

NUMERICAL MODELLING OF SHLIOMIS MODEL BASED FERROFLUID LUBRICATION PERFORMANCE IN ROUGH SHORT BEARING

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This paper has attempted to scrutinize the bearing performance of a rough short bearing assisted by a ferrofluid with the help of numerical modelling of the Shliomis model. The transverse roughness is calculated stochastically by averaging the Christensen and Tonder models. A non-zero mean is assumed for the probability density function for the random variable that determines the roughness of the bearing which is symmetrical. This attempt is made to create a more pragmatic and applicable situation. Expressions that can signify a dimensionless form of pressure and bearing load carrying capacity are found using Reynolds' equation. The load carrying capacity equation is then solved numerically with the help of Simpson's 1/3 rule to analyze the impact on the bearing system. From the graphical representation, it can be concluded that ferrofluid lubrication based on the Shliomis model can significantly neutralize the negative effects of the bearing roughness on its load carrying capacity.

Keywords: ferrofluid, roughness, load carrying capacity

1. Introduction

The slider bearing is one of the most basic and commonly used hydrodynamic bearing. The simplicity of the film thickness expression and the straightforwardness of boundary conditions can be held accountable for the same. Unlike other bearings, slider bearings do not create negative pressure which can be problematic for load bearing. This is because their film is continuous and non-diverging. Thus, they support axial loads. Many researchers have studied non-porous sliders. Christensen and Tonder (1969a,b, 1970) used a stochastic concept and came up with a new model for lubricated surfaces with striated roughness using an averaging film. They derived stochastic Reynolds' equation and used the results to study the impact of surface roughness on the load bearing capacity in a rough bearing system. Shliomis (1974) analyzed the modes of creating magnetic colloids along with the stability concerns. Patel *et al.* (2010b) worked on studying the performance of a smooth short bearing. Deheri and Patel (2011) and Patel *et al.* (2010a) analyzed the performance of a short rough bearing with a zero mean. They worked with a variety of magnetic field magnitudes in their study. Shimpi and Deheri (2010) further worked on the results of (Deheri and Patel, 2011; Patel *et al.*, 2010a) focusing on short bearings with the non-zero mean with a different form of the magnetic field magnitude. Shimpi and Deheri (2012) worked on it by including a deformation effect as well. Patel and Deheri (2013a) studied

further comparing two different types of porous structures. Patel and Deheri (2013b) extended the study of (Patel *et al.*, 2010b) by adding the aspect of slip velocity. They concluded that optimal performance can only be achieved at the minimum slip. It should be noted that the researches mentioned above used the Neuringer-Rosensweig model. Patel *et al.* (2015) conducted a different study by replacing the hydrodynamic bearing of the above-mentioned studies with a hydromagnetic porous short bearing. Additionally, it is a commonly known fact that roughness impacts the load carrying capacity substantially.

By reviewing literature concerning ferrofluid flow, one can understand that the Shliomis model displays better results than the Neuringer-Rosensweig model. Thus, this study is focused on scrutinizing the performance of ferrofluid lubrication using a short bearing based on the Shliomis model.

2. Analysis

The configuration of the short bearing system (infinitely short in the z -direction) is presented in Fig. 1. The slider has a velocity U in the x -direction. The breadth B is in the z -direction where $B \ll L$ (length). The pressure gradient $\partial p/\partial x$ can be neglected as the pressure gradient $\partial p/\partial z$ remains much larger.

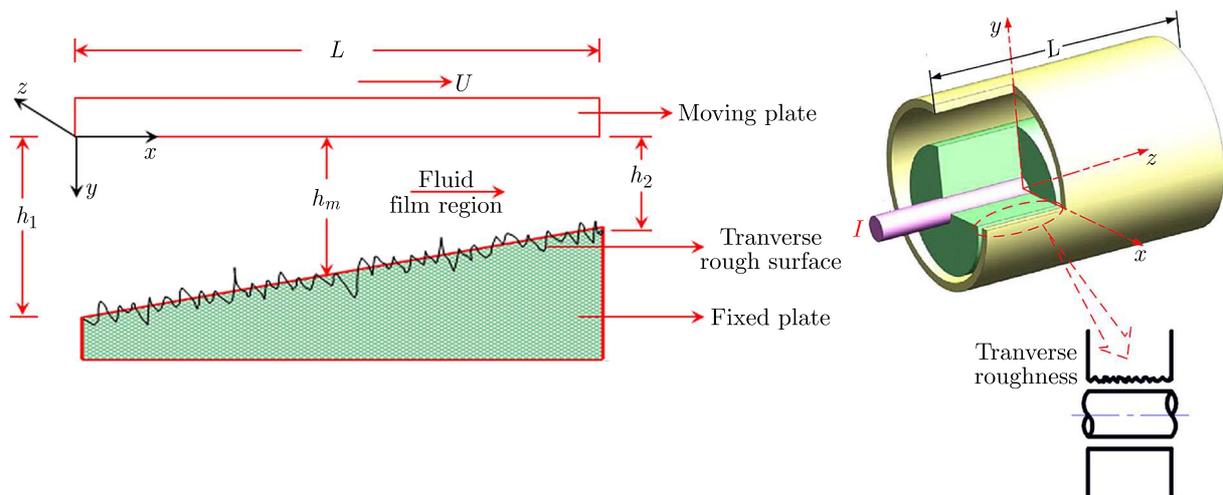


Fig. 1. Configuration of the bearing system (Patel and Deheri, 2013a)

