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EXPERIMENTAL STUDY OF THE NONLINEAR DYNAMICS OF A SMOOTH AND DISCONTINUOUS OSCILLATOR WITH DIFFERENT SMOOTHNESS PARAMETERS AND INITIAL VALUES

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The dynamic response of a nonlinear system is very sensitive to initial conditions. Both the irrational nonlinearity and the large displacement of a smooth and discontinuous (SD) oscillator have been studied in this paper. An experimental study has been conducted on a model of the SD oscillator with different initial conditions and smoothness parameters. Experimental results indicate that tiny variation in the initial displacement will lead to different kinds of vibrations, and the system exhibits a wide range of nonlinear dynamical phenomena with the change of smoothness parameters. All experimental results are in good conformity with numerical simulation results.

Keywords: SD oscillator, experimental system, nonlinear dynamic characteristic, initial value sensitivity, smoothness parameters

1. Introduction

Geometric nonlinearity is an important problem often encountered in the sciences and engineering (e.g., truss structures with large deformation or displacement, supporting systems of a large piece of dynamic equipment, vibration control of precision instruments). Its importance also extends to practical issues, for instance, building a theoretical model of geometric nonlinear problems, developing an effective approach to solve these problems, and accurately describing the global and local dynamic behavior of such systems. The theory of smooth and discontinuous (SD) oscillators with smooth and discontinuous characteristics was first developed by Cao *et al.* (2006, 2008a,b,c). Since then, the SD oscillator as a typical geometric nonlinear system has drawn much attention from researchers throughout the world. The continuous change of smoothness parameters achieves a transition between two different characteristics. Further study is required on the dynamic behavior of the SD oscillator to fully understand the dynamic characteristics of nonsmooth systems.

In 1973, a simply-supported arch beam as a mass-spring system was proposed by Thompson and Hunt (1973) for the first time. It was applied to analyze nonlinear geometric problems successfully, such as buckling. Later on, other research was carried out on the study of nonlinear dynamic problems from smooth dynamic systems to nonsmooth systems (Filippov, 1988; Kunze, 2000; Shaw and Holmes, 1983).

To solve the transition from the smooth dynamic system to the noncontinuous dynamic system, Cao, Wiercigroch and others devised an SD oscillator dynamic system whose behavior depends on the continuous variation of a smoothness parameter α. When $\alpha > 0$, the system is characterized as being smooth; when $\alpha = 0$, the system is characterized by a discontinuity. This parameter provides a smooth transition from a smooth dynamic system to a discontinuous dynamic system (Cao *et al.*, 2006; Cao *et al.*, 2008a,b,c). An early mechanics model of an SD system is shown in Fig. 1. The differential equation for its motion is

$$m\ddot{X} + 2kX\Big(1 - \frac{L}{\sqrt{X^2 + l^2}}\Big) = F_0 \cos(\omega t) \tag{1.1}$$

where L is the original length of the spring, and X is the mass displacement.

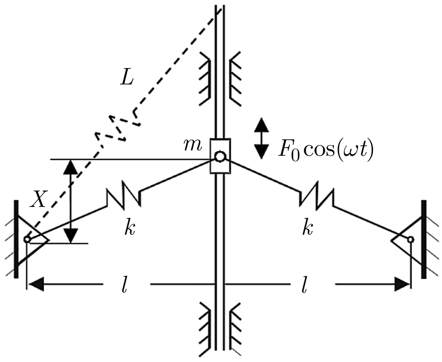

Fig. 1. Dynamic model of an SD oscillator

In terms of

$$\omega_0^2 = \frac{2k}{m} \qquad x = \frac{X}{L} \qquad \alpha = \frac{l}{L}$$

Because the negative l makes no sense in the experimental system, so $\alpha \geqslant 0$.

Equation (1.1) can be written as

$$\ddot{x} + \omega_0^2 x\Big(1 - \frac{1}{\sqrt{x^2 + \alpha^2}}\Big) = \frac{F_0}{mL} \cos(\omega t) \tag{1.2}$$

when $\alpha = 0$, dynamical system (1.2) is transformed into

$$\ddot{x} + \omega_0^2[x - \operatorname{sgn}(x)] = \frac{F_0}{mL} \cos(\omega t) \tag{1.3}$$

The discrete system is gained directly by the continuous smooth system through the parameter α changing to zero. Dynamical system (1.2) and dynamical system (1.3) are referred to as the SD oscillator, and the disturbance of the vibrator system attractor is called the SD attractor.

The wealthy nonlinear dynamic behavior of the SD oscillator has been studied in depth. Different kinetic behavior of the system were shown in the continuous changes of the parameter α (Cao *et al.*, 2008a,b,c). Fork-type bifurcation, single Hopf bifurcation and double Hopf bifurcation, homoclinic bifurcation, and closed-orbit bifurcation around the equilibrium points were discussed in (Cao *et al.*, 2008a,b,c; Tian *et al.*, 2010a,b). The response of the SD oscillator was derived under constant excitation (Tian *et al.*, 2012). Han *et al.* (2012) and Cao *et al.* (2012) studied the strong irrational nonlinear oscillator stability and bifurcation. By constructing both an elliptic function and a hyperbolic function, the analytical solution was obtained. The SD system with contact and friction constraint nonlinear dynamic behavior was been studied (Légar *et al.*, 2012; Zhang *et al.*, 2014). However, most of those theoretical studies were conducted through numerical simulations or analytical methods, and few scholars conducted the research by constructing an experimental system. An experimental system to research the dynamical behavior of the SD oscillator has been constructed in this paper.

The literature about constructing the corresponding experimental system with nonlinear dynamics to study nonlinear behavior is sparse. By constructing a small building model with characteristics of a single potential well, a nonlinear vibration response was studied (Gourdon *et al.*, 2007). The dynamical characteristics of a shock absorber with cubic nonlinear stiffness coefficients and two-degree-of-freedom were studied experimentally. The mechanical model of the system contained description of how a quasi-zero stiffness characteristic is achieved (Gatti *et al.*, 2010). The nonlinear vibration of a viscoelastic belt was been studied experimentally (Zhang *et al.*, 2007). However, owing to the difficulty in selecting parameters and designing an experiment, the relevant dynamic experimental research of the SD oscillator yielded fewer results. So the experimental system was constructed to study nonlinear phenomena of the SD oscillator.

