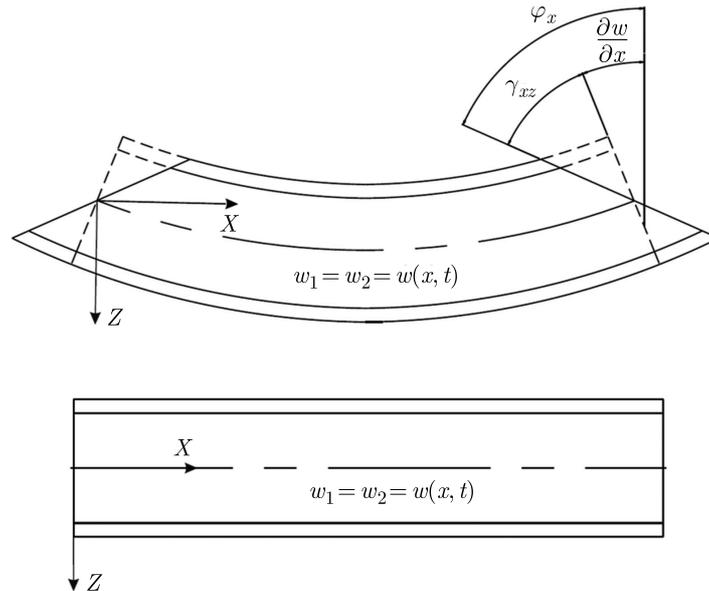


Fig. 3. Timoshenko beam model

takes into account flexural, shear and longitudinal deformations, the potential energy has the following form

$$\begin{aligned} \pi &= \frac{1}{2} \iiint_V (\sigma_{xx}\varepsilon_{xx} + \sigma_{zx}\varepsilon_{zx}) dV \\ &= \frac{1}{2} \int_0^l \left[E_1 I_1 \left(\frac{\partial \varphi}{\partial x} \right)^2 + k A_1 G_1 \left(\frac{\partial w}{\partial x} - \varphi \right)^2 + E_2 I_2 \left(\frac{\partial \varphi}{\partial x} \right)^2 + k A_2 G_2 \left(\frac{\partial w}{\partial x} - \varphi \right)^2 \right] dx \end{aligned} \quad (3.3)$$

However, the kinetic energy taking into account the inertia of the beam resulting from its rotation around its axis takes the following form

$$T = \frac{1}{2} \int_0^l \left[\rho_1 A_1 \left(\frac{\partial w}{\partial t} \right)^2 + \rho_1 I_1 \left(\frac{\partial \varphi}{\partial t} \right)^2 + \rho_2 A_2 \left(\frac{\partial w}{\partial t} \right)^2 + \rho_2 I_2 \left(\frac{\partial \varphi}{\partial t} \right)^2 \right] dx \quad (3.4)$$

In further calculations, the natural vibration frequencies of the considered composite beam were determined assuming that the beam is of length of $l = 1$ m. Therefore, using Hamilton principle (3.5), the following formula for the composite beam was obtained

$$\delta \int_{t_1}^{t_2} (\pi - T) dt = 0 \quad (3.5)$$

and

$$\begin{aligned} \delta \int_{t_1}^{t_2} \int_0^l \left\{ \left[(E_1 I_1 + E_2 I_2) \frac{\partial \varphi}{\partial x} + k (A_1 G_1 + A_2 G_2) \left(\frac{\partial w}{\partial x} - \varphi \right) \right]^2 \right. \\ \left. - \left[(\rho_1 A_1 + \rho_2 A_2) \left(\frac{\partial w}{\partial t} \right)^2 + (\rho_1 I_1 + \rho_2 I_2) \left(\frac{\partial \varphi}{\partial t} \right)^2 \right] - f w \right\} dx dt = 0 \end{aligned} \quad (3.6)$$

Based on obtained formula (3.6), it can be concluded that it is possible to derive equivalent coefficients for a composite beam based on known values of coefficients for individual beams – steel and polymer concrete beams.