The thickness h is considered as

$$h = h_m + h_s \quad (2.1)$$

where h_m is taken as (Patel and Deheri, 2013a)

$$h_m = h_2 \left[1 + m \left(1 - \frac{x}{L} \right) \right] \quad m = \frac{h_1 - h_2}{h_2}$$

using the works of (Christensen and Tonder, 1969a,b, 1970). Also, the study uses h_s using the probability density function (Christensen and Tonder, 1969a,b, 1970)

$$f(h_s) = \begin{cases} \frac{35}{32c} \left(1 - \frac{h_s^2}{c^2} \right)^3 & -c \leq h_s \leq c \\ 0 & \text{elsewhere} \end{cases} \quad (2.2)$$

with c being the maximum deviation from the mean film thickness. α , σ and ε are given by the relationships (Christensen and Tonder, 1969a,b, 1970)

$$\alpha = E(h_s) \quad \sigma^2 = E(h_s - \alpha)^2 \quad \varepsilon = E(h_s - \alpha)^3 \quad (2.3)$$

where the expectancy operator is given by

$$E(\cdot) = \int_{-c}^c (\cdot) f(h_s) dh_s \quad (2.4)$$

Actually, magnetic fluids or ferrofluids have a constant nature and are a type of colloidal suspensions that possess extremely superior magnetic particles in a viscous fluid. We can use an external magnetic field to position, limit or monitor these fluids as required. This, in turn, increases the fluid effective viscosity. This research has substantially contributed to an increased application of these magnetic fluids in bearing systems as lubricating agents. A noteworthy fact here is that major studies in the field use the Neuringer-Rosensweig model suggesting that magnetization vector is parallel to the applied magnetic field. Since the Shliomis model considers particle rotation, it overcomes this limitation.

Shliomis (1972, 1974) proposed that a change in the applied magnetic field can have a two-way implication on the particles in the magnetic fluid. Either, rotation of such particles is impacted or the magnetic moment changes in them. τ_B (Brownian relaxation time parameter) is used to derive particle rotation and τ_S (magnetic moment relaxation time parameter) suggests the intrinsic process of rotation. Considering a steady flow while overlooking the inertial and second derivatives of \mathbf{S} , the equations of flow becomes

$$-\nabla p + \eta \nabla^2 \mathbf{q} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{1}{2\tau_s} \nabla \times (\mathbf{S} - I\boldsymbol{\Omega}) = \mathbf{0} \quad (2.5)$$

where

$$\begin{aligned} \boldsymbol{\Omega} &= \frac{1}{2} \nabla \times \mathbf{q} & \mathbf{S} &= I\boldsymbol{\Omega} + \mu_0 \tau_s (\mathbf{M} \times \mathbf{H}) \\ \mathbf{M} &= M_0 \frac{\mathbf{H}}{H} + \frac{\tau_B}{I} (\mathbf{S} \times \mathbf{M}) & \nabla \times \mathbf{H} &= \mathbf{0} \end{aligned} \quad (2.6)$$

and

$$\nabla(\mathbf{H} + \mathbf{M}) = \mathbf{0} \quad (2.7)$$

The following can be derived by combining all the above mentioned equations; in the light of the procedure given in (Shliomis, 1974)

$$\begin{aligned} -\nabla p + \eta \nabla^2 \mathbf{q} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{1}{2} \mu_0 \nabla \times (\mathbf{M} \times \mathbf{H}) &= \mathbf{0} \\ \mathbf{M} = M_0 \frac{\mathbf{H}}{H} + \tau_B \boldsymbol{\Omega} \times \mathbf{M} + \frac{\mu_0 \tau_B \tau_s}{I} (\mathbf{M} \times \mathbf{H}) \times \mathbf{M} \end{aligned} \quad (2.8)$$

Langevin's parameter $\xi > 1$ is used for a strong magnetic field, the above equation (Shliomis, 1972, 1974; Bhat, 2003) changes to

$$\mathbf{M} = \frac{M_0}{H} [\mathbf{H} + \tau_B (\boldsymbol{\Omega} \times \mathbf{H})] \quad (2.9)$$

with

$$\bar{\tau} = \frac{6\eta\phi}{nk_B T(1 + \xi \coth \xi)} \quad (2.10)$$

where

$$M_0 = n\mu \left(\coth \xi - \frac{1}{\xi} \right) \quad H = \frac{k_B T \xi}{\mu_0 \mu} \quad (2.11)$$

For a deferment of spherical particles

$$\frac{I}{\tau_s} = 6\eta\phi \quad \tau_B = \frac{3\eta V}{k_B T} \quad (2.12)$$

In view of the discussion (Shliomis, 1974), the flow remains in the xz -plane while the magnetic field is taken in the y -direction by making use of the assumptions for $U = (u, v, w)$ and $\mathbf{H} = [H_x, H_y, H_z]$ where in $u, w \gg v$ and $H_y \gg H_x, H_z$. In the light of the boundary conditions of the magnetic field components at the plates, it can be inferred that $H_y \approx H_0$ while H_x, H_z remain negligible in comparison with H_0 . Therefore, for the axially symmetric flow, the associated uniform magnetic field may be represented by $\mathbf{H} = [0, H_0, 0]$.