Based on the homemade SD experimental system of kinetic characteristics, this article focuses on the influence of the initial value and smoothness parameters and the SD system nonlinear motion state, verifying many nonlinear dynamic phenomena calculated by numerical simulation. The results reveal sensitivity and complexity of the transition parameter existing in the nonlinear system.

2. SD oscillator experimental system

2.1. Experimental device

In this paper, an experimental model with SD oscillator characteristics is designed based on the early simplified mechanical model of the SD oscillator comprising an initial arch beam shown in Fig. 2.

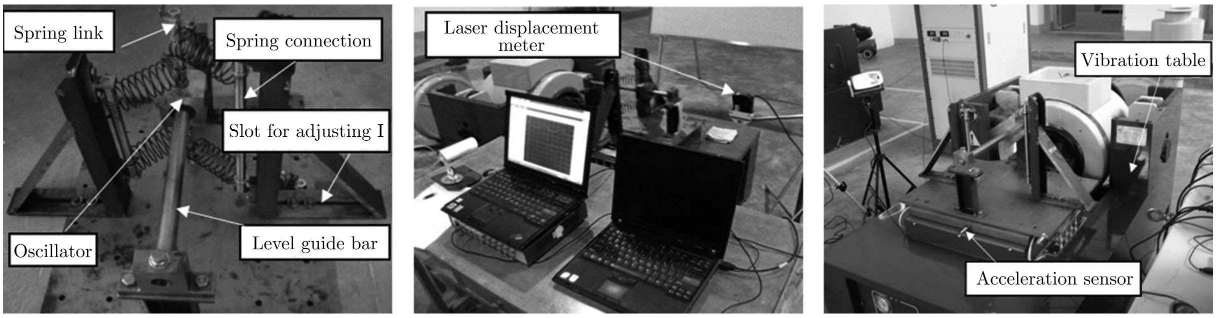

Fig. 2. SD oscillator experimental system

In the experimental SD oscillator device, the upper and lower ends of the oscillator centroid are fixed with spring connecting rods. The connecting rod and the spring are provided with a sheath that is rotatable about the rod. In addition, the upper and lower springs are arranged symmetrically and the other end of the spring is connected to a hinged post. According to the experimental requirement, the connection l can be adjusted by moving the clamped screw along the centerline of the post to obtain different values of the smoothness parameter α. Moreover, the oscillator is installed in a horizontal guide rod. To reduce friction damping, the vibrator is fitted with a linear sliding bearing. The spring connecting post and the horizontal guide bracket are rigidly connected to the baseplate (as shown in Fig. 2), and the whole experimental device is fixed to the level of the vibration table.

2.2. Laboratory equipment and experimental processes

The SD oscillator experimental system is fixed onto an electric vibration table and a horizontal sliding table. The oscillator can slide linearly along the horizontal guide rod. Stretching deformation of the spring connecting the SD oscillator only occurs in the process of oscillator movement. The main equipment of the experiment comprises ES-10 electric vibrating table, SD-PUMA vibration controller, CA-YD-186 acceleration transducer, and LKG-3001 laser displacement meter. The displacement of the oscillator during its movement is simultaneously measured by the laser displacement meter.

The experiment consists of three parts:

(1) Ascertain the stiffness, damping, natural frequency and other kinetic parameters of the system by damped free vibration, providing the basis for the simulation of the experimental system.
(2) Observe the status of the oscillator system responding to driving forces under through experiments in different initial positions.
(3) Research the vibrator nonlinear motion characteristics of the system under different conditions of excitation parameters by changing smoothness parameters of the system.

3. Differential equations of motion of the SD oscillator and numerical analysis

Based on the SD oscillator experiment, a corresponding model of the experimental apparatus was built as shown in Fig. 3.

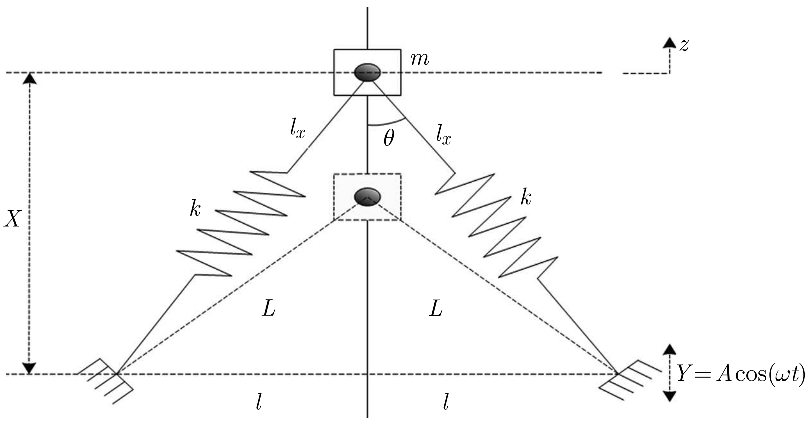

Fig. 3. SD oscillator experimental model

The absolute displacement of the oscillator is given by

$$Z = X + Y \tag{3.1}$$

where X is the relative displacement of the oscillator m for the fixed support, Y is the implicated displacement of the horizontal sliding table of the vibrating table. Then, the undamped differential equation of motion for the SD oscillator is

$$m(\ddot{X} + \ddot{Y}) + 2k\Big(\sqrt{X^2 + l^2} - L\Big)\frac{X}{\sqrt{X^2 + l^2}} = 0 \tag{3.2}$$

If we assume that the damping system is viscous and that the damping coefficient is c, the motion amplitude is A, and the frequency is ω, then $Y = A\cos(\omega t)$, and the damped differential equation of motion for the SD oscillator is

$$m\ddot{X} + c\dot{X} + 2kX\Big(\sqrt{X^2 + l^2} - L\Big)\frac{1}{\sqrt{X^2 + l^2}} = mA\omega^2\cos(\omega t) \tag{3.3}$$

If we let

$$\alpha = \frac{l}{L} \qquad x = \frac{X}{L} \qquad \omega_0^2 = \frac{k}{m} \qquad t = \frac{\tau}{\omega_0}$$

where ω_0 is the natural frequency of the system, then equation (3.3) becomes

$$x'' + \frac{c}{m\omega_0}x' + 2x\Big(1 - \frac{1}{\sqrt{x^2+\alpha^2}}\Big) = \frac{A\omega^2 \cos\frac{\omega\tau}{\omega_0}}{L\omega_0^2} \tag{3.4}$$