Then, using integration through parts and ordering equation (3.6) for a homogeneous beam, the following differential equations of motion are obtained in relation to w and φ

$$\begin{aligned} & -k(A_1G_1 + A_2G_2)\frac{\partial^2 w}{\partial x^2} + k(A_1G_1 + A_2G_2)\frac{\partial \varphi}{\partial x} + (\rho_1A_1 + \rho_2A_2)\frac{\partial^2 w}{\partial t^2} = 0 \\ & - (E_1I_1 + E_2I_2)\frac{\partial^2 \varphi}{\partial x^2} - k(A_1G_1 + A_2G_2)\frac{\partial w}{\partial x} + k(A_1G_1 + A_2G_2)\varphi \\ & + (\rho_1I_1 + \rho_2I_2)\frac{\partial^2 \varphi}{\partial t^2} = 0 \end{aligned} \quad (3.7)$$

After differentiation in relation to x and appropriate substitution, the equation of motion in form of (3.8) is obtained

$$\begin{aligned} & (E_1I_1 + E_2I_2)\frac{\partial^4 w}{\partial x^4} + (\rho_1A_1 + \rho_2A_2)\frac{\partial^2 w}{\partial t^2} \\ & - (\rho_1I_1 + \rho_2I_2)\left[1 + \frac{(E_1I_1 + E_2I_2)(\rho_1A_1 + \rho_2A_2)}{k(A_1G_1 + A_2G_2)}\right]\frac{\partial^4 w}{\partial x^2 \partial t^2} \\ & + \frac{(\rho_1I_1 + \rho_2I_2)(\rho_1A_1 + \rho_2A_2)}{(A_1G_1 + A_2G_2)}\frac{\partial^4 w}{\partial t^4} = 0 \end{aligned} \quad (3.8)$$

The elements of this equation may be expressed as $\pm\lambda_1, \pm\lambda_2$. The general solution for natural vibration analysis takes the following form

$$W(x, t) = w(x)q(t) \quad (3.9)$$

After separating the variables, the individual solutions take the following form

$$\begin{aligned} W(x) &= C_1 \sinh \frac{\lambda_1 x}{l} + C_2 \cosh \frac{\lambda_1 x}{l} + C_3 \sin \frac{\lambda_2 x}{l} + C_4 \cos \frac{\lambda_2 x}{l} \\ \varphi(x) &= D_1 \sinh \frac{\lambda_1 x}{l} + D_2 \cosh \frac{\lambda_1 x}{l} + D_3 \sin \frac{\lambda_2 x}{l} + D_4 \cos \frac{\lambda_2 x}{l} \\ q(t) &= A \sin(\omega t) + B \cos(\omega t) \end{aligned} \quad (3.10)$$

Constants C_m and D_m are correlated. Boundary conditions (3.11) for a free-free beam result from the fact that the bending moments and shear forces at both ends of the beam equal zero

$$\begin{aligned} \frac{\partial \varphi}{\partial x}(x, t) &= 0 \quad x = 0, l \\ (E_1I_1 + E_2I_2)\left(\frac{\partial w}{\partial x}(x, t) - \varphi(x, t)\right) &= 0 \quad x = 0, l \end{aligned} \quad (3.11)$$

After substituting the boundary conditions at the one end of the beam $x = 0$, we obtain

$$\lambda_1^2 C_1 - \lambda_2^2 C_3 = 0 \quad \lambda_1^3 C_2 - \lambda_2^3 C_4 = 0 \quad (3.12)$$

However, by substituting boundary conditions at the other end of the beam $x = l$, we obtain

$$\begin{aligned} \lambda_1^2 C_1 \cosh \lambda_1 + \lambda_1^2 C_2 \sinh \lambda_1 - \lambda_2^2 C_3 \cos \lambda_2 - \lambda_2^2 C_4 \sin \lambda_2 &= 0 \\ \lambda_1^3 C_1 \sinh \lambda_1 + \lambda_1^3 C_2 \cosh \lambda_1 + \lambda_2^3 C_3 \sin \lambda_2 - \lambda_2^3 C_4 \cos \lambda_2 &= 0 \end{aligned} \quad (3.13)$$

After determining the integral constants and substituting them into equation (3.10), we obtain equation (3.14) allowing one to determine the form of natural vibrations

$$\begin{aligned} w(x) &= C_n \left[\frac{\lambda_2^3}{\lambda_1^3} \sinh(\lambda_1 x) + \sin(\lambda_2 x) \right. \\ & \left. + \frac{\sin(\lambda_2 x) - \sinh(\lambda_1 x)}{\cosh(\lambda_1 x) - \cos(\lambda_2 x)} \left(\frac{\lambda_2^2}{\lambda_1^2} \cosh(\lambda_1 x) + \cos(\lambda_2 x) \right) \right] \end{aligned} \quad (3.14)$$