Equations (2.8) and (2.9) develop into the following (Majumdar, 2008)

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left(\eta(1 + \tau) \frac{\partial u}{\partial y} \right) \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = \frac{\partial}{\partial y} \left(\eta(1 + \tau) \frac{\partial w}{\partial y} \right) \quad (2.13)$$

and

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (2.14)$$

where (referring to Shliomis, 1972)

$$\tau(\phi, \xi) = \frac{3}{2} \phi \frac{\xi - \tanh \xi}{\xi + \tanh \xi} \quad \eta = \eta_0 \left(1 + \frac{5}{2} \phi \right) \quad (2.15)$$

Solving equation (2.13) under no slip boundary conditions (Majumdar, 2008)

$$\begin{aligned} \text{at } y = 0 : \quad & u = u_b \quad w = w_b \\ \text{at } y = h : \quad & u = u_a \quad w = w_a \end{aligned}$$

one can find

$$\begin{aligned} u &= \frac{1}{2\eta(1 + \tau)} \frac{\partial p}{\partial x} y(y - h) + \frac{h - y}{h} u_b + \frac{y}{h} u_a \\ w &= \frac{1}{2\eta(1 + \tau)} \frac{\partial p}{\partial z} y(y - h) + \frac{h - y}{h} w_b + \frac{y}{h} w_a \end{aligned} \quad (2.16)$$

Putting the velocity components expression into continuity equation (2.14) and integrating under the conditions $v = v_b$ at $y = 0$ and $v = v_a$ at $y = h$ gives rise to

$$\begin{aligned} \rho(v_a - v_b) &= -\frac{1}{2} \left\{ \int_0^h \frac{\partial}{\partial x} \left[\frac{\rho}{\eta(1 + \tau)} \frac{\partial p}{\partial x} y(y - h) \right] dy + \int_0^h \frac{\partial}{\partial z} \left[\frac{\rho}{\eta(1 + \tau)} \frac{\partial p}{\partial z} y(y - h) \right] dy \right\} \\ &\quad - \int_0^h \frac{\partial}{\partial x} \left\{ \rho \left[\left(\frac{h - y}{h} \right) u_b + \frac{y}{h} u_a \right] \right\} dy - \int_0^h \frac{\partial}{\partial z} \left\{ \rho \left[\left(\frac{h - y}{h} \right) w_b + \frac{y}{h} w_a \right] \right\} dy - h \frac{\partial \rho}{\partial t} \end{aligned} \quad (2.17)$$

In reference to the experiment through the utilization of relation (Majumdar, 2008)

$$\int_0^h \frac{\partial}{\partial x} [f(x, y, z)] dz = \frac{\partial}{\partial x} \int_0^h f(x, y, z) dz - f(x, y, h) \frac{\partial h}{\partial x} \quad (2.18)$$

one obtains for constant velocities

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta(1+\tau)} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\rho h^3}{12\eta(1+\tau)} \frac{\partial p}{\partial z} \right) = \frac{U}{2} \frac{\partial}{\partial x} (\rho h) + \frac{\partial}{\partial t} (\rho h) \tag{2.19}$$

where

$$U = u_a + u_b \tag{2.20}$$

Generalized Reynolds' equation (2.19) turns in the state of equilibrium, which brings (Majumdar, 2008)

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta(1+\tau)} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\rho h^3}{12\eta(1+\tau)} \frac{\partial p}{\partial z} \right) = \frac{U}{2} \frac{\partial}{\partial x} (\rho h) \tag{2.21}$$

Modified two dimensional Reynolds' equation for an incompressible lubricant is

$$\frac{\partial}{\partial x} \left(\frac{h^3}{12\eta(1+\tau)} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{12\eta(1+\tau)} \frac{\partial p}{\partial z} \right) = \frac{U}{2} \frac{\partial h}{\partial x} \tag{2.22}$$

In the x -direction, the flow because of pressure gradient in the variation of pressure can be avoided when the bearing is short. In this case, one dimensional equation (2.22) leads to

$$\frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = 6\eta(1+\tau)U \frac{\partial h}{\partial x} \tag{2.23}$$

which is Reynolds' equation (Basu *et al.*, 2005; Majumdar, 2008; Shimpi and Deheri, 2010) modified according to the general hydrodynamic lubrication assumptions.

According to the stochastically average process (Shimpi and Deheri, 2010), (2.23) becomes

$$\begin{aligned} \frac{\partial}{\partial z} \left(E(h^3) \frac{\partial p}{\partial z} \right) &= 6\eta(1+\tau)U \frac{\partial}{\partial x} (E(h)) \\ \frac{\partial}{\partial z} \left(g(h_m, \alpha, \sigma, \varepsilon) \frac{\partial p}{\partial z} \right) &= 6\eta(1+\tau)U \frac{\partial}{\partial x} (h_m + \alpha) \end{aligned} \tag{2.24}$$

where

$$g(h_m, \alpha, \sigma, \varepsilon) = h_m^3 + 3\alpha h_m^2 + 3(\sigma^2 + \alpha^2)h_m + 3\sigma^2\alpha + \alpha^3 + \varepsilon \tag{2.25}$$