Then the SD oscillator equation of motion is given by (3.5)

$$x'' + \eta x' + 2x\Big(1 - \frac{1}{\sqrt{x^2+\alpha^2}}\Big) = F_0 \cos\frac{\omega\tau}{\omega_0} \tag{3.5}$$

where $\eta = c/(m\omega_0)$ and $F_0(\omega) = A\omega^2/(L\omega_0^2)$. Let the disturbance of the SD oscillator equation of motion (3.5) be written as the state equations

$$x' = y \qquad y' = -\eta y - 2x\Big(1 - \frac{1}{\sqrt{x^2+\alpha^2}}\Big) + F_0 \cos\frac{\omega\tau}{\omega_0} \tag{3.6}$$

When $\alpha > 1$, the corresponding system has a stable equilibrium (center). It is a single well system. When $\alpha < 1$, the corresponding system exists as three balance points, including two stable equilibrium points (center) and an unstable equilibrium point (saddle point) situated at the boundary of two attractor domains. The system is a double well system. According to Fig. 4, the attractor of the SD oscillator is a stable equilibrium point.

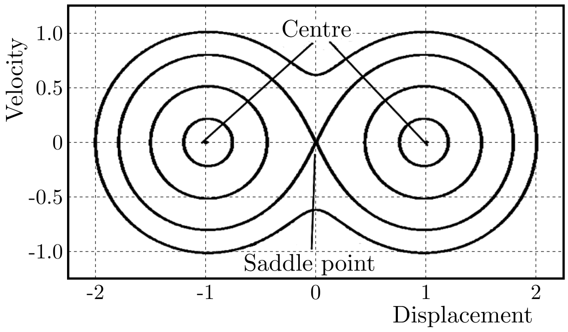

Fig. 4. Phase diagram in the displacement-velocity for the unperturbed system: ($\eta = 0$, $F = 0$, $0 < \alpha < 1$)

3.1. Influence of initial value on the SD oscillator system

The same parameters are applied in numerical calculations and experiments. The oscillator mass m is assumed to be 2.63 kg and it is connected by four springs. Each spring is 0.0544 kg, with stiffness K of 500 N/m, and an unstretched length L of 0.12 m. Moreover, the damping c is obtained by the attenuation vibration curve, by taking $c = 3060$ Ns/m.

To study the SD oscillator system sensitivity to the initial value, a numerical simulation of the vibration response of the system was conducted according to the given parameters in the experimental scheme. By making use of a time-domain plot, frequency-domain plot, oscillator phase track diagram, Poincaré cross section figure is made to analyze the simulation results. The smoothness parameter α is set to 0.67, the excitation amplitude A is set to 0.03 m, and the excitation frequency f is set to 3 Hz. The displacement of the origin of the SD oscillator is calculated as the distance X of the oscillator from the spring original length to the distance of the two connections stud attachment points O. As shown in Fig. 3, when the oscillator system is in the state of static balance, $X = 0.090$ m. Then the initial velocity is $V = 0.565$ m/s and the selected initial displacement values used in the simulation calculation are $X_0 = 0.090$ m, $X_0 = 0.0925$ m, and $X_0 = 0.095$ m, respectively. When $X_0 = 0.090$ m, the simulation result is shown as Fig. 5a.

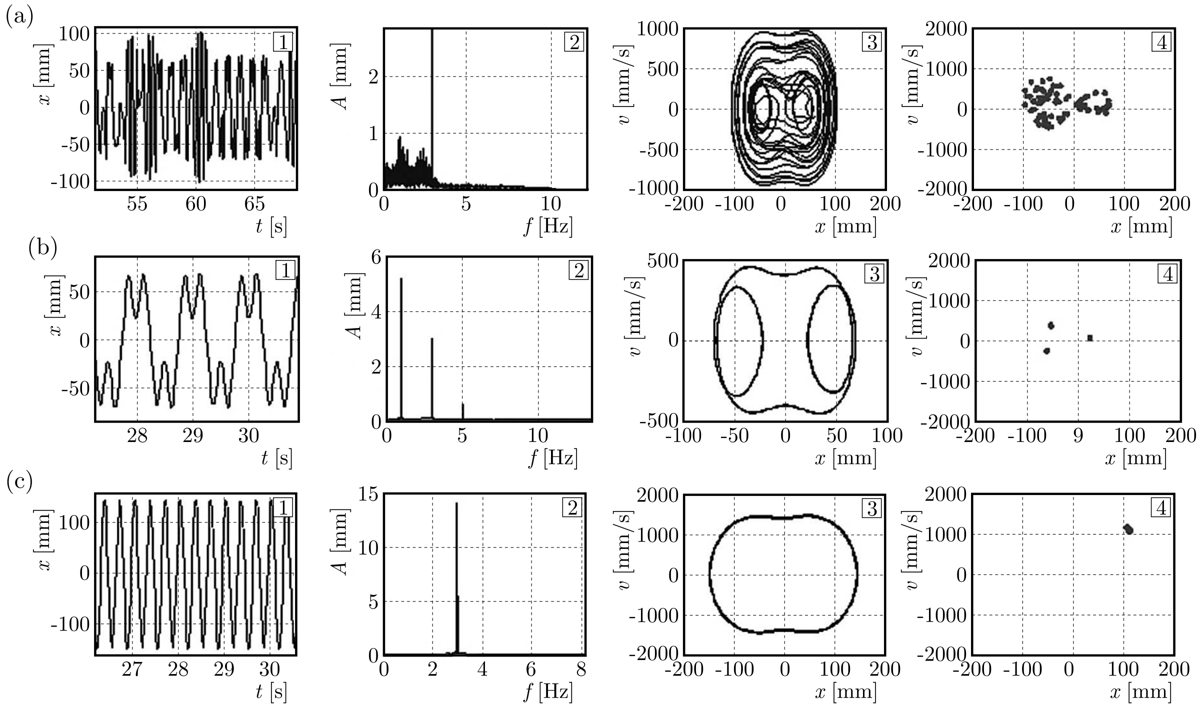

Fig. 5. SD oscillator movement response ($\alpha = 0.667$, $m = 2.63\,\mathrm{kg}$, $L = 0.120\,\mathrm{m}$, $K = 500\,\mathrm{N/m}$, $c = 3060\,\mathrm{Ns/m}$, $f = 3\,\mathrm{Hz}$, $A = 0.030\,\mathrm{m}$, $V = 0.565\,\mathrm{m/s}$, $l = 0.080\,\mathrm{m}$): (a) $X_0 = 0.090\,\mathrm{m}$, (b) $X_0 = 0.0925\,\mathrm{m}$, (c) $X_0 = 0.095\,\mathrm{m}$; 1 – time domain, 2 – frequency spectrum diagram, 3 – phase diagram, 4 – poinkale section

From the simulation results one can see that the response in the time domain of the SD oscillator is an irregular curve and the response in the frequency domain is continuous. The phase trajectory exhibits quasi-random characteristics, and the Poincaré map indicates an SD attractor. Obviously, the system is in a state of chaotic movement at this time.