After defining the matrix of coefficients of integral constants, the characteristic equation can be finally presented in the form

$$2(1 - \cosh \lambda_1 \cos \lambda_2) + \left(\frac{\lambda_1}{\lambda_2} - \frac{\lambda_2}{\lambda_1}\right) \sinh \lambda_1 \sin \lambda_2 = 0 \quad (3.15)$$

The approximate solution for this equation are elements in the following form

$$\lambda_i = (2n - 1) \frac{\pi}{2} \quad (3.16)$$

On the other hand, the natural vibration frequencies are determined from

$$\begin{aligned} \frac{\rho_1 I_1 + \rho_2 I_2}{k(A_1 G_1 + A_2 G_2)} \omega_n^4 - \left[1 + \frac{\lambda_i^2 (\rho_1 I_1 + \rho_2 I_2)}{l^2 (\rho_1 A_1 + \rho_2 A_2)} + \frac{\lambda_i^2 (E_1 I_1 + E_2 I_2)}{l^2 k (A_1 G_1 + A_2 G_2)} \right] \omega_n^2 \\ + \frac{\lambda_i^4 (E_1 I_1 + E_2 I_2)}{l^4 (\rho_1 A_1 + \rho_2 A_2)} = 0 \end{aligned} \quad (3.17)$$

Owing to the obtained dependencies, it is possible to determine the natural vibration frequencies and the form of natural vibrations for beams with an identical neutral axis, among others, of the steel-polymer concrete beam in question. Determining the coefficients of equation (3.8), the natural vibration frequencies and mode shapes were obtained. They were then compared with the results obtained experimentally and using the finite element method – Table 4 and Fig. 6.

3.2. Beam model with the use of the finite element method

In engineering practice, finite element methods are commonly used to assess dynamic properties of structural elements. Therefore, a FEM model of the beam under consideration was constructed in order to compare the results of numerical calculations with the analytical model developed according to the Timoshenko theory, presented in Section 3.1.

Discretization of the geometric model of the beam was carried out using the Midas NFX pre-processor (Midas, 2011). The calculation area was divided with the use of eight-node, six-sided, isoparametric finite CHEXA elements and six-node, five-sided, isoparametric CPENTA elements. The applied finite elements were characterized by linear shape functions and three translation degrees of freedom in each node. A structural grid was used in the process of discretization. The construction of the grid was supported by the analysis of its quality, considering the coefficient of shape and slope. The use of a spatial finite element grid was dictated by the desire to accurately reproduce geometry of the beam and contact surfaces between steel and polymer concrete. The contact of the steel profile with the polymer concrete filling was modeled by the coincidence of nodes. In summary, the developed model consisted of 19600 elements and had 21985 degrees of freedom; the discrete model is shown in Fig. 4 (Dunaj *et al.*, 2019, 2020).

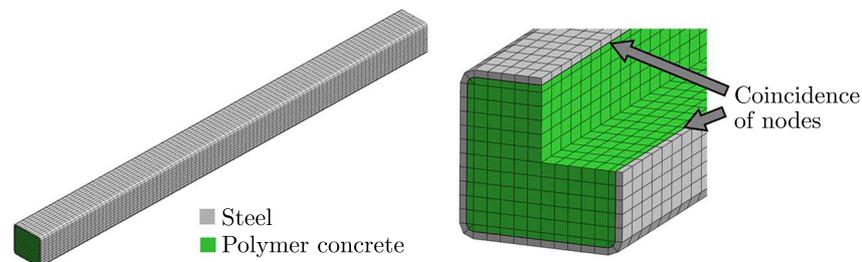


Fig. 4. Discrete model of a steel-polymer concrete beam

The Timoshenko beam model for free vibrations describes flexural vibrations, so that its direct comparison with the spatial model developed using the finite element method is possible

once the appropriate boundary conditions for the FEM model have been defined. Therefore, the degrees of freedom associated with the ability to move in the Y -axis and rotate about the X -axis and the Z -axis have been blocked. Then, for the so defined boundary conditions, values of natural vibration frequencies and corresponding forms of natural beam vibrations were determined – calculations were carried out with the use of Nastran Solver (SOL103). This task was implemented by solving the own problem formulated in the following way

$$(\mathbf{K} - \omega_i^2 \mathbf{M}) \Phi_i = \mathbf{0} \quad (3.18)$$

where: \mathbf{M} – inertia matrix of the model, ω_i – i -fold value of natural vibration frequency, Φ_i – i -fold mode shape vector.