The following dimensionless quantities are used

$$\begin{aligned} \bar{X} &= \frac{x}{L} & \bar{Z} &= \frac{z}{B} & \bar{h} &= \frac{h_m}{h_2} = 1 + m(1 - \bar{X}) \\ \bar{g}(\bar{h}, \bar{\alpha}, \bar{\sigma}, \bar{\varepsilon}) &= \frac{g(h_m, \alpha, \sigma, \varepsilon)}{h_2^3} & \bar{\alpha} &= \frac{\alpha}{h_2} & \bar{\sigma} &= \frac{\sigma}{h_2} \\ \bar{\varepsilon} &= \frac{\varepsilon}{h_2^3} & \bar{p} &= \frac{ph_2^3}{\eta_0 UB^2} & \bar{L} &= \frac{L}{h_2} & \bar{B} &= \frac{B}{h_2} \end{aligned} \tag{2.26}$$

The associated boundary conditions (Lin *et al.*, 2013; Patel *et al.*, 2012; Deheri and Patel, 2011) are

$$\bar{p} = 0 \quad \text{at} \quad \bar{Z} = \pm \frac{1}{2} \quad \text{and} \quad \frac{d\bar{p}}{d\bar{Z}} = 0 \quad \text{at} \quad \bar{Z} = 0 \tag{2.27}$$

With the aid of (2.27), the pressure distribution in a non-dimensional form comes out to be

$$\bar{p} = \frac{3m(1 + 2.5\phi)(1 + \tau)}{\bar{L}\bar{g}(\bar{h}, \bar{\alpha}, \bar{\sigma}, \bar{\varepsilon})} \left(\frac{1}{4} - \bar{Z}^2 \right) \tag{2.28}$$

where

$$\bar{g}(\bar{h}, \bar{\alpha}, \bar{\sigma}, \bar{\varepsilon}) = \bar{h}^3 + 3\bar{\alpha}\bar{h}^2 + 3(\bar{\sigma}^2 + \bar{\alpha}^2)\bar{h} + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon} \tag{2.29}$$

The load bearing capacity in the dimensionless form (Patel and Deheri, 2013a) is obtained

$$\bar{W} = \frac{h_2^3 W}{\eta_0 U B^4} \quad \bar{W} = \frac{m(1 + 2.5\phi)(1 + \tau)}{2\bar{B}} \int_0^1 \frac{d\bar{X}}{\bar{g}(\bar{h}, \bar{\alpha}, \bar{\sigma}, \bar{\varepsilon})} \tag{2.30}$$

where the load bearing capacity is calculated using (Patel and Deheri, 2013a)

$$W = \int_{-\frac{B}{2}}^{\frac{B}{2}} \int_0^L p(x, z) dx dz \tag{2.31}$$

3. Results and discussion

The final values of \bar{W} using different parameters are plotted graphically. Figures 2a-2f display the changes in \bar{W} corresponding to different values of τ . They suggest that magnetization leads