When other parameters are held constant, with the initial displacement $X_0 = 0.0925\,\mathrm{m}$, the oscillator deviates from the equilibrium position, sliding along the guide bar by 0.0025 m. The simulation result is shown in Fig. 5b. In this circumstance, the oscillator system responds to periodic motion, and the fundamental frequency of the spectrum is 1/3 of the excitation frequency. The spectrum and the Poincaré section are displayed as period-3 motion.

When other parameters are held constant, and the initial displacement is $X_0 = 0.095\,\mathrm{m}$, the oscillator again deviates from the equilibrium position, sliding along the guide bar by 0.005 m. The simulation result is shown in Fig. 5c. At this time, system is undergoing cycle (period-1) movement.

The result of the numerical simulations shows that the SD system has strong sensitivity to the initial value. When smoothness parameters and external forces are kept the same, small changes in the initial displacement can drive the SD oscillator system into three different motion states.

3.2. Influence of smoothness parameters on the SD oscillator motion state

The SD oscillator smooth characteristic parameter reflects the system of smooth and continuous features.

To study the effect of smoothness parameter variation on motion of the oscillator system, we choose the smoothness parameter α of the corresponding system to 0.667, 1, and 1.08, respectively, then l was set to 0.08, 0.12, and 0.13 m, respectively. At this time, the corresponding damping values c for the system were 3.004, 0.371, and 0.679, respectively, and the other parameters remained constant. The excitation amplitude was set to 0.03 m and the excitation frequency f was 1.5-6 Hz. The bifurcation diagrams of the system under these four parameters obtained by the simulation are shown in Fig. 6

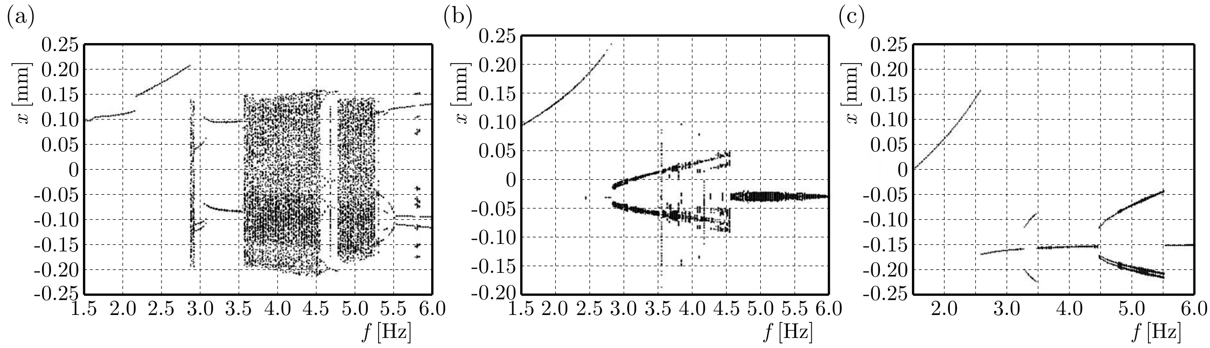

Fig. 6. Bifurcation diagram for displacement versus frequency ($X_0 = 0.0925\,\mathrm{m}$, $m = 2.63\,\mathrm{kg}$, $K = 500\,\mathrm{N/m}$, $A = 0.03\,\mathrm{m}$, $V = 0.565\,\mathrm{m/s}$, $f = 1.5\text{-}6\,\mathrm{Hz}$): (a) $\alpha = 0.67$, (b) $\alpha = 1.0$, (c) $\alpha = 1.08$