3.3. Experimental verification of results

In order to verify the accuracy of the developed model, an impulse test was carried out for a steel-polymer concrete beam with a cross-section of 70×70 mm, 3 mm steel wall thickness and length of 1000 mm. The tested beam was suspended on steel cables in order to provide the best possible representation of the conditions of the free edge type. A diagram of the test stand is shown in Fig. 5. The object was excited to vibrate in two mutually perpendicular directions $+X$ and $-Z$ (Fig. 5.) using a PCB 086C0 modal hammer. The measurement of the system response was carried out in 56 points with the use of triaxial ICP sensors manufactured by PCB, model 356A01. The signals were acquired using a Scadas Mobile Vibco analyzer. Selected parameters of the signal acquisition are presented in Table 3.

Table 3. Signal acquisition parameters

Parameter	Value
Sampling frequency	4096 Hz
Frequency resolution	0.5 Hz
Signal acquisition time	2 s
Frequency response function estimator	H_1
Number of averages	10
Scaling factor	global

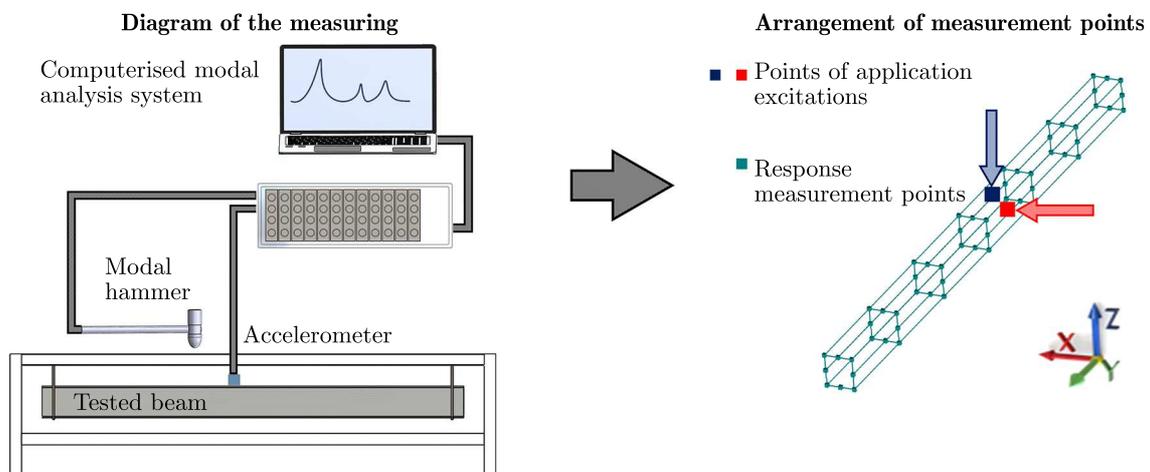


Fig. 5. Diagram of a test stand for testing dynamic properties of a steel-polymer concrete beam

As a result of the conducted impulse test, 112 frequency functions of the transition were determined. A modal model was built on their basis. The Polymax algorithm was used to estimate its parameters (Peeters *et al.*, 2004). The estimated modal model was validated using the MAC indicator.

3.4. Comparison of the developed models

The comparison of eigenvalues for the Timoshenko model, Euler-Bernoulli model, FEM model and experimental results, supplemented by the relative error values related to the natural vibration frequency determined on the basis of experimental studies, is presented in Table 4. The value of the relative error δ is defined as follows

$$\delta = \left| \frac{\omega_{exp} - \omega_{model}}{\omega_{exp}} \right| \cdot 100\% \quad (3.19)$$

where: ω_{exp} – experimentally determined natural frequency, ω_{model} – natural frequency determined based on the Timoshenko (T), Euler-Bernoulli (E-B) and finite element model (FEM).

Table 4. Parameters of the modal model of a steel-polymer concrete beam

Mode shape	Experim. data	Timoshenko	Relative error δ_T	Euler-Bernoulli	Relative error δ_{E-B}	FEM	Relative error δ_{FEM}
1	339 Hz	338 Hz	0.3%	347 Hz	2.4%	338 Hz	0.3%
2	899 Hz	899 Hz	< 0.1%	963 Hz	7.1%	915 Hz	1.7%
3	1669 Hz	1665 Hz	0.2%	1889 Hz	13.2%	1755 Hz	5.2%
4	2572 Hz	2581 Hz	0.4%	3123 Hz	21.4%	2833 Hz	10.1%
5	3589 Hz	3597 Hz	0.2%	4665 Hz	30.0%	4124 Hz	14.8%

A comparison of selected forms of beam vibrations determined by calculation and experimentation is presented in Fig. 6.