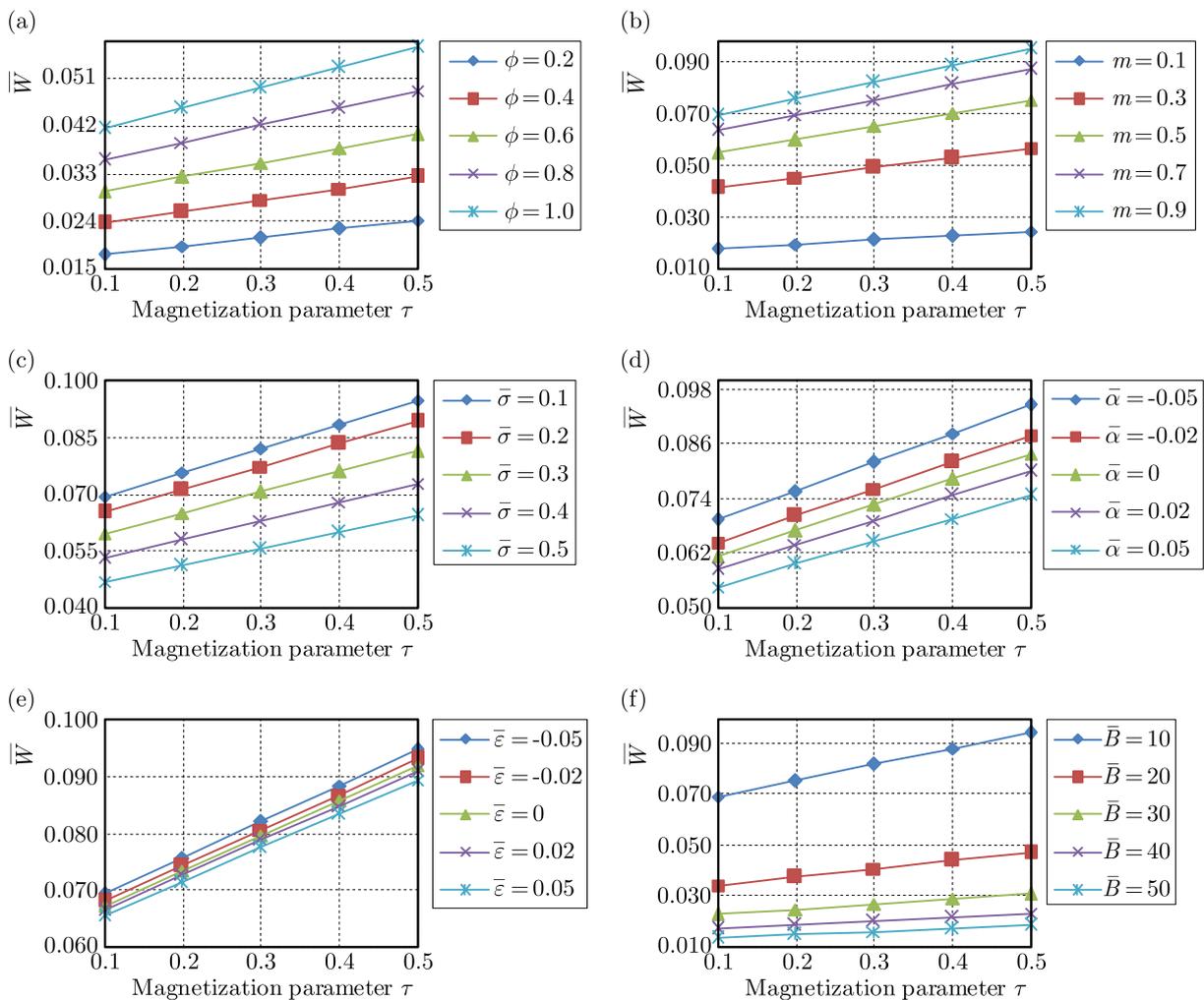


Fig. 2. Trends of \bar{W} concerned with τ and: (a) ϕ , (b) m , (c) $\bar{\sigma}$, (d) $\bar{\alpha}$, (e) $\bar{\varepsilon}$, (f) \bar{B}

to a substantial increase in \bar{W} . It is noticed from Fig. 2e that the value of maximum \bar{W} derived is 0.094 at a smaller value of $\bar{\varepsilon} = -0.05$ with regards to τ .

From Figs. 3a-3e, it can be suggested that with an increase in \bar{W} , ϕ shifts from 0.2 to 1.