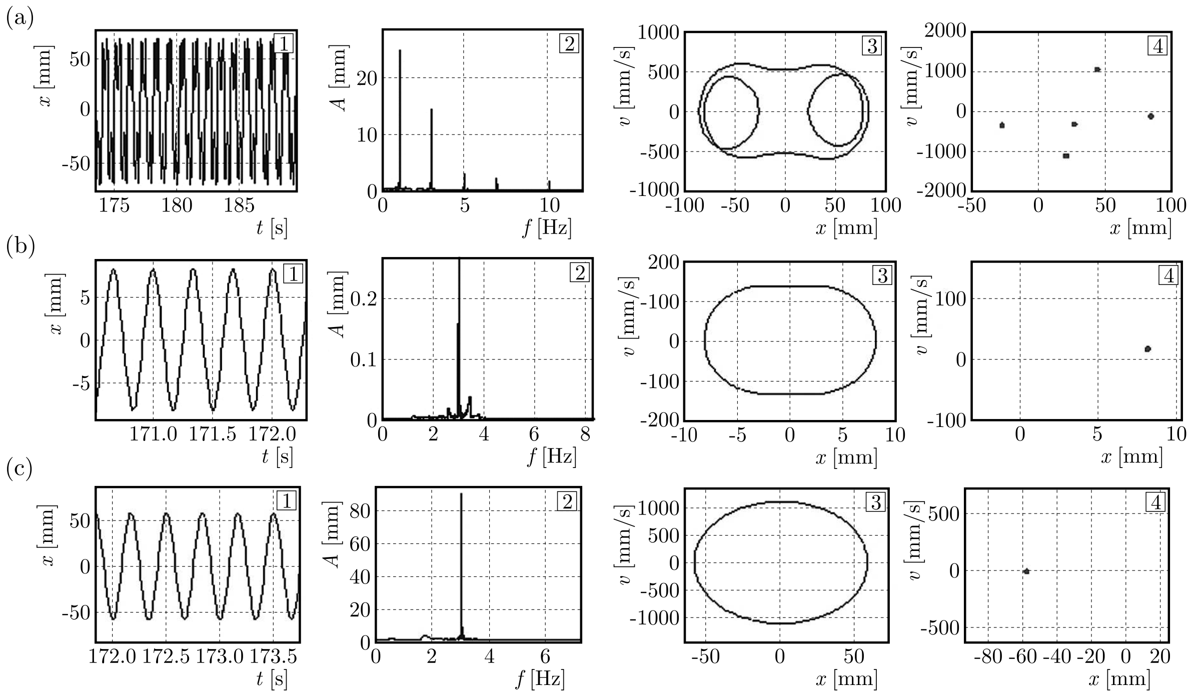

Fig. 7. SD oscillator movement response as $f = 3\,\mathrm{Hz}$ ($X_0 = 0.0925\,\mathrm{m}$, $m = 2.63\,\mathrm{kg}$, $K = 500\,\mathrm{N/m}$, $A = 0.030\,\mathrm{m}$, $V = 0.565\,\mathrm{m/s}$): (a) $\alpha = 0.67$, (b) $\alpha = 1.0$, (c) $\alpha = 1.08$; 1 – time domain, 2 – frequency spectrum diagram, 3 – phase diagram, 4 – poinkale section

One can conclude from these diagrams that the smoothness parameter has a great influence on the bifurcated structure of the SD system. Meanwhile, depending on the value of the smoothness parameter, the corresponding system exhibits four types of motion: periodic motion, double periodic motion, periodic bifurcation motion, and chaotic motion, thus demonstrating abundant nonlinear dynamic phenomena.

When the excitation amplitude remains constant and the excitation frequency increases to 3 Hz, the system response for the above three of smoothness parameters is shown in Fig. 7.

According to Fig. 7 and the bifurcation diagram, when $\alpha = 0.67$, the system is undergoing period-5 movement and the oscillator has a 1/5 subharmonic resonance; when $\alpha = 1.0$, a critical system appears with coexisting multiple solutions. There is a syntonic response that is the same as the external excitation frequency in the low-frequency chaotic solution, and the amplitude of the system response is also small. This system can ably inhibit the input excitation. When $\alpha = 1.08$, the oscillator exhibits period-1 movement.

If one continues to increase the excitation frequency to 5 Hz, the system responses of the four groups above under the same smoothness parameter are shown as Fig. 8.

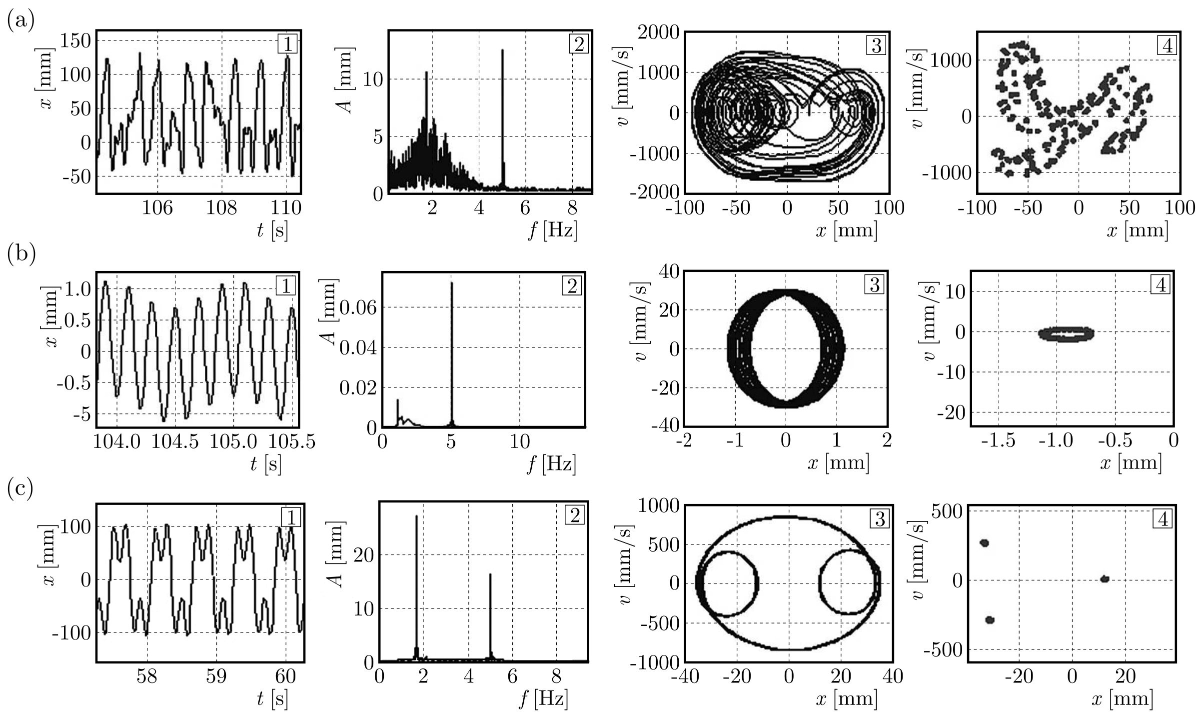

Fig. 8. SD oscillator movement response as $f = 5$ Hz ($X_0 = 0.0925$ m, $m = 2.63$ kg, $K = 500$ N/m, $A = 0.030$ m, $V = 0.565$ m/s): (a) $\alpha = 0.67$, (b) $\alpha = 1.0$, (c) $\alpha = 1.08$; 1 – time domain, 2 – frequency spectrum diagram, 3 – phase diagram, 4 – poinkale section

According to Fig. 8 and the bifurcation diagram, when $\alpha = 0.67$ and the excitation frequency is 5 Hz, the system is in the state of chaotic motion. When $\alpha = 1.0$, the critical system is in a mixed state of quasi-periodicity, and the period-1 movement and the amplitude of the system response is also small. When $\alpha = 1.08$, a 1/3 subharmonic resonance occurs in the system.

4. Experimental analysis of SD oscillator nonlinear motion

To verify the numerical simulation results, an experimental study of the SD oscillator was conducted. First, the experimental setup was fixed onto the electric vibration table level slider. The length L of the spring was 0.120 m and the horizontal distance between the two ends of the spring was $2l = 0.1608$ m. The SD oscillator was initially located at a stable equilibrium point and the two stable equilibrium points were 180 mm apart. The central point of the oscillator and saddle point is presented in Fig. 4. By using the laser displacement sensor, the movement of the SD oscillator could be captured. Then, MATLAB was applied to analyze the captured oscillator displacement time series.