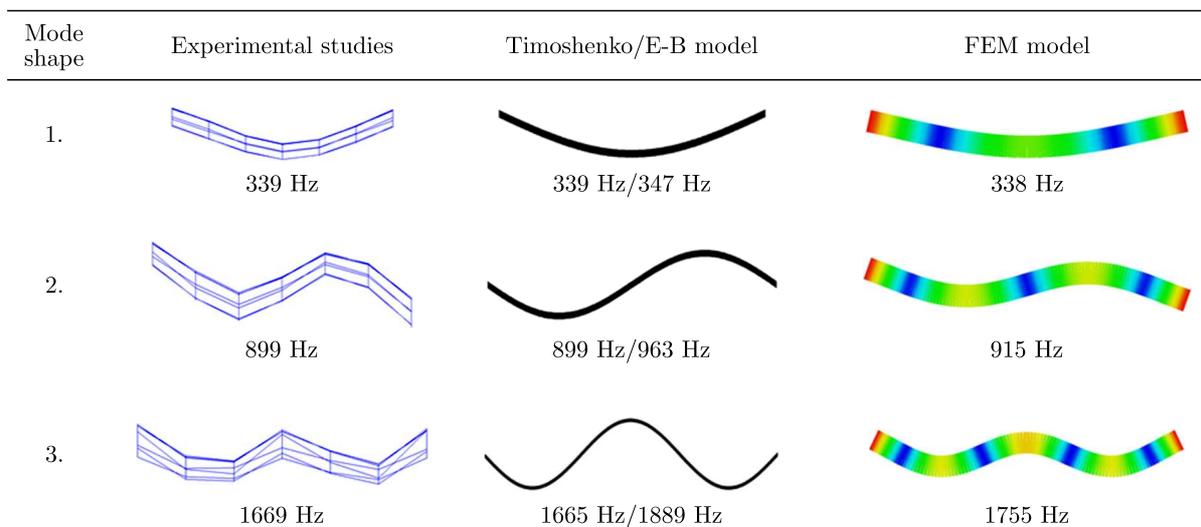


Fig. 6. Comparison of selected mode shapes determined by calculation and experimentation for a steel-polymer concrete beam

4. Findings

Summarizing the studies presented in this paper, it can be stated that:

- for the proposed analytical model of a steel-polymer concrete beam, a high consistency of the values of the natural vibration frequency was achieved – the relative error does not exceed 0.4%, on average 0.2%;
- comparing the proposed model with the model built according to the Euler-Bernoulli beam theory it can be seen that taking into account the shear phenomenon causes a significant increase in the accuracy of the results – relative errors for the natural vibration frequencies in the E-B model reach even 30%, on average 14.8%;
- the proposed model has a higher accuracy of representation of the values of the natural frequencies than the FEM model analysed with 21,985 degrees of freedom, which is particularly evident at higher frequencies;
- it is demonstrated – formula (3.6) – that it is possible to build a reliable model of a composite steel-polymer concrete beam using a homogeneous beam model with equivalent stiffness.

5. Conclusions

To sum up, this paper presents the issues of modeling the dynamic properties of steel-polymer concrete beams. The conducted experimental and model tests contribute to a deeper understanding of the phenomena occurring in a complex composite beam. The presented analytical model, developed according to the Timoshenko beam theory, is characterized by high accuracy of mapping (in relation to experimental results) of natural vibration frequencies and natural vibration forms.

Moreover, comparing the presented model with the results of the analysis of the model developed according to the Euler-Bernoulli theory, it was shown that taking into account the shear in the case of modeling of steel-polymer concrete beams allows for a significant increase in the accuracy of mapping the values of natural vibration frequencies.

The presented model shows great application potential in the case of the assessment of dynamic properties of steel-polymer concrete beams constituting the basic structural component of hybrid bodies of technological machines, assembly lines, vibratory machines, etc. The presented model may be a good alternative to the models currently used in engineering practice and developed in accordance with the finite element method.

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