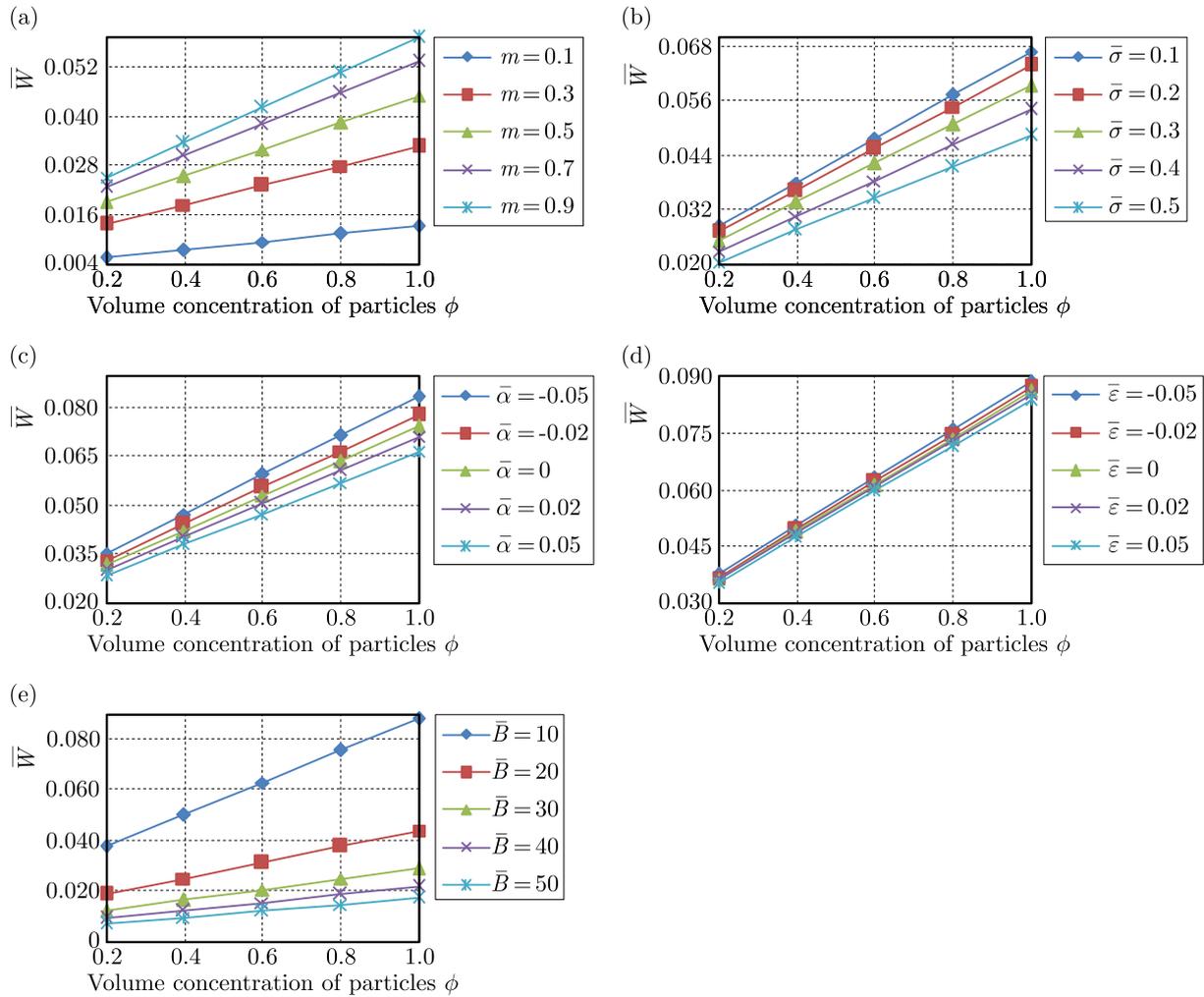


Fig. 3. Trends of \bar{W} concerned with ϕ and: (a) m , (b) $\bar{\sigma}$, (c) $\bar{\alpha}$, (d) $\bar{\varepsilon}$, (e) \bar{B}

Figures 4a-4d display the impact of the aspect ratio on \bar{W} and suggests that the aspect ratio causes a sharp increase in \bar{W} .

The decrease in the value of \bar{W} in Figs. 5a-5c displays the unfavorable impact of standard deviation on the bearing system performance.

Figures 6a and 6b establish an inverse relationship between the variance and \bar{W} suggesting that positive variance reduces \bar{W} while negative variance increases it. Under the impact of skewness, \bar{W} changes according to Fig. 7.

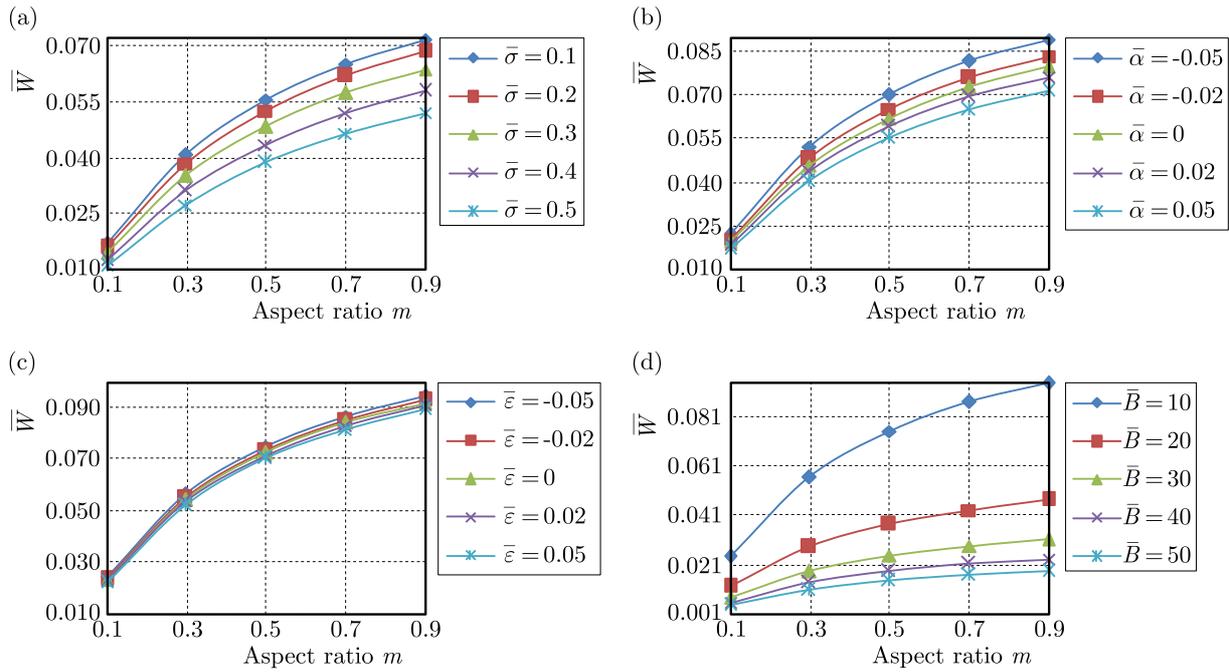


Fig. 4. Trends of \bar{W} concerned with m and: (a) $\bar{\sigma}$, (b) $\bar{\alpha}$, (c) $\bar{\epsilon}$, (d) \bar{B}