According to the experimental requirements, the distance of the upright rod that connects the oscillator and the springs can be adjusted. The smoothness parameter of the system was also changeable. To facilitate comparison with the simulation calculation results, the experimental system for parameter selection was in accordance with the simulation calculation parameters.

4.1. Influence of initial value on the SD oscillator system

The initial value of the oscillator movement affects the experiment. By choosing the smoothness parameter α as 0.67, the excitation amplitude A as 30 mm, the vibration excitation frequency as 2.95 Hz, and the initial velocity of the oscillator as 56.5 cm/s, experiments were conducted for the case of oscillator initial displacements X_0 of 90, 92.5, and 95 mm, separately. Then the oscillator response data were collected when the oscillator initial displacement was constant and the oscillator motion was stable. Finally, the oscillator response could be converted to a graph from the data.

From the graphs in Fig. 9, one can see that the state of the system motion is different when the initial value is changed. For example, chaotic movement, similar period-3 movement, and similar period 1 movement are shown in Fig. 9. The experimental results and those of the simulation calculation are basically identical. The experimental results show that a tiny change in the initial displacement makes the SD oscillator system exhibit three different motion states when smoothness parameters and external forcing conditions are the same. Meanwhile, the stable solution of the system and the initial value have a strong relationship.

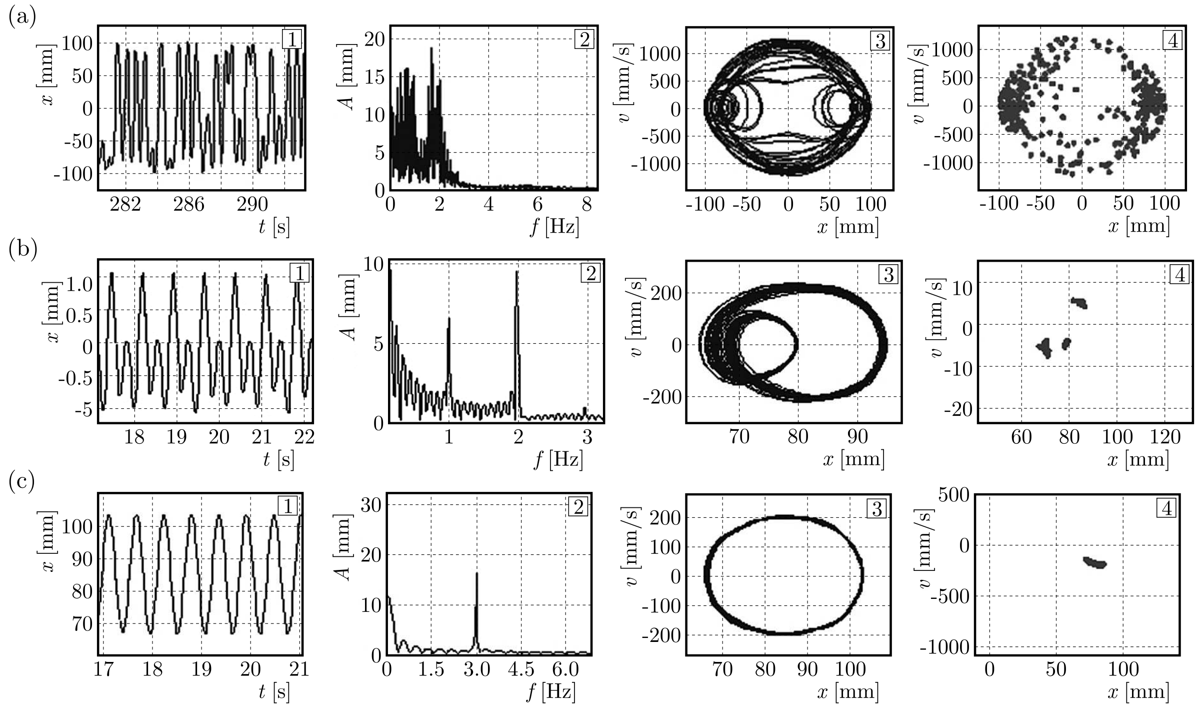

Fig. 9. SD oscillator movement testing response as $f = 2.95$ Hz ($m = 2.63$ kg, $L = 0.12$ m, $K = 500$ N/m, $c = 3060$ Ns/m, $A = 0.03$ m, $V = 0.0565$ m/s, $l = 0.08$ m): (a) $X_0 = 0.090$ m, (b) $X_0 = 0.0925$ m, (c) $X_0 = 0.095$ m; 1 – time domain, 2 – frequency spectrum diagram, 3 – phase diagram, 4 – poinkale section

Because the accuracy of the experimental model is limited, the excitation frequency deviation is slight at a similar motion state. At the same time, chaotic motion is a seemingly random movement, so through an experiment it is difficult to ensure that, at the end of each cycle, the oscillator is exactly at the same motion state. After several vibrations, the response of the system state will deviate from the theoretical value but the motion properties are basically identical.

4.2. Influence of smoothness parameters on the oscillator motion state

SD oscillator dynamical behavior depends on the continuous variation of the smoothness parameter. The system exhibits discontinuous characteristics when $\alpha = 0$, but an experimental system is difficult to achieve when $\alpha = 0$. This article focuses on the motion response of the system when $\alpha > 0$. The parameters of the experimental system are the same as those of the numerical simulation. The SD oscillator system was subjected to the vibration excitation of vibration table and then the vibration was started. To study the influence of the smoothness parameter, $\alpha = 0.67, 1.0$, and 1.08 were selected for the experiment. When the vibration frequency of the vibration table was $f = 3$ Hz, the motion data of the sample were analyzed, and the system response is shown in Fig. 10.

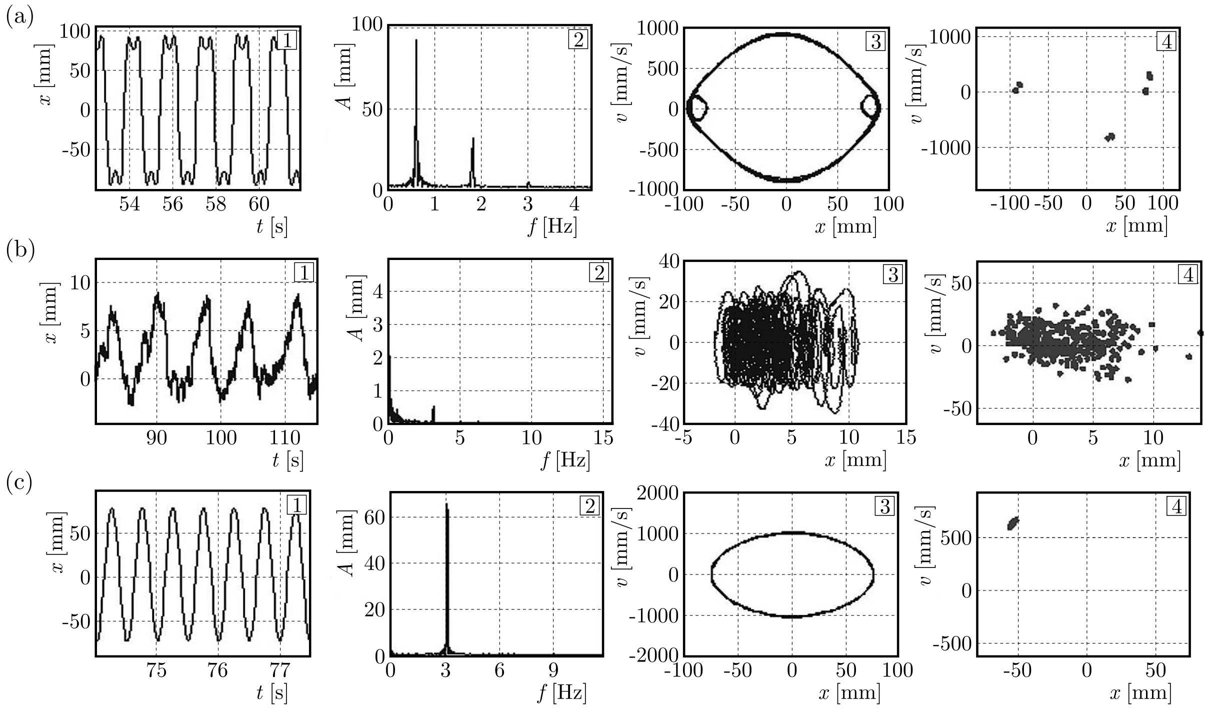

Fig. 10. Oscillator movement testing response as $f = 3$ Hz ($m = 2.63$ kg, $L = 0.12$ m, $K = 500$ N/m, $c = 3060$ Ns/m, $A = 0.03$ m, $V = 0.0565$ m/s, $l = 0.08$ m): $\alpha = 0.67$, (b) $\alpha = 1$, (c) $\alpha = 1.08$; 1 – time domain, 2 – frequency spectrum diagram, 3 – phase diagram, 4 – poinkale section

As seen in Fig. 10, when the smoothness parameter $\alpha = 0.67$ and $\alpha = 1.08$, the system undergoes period-5 and period-1 movement, respectively. And when $\alpha = 1$, a periodic solution and chaos coexist in the critical system. The oscillator amplitude is small and the system has a good inhibitory effect on the external excitation. The experimental results and simulation results are basically identical.

If one continues to increase the excitation frequency of the vibrating table to 5 Hz, the stable motion responses of the two groups of smoothness parameters of subsystem vibration are shown in Fig. 11.

According to Fig. 11, the excitation amplitude is 30 mm and the excitation frequency is 5 Hz. When $\alpha = 0.67$, the system is in chaotic motion. When $\alpha = 1.0$, the oscillator response amplitude is small, and the system exhibits a low-frequency peak and quasi-periodic motion. The system has a good inhibitory effect on the external excitation. The experimental results and numerical simulation calculation results are basically the same. When $\alpha = 1.08$, the oscillator gains a large amount of energy and the response amplitude exceeds the experimental model design, so valid data could not be collected.

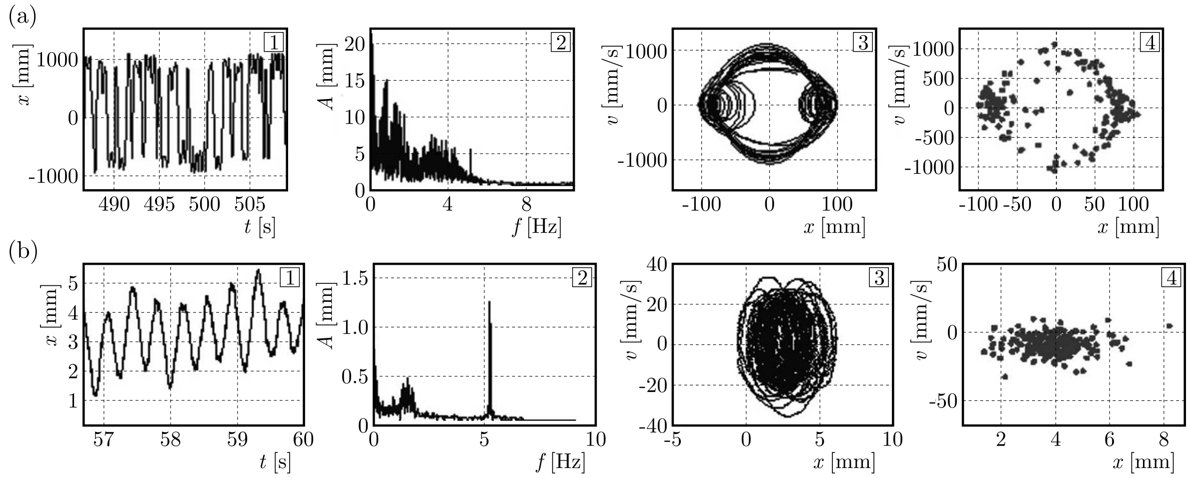

Fig. 11. The chaotic motion of the SD oscillator movement testing response ($f = 5\,\text{Hz}$, $m = 2.63\,\text{kg}$, $L = 0.12\,\text{m}$, $K = 500\,\text{N/m}$, $c = 3060\,\text{Ns/m}$, $A = 0.03\,\text{m}$, $V = 0.0565\,\text{m/s}$, $l = 0.08\,\text{m}$): (a) $\alpha = 0.67$, (b) $\alpha = 1.0$; 1 – time domain, 2 – frequency spectrum diagram, 3 – phase diagram, 4 – poinkale section

5. Conclusions

The geometric nonlinear large deformation characteristics of an SD oscillator were detected. Influence of the initial value and smoothness parameters on the nonlinear oscillator system was discussed via numerical and experimental research. The smoothness parameter of the experimental system can be adjusted by changing the geometric shape. The system can be used to study the nonlinear dynamic response of the SD oscillator under different excitation amplitudes and frequencies.