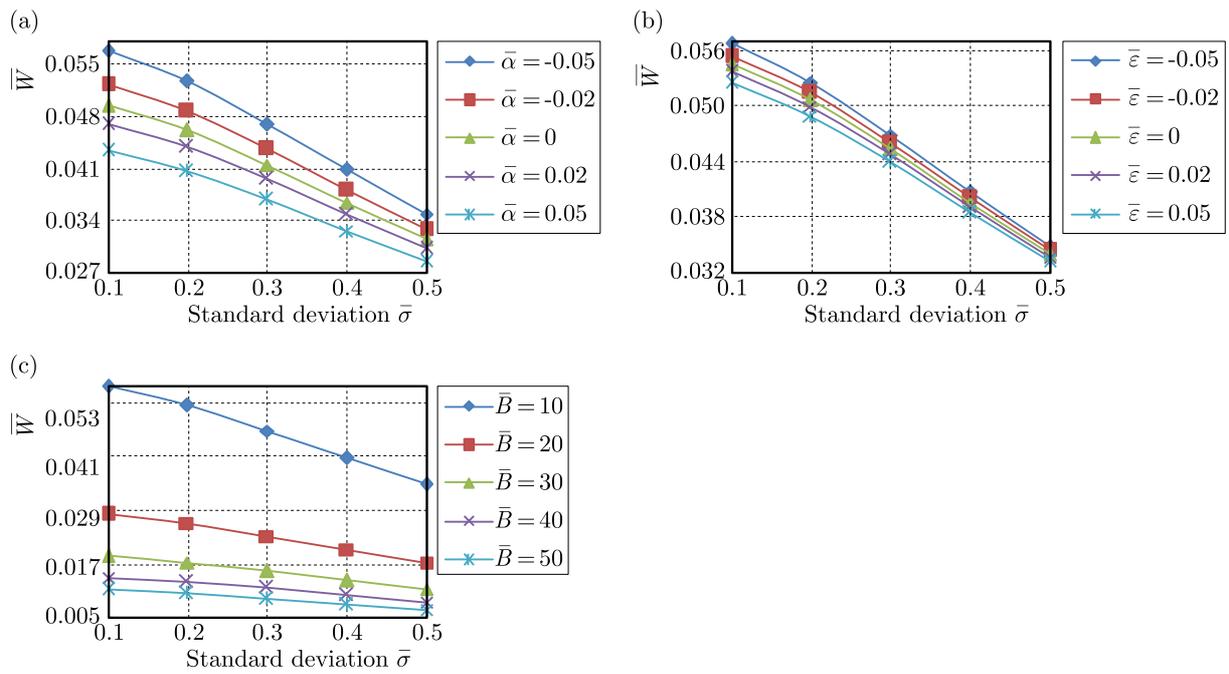


Fig. 5. Trends of \bar{W} concerned with $\bar{\sigma}$ and: (a) $\bar{\alpha}$, (b) $\bar{\epsilon}$, (c) \bar{B}

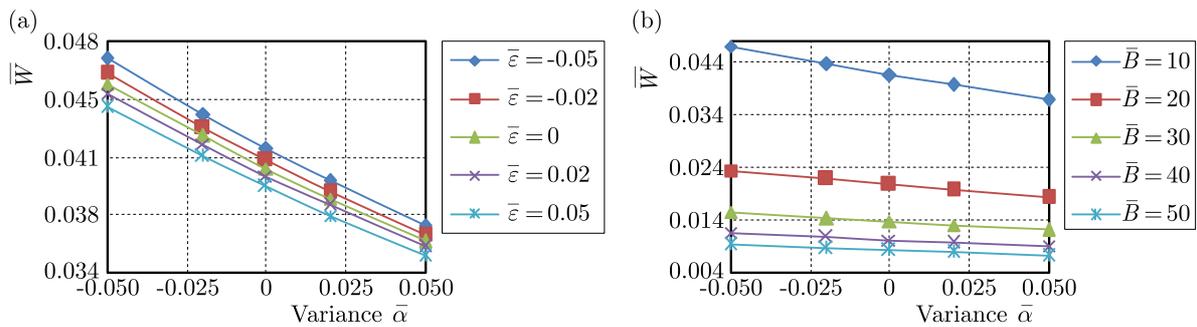


Fig. 6. Trends of \bar{W} concerned with $\bar{\alpha}$ and: (a) $\bar{\epsilon}$, (b) \bar{B}

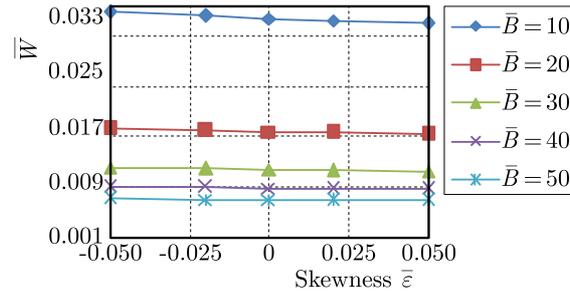


Fig. 7. Trends of \bar{W} concern with $\bar{\varepsilon}$ and \bar{B}

4. Validation

The validation of the conclusion of this paper has been achieved by using the following comparison sets used in other publication. Tables 1 to 5 depicts the increase in the load carrying capacity over 14%.

Table 1. Comparison of \bar{W} calculated for $\bar{\alpha}$

Quantity	Load bearing capacity (calculated for $\mu^* = 0.09, \bar{\sigma} = 0.1, \bar{\varepsilon} = -0.05, \phi = 0.1, \tau = 0.1, m = 0.3, \bar{B} = 10, \bar{L} = 0.5$)			
	$\bar{\alpha}$	Result of the current study	Result of Patel and Deheri (2013a)	increase [%]
-0.05		0.0148500	0.0115134	28.98
-0.02		0.0136554	0.0106446	28.28
0		0.0129308	0.0101176	27.80
0.02		0.0122575	0.0096279	27.31
0.05		0.0113339	0.0089562	26.54