As a nonlinear system, the SD oscillator, under the action of external excitation, exhibited nonlinear characteristics that were strongly sensitive to the initial value. Small changes of the initial displacement led to entirely different motion states. The system presented a complex, rich dynamic behavior, including both periodic and chaotic motion.

Changing the smoothness parameter α determined the dynamic behavior of the system. SD oscillator systems with different excitation frequency parameters under certain parameter conditions exhibited different dynamic characteristics, ranging from cyclic motion to chaotic motion, to quasi-periodic motion, and other complicated nonlinear dynamic phenomena.

The response characteristics of the SD experimental system were basically identical to those found through numerical simulation under the corresponding parameters. Several typical nonlinear response characteristics of the numerical solution replicated the corresponding performance aspects observed in the experiment and the transition rules between various states were the same. In the experiments, the system appeared to exhibit transient chaos, with the oscillator performing complicated nonlinear motion between two attractor domains.

With the increase of excitation frequency, the response of the system became complex and rich. With the increase of system energy, the oscillator changed from small periodic motion to large periodic motion and from cyclic motion to chaotic motion. Especially, the transitions between periodic chaotic motion showed that there were bifurcation motions in the system, which was verified by numerical calculation.

Acknowledgments

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References

1. Cao Q., Wang D., Chen Y., Wiercigroch M., 2012, Irrational elliptic functions and the analytical solutions of SD oscillator, *Journal of Theoretical and Applied Mechanics*, **50**, 3, 701-715
2. Cao Q., Wiercigroch M., Pavlovskaia E.E., Grebogi C., Thompson J.M., 2006, Archetypal oscillator for smooth and discontinuous dynamics, *Physical Review E*, **74**, 1-5, DOI: 10.1103/PhysRevE.74.046218
3. Cao Q., Wiercigroch M., Pavlovskaia E.E., Grebogi C., Thompson J.M., 2008a, The limit case response of the archetypal oscillator for smooth and discontinuous dynamics, *International Journal of Nonlinear Mechanics*, **43**, 462-473, DOI: 10.1016/j.ijnonlinmec.2008.01.003
4. Cao Q., Wiercigroch M., Pavlovskaia E.E., Thompson J.M., Grebogi C., 2008b, Piecewise linear approach to an archetypal oscillator for smooth and discontinuous dynamics, *Philosophical Transactions of the Royal Society –Mathematical Physical and Engineering Sciences*, **1865**, 366, 635-652, DOI: 10.1098/rsta.2007.2115
5. Cao Q., Xiong Y., Wiercigroch M., 2008c, Resonances of the SD oscillator due to the discontinuous phase, *Journal of Applied Analysis and Computation*, **1**, 2, 183-191
6. Filippov A.F., 1988, *Differential Equations with Discontinuous Right-hand Sides*, Dordrecht, The Netherlands: Kluwer Academic Publishers
7. Gatti G., Kovacic I., Brennan M.J., 2010, On the response of a harmonically excited two degree-of-freedom system consisting of a linear and a nonlinear quasi-zero stiffness oscillator, *Journal of Sound of Vibration*, **329**, 1832-1835, DOI: 10.1016/j.jsv.2009.11.019
8. Gourdon E., Alexander N.A., Taylor C.A., Lamarque C.H., Pernot S., 2010, Nonlinear energy pumping under transient forcing with strongly nonlinear coupling: Theoretical and experimental results, *Journal of Sound of Vibration*, **300**, 2007, 522-551, DOI: 10.1016/j.jsv.2006.06.074
9. Han Y.W., Cao Q.J., Chen Y.S., Wiercigroch M., 2012, A novel smooth and discontinuous oscillator with strong irrational nonlinearities, *Science China-Physical Mechanics and Astronomy*, **55**, 10, 1832-1843, DOI: 10.1007/s11433-012-4880-9
10. Kunze M., 2000, *Non-Smooth Dynamical Systems*, New York: Springer-Verlag
11. Légar A., Pratt E., Cao Q., 2012, A fully nonlinear oscillator with contact and friction, *Nonlinear Dynamics*, **70**, 511-522
12. Shaw S.W, Holmes P.J., 1983, A periodically forced piecewise linear oscillator, *Journal of Sound and Vibration*, **90**, 1, 129-155, DOI: 10.1016/0022-460X(83)90407-8
13. Thompson J.M.T., Hunt G.W., 1973, *A General Theory of Elastic Stability*, London: John Wiley & Sons
14. Tian R.L., Cao Q.J., Li Z.X., 2010a, Hopf bifurcations for the recently proposed smooth--and-discontinuous oscillator, *Chinese Physical Letters*, **27**, 7, 074701-074704, DOI: 10.1088/0256-307x/27/7/074701
15. Tian R.L., Cao Q.J., Yang S.P., 2010b, The codimension-two bifurcation for the recent proposed SD oscillator, *Nonlinear Dynamics*, **59**, 1-2, 19-27, DOI: 10.1007/s11071-009-9517-9
16. Tian R.L., Wu Q.L., Liu Z.J., Yang X.W., 2012, Dynamic analysis of the smooth-and--discontinuous oscillator under constant excitation, *China Physical Letters*, **29**, 8, 084706-084711, DOI: 10.1088/0256-307x/29/8/084706
17. Tian R.L., Wu Q.L., Yang X.W., Si C.D., 2013, Chaotic threshold for the smooth-and--discontinuous oscillator under constant excitations, *European Physical Journal Plus*, **128**, 7, 801-812, DOI: 10.1140/epjp/i2013-13080-6
18. Zhang H.X., Zhang W., Yao M.H., et al., 2007, Experimental research on nonlinear vibration of an axially moving visco-elastic belt, *Journal of Dynamics and Control*, **5**, 4, 361-364
19. Zhang Y., Luo G., Cao Q., Lin M., 2014, Wada basin dynamics of a shallow arch oscillator with more than 20 coexisting low-period periodic attractors, *International Journal of Non-Linear Mechanics*, **58**, 151-161, DOI: 10.1016/j.ijnonlinmec.2013.09.009

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