Table 2. Comparison of \bar{W} calculated for $\bar{\sigma}$

Quantity	Load bearing capacity (calculated for $\mu^* = 0.09, \bar{\alpha} = 0.02, \bar{\varepsilon} = -0.05, \phi = 0.1, \tau = 0.1, m = 0.3, \bar{B} = 10, \bar{L} = 0.5$)			
	$\bar{\sigma}$	Result of the current study	Result of Patel and Deheri (2013a)	increase [%]
0.1		0.0122575	0.0096279	27.31
0.2		0.0114705	0.0090555	26.66
0.3		0.0103649	0.0082515	25.61
0.4		0.0091364	0.0073581	24.16
0.5		0.0079318	0.0064819	22.36

Table 3. Comparison of \bar{W} calculated for $\bar{\varepsilon}$

Quantity	Load bearing capacity (calculated for $\mu^* = 0.09, \bar{\sigma} = 0.1, \bar{\alpha} = 0.02, \phi = 0.1, \tau = 0.1, m = 0.3, \bar{B} = 10, \bar{L} = 0.5$)			
	$\bar{\varepsilon}$	Result of the current study	Result of Patel and Deheri (2013a)	increase [%]
-0.05		0.0122575	0.0096279	27.31
-0.02		0.0120166	0.0094527	27.12
0		0.0118614	0.0093399	26.99
0.02		0.0117104	0.0092300	26.87
0.05		0.0114912	0.0090706	26.68

Table 4. Comparison of \bar{W} calculated for m

Quantity	Load bearing capacity (calculated for $\mu^* = 0.09, \bar{\sigma} = 0.1, \bar{\varepsilon} = 0.02, \phi = 0.1, \tau = 0.1, \bar{\alpha} = 0.02, \bar{B} = 10, \bar{L} = 0.5$)		
	Result of the current study	Result of Patel and Deheri (2013a)	increase [%]
m			
0.1	0.0050043	0.0043529	14.96
0.3	0.0117104	0.0092300	26.87
0.5	0.0158152	0.0122153	29.46
0.7	0.0184708	0.0141467	30.56
0.9	0.0202714	0.0154562	31.15

Table 5. Comparison of \bar{W} calculated for \bar{B}

Quantity	Load bearing capacity (calculated for $\mu^* = 0.09, \bar{\sigma} = 0.1, \bar{\varepsilon} = 0.02, \phi = 0.1, \tau = 0.1, m = 0.3, \bar{\alpha} = 0.02, \bar{L} = 0.5$)		
	Result of the current study	Result of Patel and Deheri (2013a)	increase [%]
\bar{B}			
10	0.0117104	0.009230	26.87
20	0.0058552	0.0046150	26.87
30	0.0039035	0.0030767	26.87
40	0.0029276	0.0023075	26.87
50	0.0023421	0.0018460	26.87

5. Conclusion

The impact of ferrofluid lubrication on the load bearing capacity of a short bearing system with a rough surface is studied. From the numerical computations performed, the analysis has yielded the following conclusions:

- Shliomis' ferrofluid flow provides relevant insights on the impact of rotations of the carrier liquid and magnetic particles. Furthermore, a varying magnetic field provides the benefit of creating the maximum field according to the necessary contact area of the bearing.
- This work is crucial because it provides more freedom than (Verma, 1986) and (Prajapati, 1994) regarding the magnitude.
- Standard deviation is the most important parameter in determining the performance of a bearing system of this type.
- Negatively skewed roughness aids the load carrying capacity and boosts the performance. Another thing to be understood is that a constant magnetic field shows a positive effect on the bearing capacity in the Shliomis model while the same is not true for the Neuringer-Rosensweig ferrofluid flow model.
- Furthermore, this article can create a new pathway for ensuring maximum utilization of the bearing system. It also clearly proposes that by managing the lubricant loss, the life span of the load bearing system can be increased substantially.

Nomenclature

- B – breadth of bearing
 g – function of different parameters
 h, h_m – film thickness and mean film thickness [mm]
 h_s – deviation from mean level

h_1, h_2	–	maximum and minimum film thickness
H, H_0	–	magnitude of \mathbf{H} and constant magnetic field
I	–	sum of moments of inertia of particle per unit volume [Ns^2/m^2]
k_B	–	Boltzmann constant [J/K]
L	–	length of bearing, $B \ll L$
m	–	aspect ratio
\mathbf{M}	–	magnetization vector
M_0	–	equilibrium magnetization [A/m]
n	–	number of particles per unit [m^{-3}]
p	–	pressure in film region [N/m^2]
\mathbf{q}	–	fluid velocity, $\mathbf{q} = [u, v, w]$
\mathbf{S}	–	internal angular momentum vector
T	–	temperature [K]
u, v, w	–	velocity components in x, y and z directions
u_a, v_a, w_a	–	velocity components of upper surfaces in x, y and z directions
u_b, v_b, w_b	–	velocity components of lower surfaces in x, y and z directions
V	–	volume of particle
W	–	load capacity [N]
x, z	–	bearing width and length coordinates
y	–	fluid film thickness coordinate
α	–	variance [mm]
ε	–	skewness [mm^3]
η, η_0	–	viscosity of suspension and of main liquid [Ns/m^2]
μ	–	magnetic moment of particle
μ_0	–	permeability of free space [N/A^2]
ρ	–	lubricant density
σ	–	standard deviation [mm]
ϕ	–	volume concentration of particles, $\phi = nV$
τ	–	magnetization parameter
Ω	–	fluid vorticity
$\overline{(\cdot)}$	–	non-dimensional quantity

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