

## THEORETICAL SIMULATION OF TEMPERATURE DISTRIBUTION IN A GUN BARREL BASED ON THE DPL MODEL

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In this paper, an exact closed form solution is introduced for the heat conduction equation in cylindrical coordinates under consecutive inner time dependent surface heat flux by both the Fourier and dual-phase-lag (DPL) models. The solution is used to calculate the temperature distribution in a gun barrel subjected to single and consecutive shoots, and the results are compared with literature. The parametrical study is done using the analytical solution to show the effect of shooting frequency which leads to different heat power from each fire shoot and temperature distribution. The result shows good ability of analytical solution for estimation of temperature distribution in the gun barrel, especially under consecutive shoots in which unexpected incidents such as barrel melting is so probable. The closed form solution can be applied for verification of other numerical works in this area.

*Keywords:* DPL model, closed form solution, gun barrel, exponential decaying heat flux

### 1. Introduction

According to literature, the classical Fourier theory can estimate an acceptable thermal distribution in most engineering materials but the thermal propagation velocity in this model is infinite, thus in some applications such as great heat flux on a surface or small time of interaction this model must be improved. Determination of temperature distribution in a gun barrel that is exposed to great heat flux in a short period is one of the mentioned phenomena. At first, the non-Fourier heat flux equation as a modified model was presented by Cattaneo (1958), Vernotte (1961) (C-V relation) as a single phase model, and Tzou (1995, 1997) proposed a new concept of dual-phase-lag (DPL) model in which the heat flux and temperature gradient could be simulated with lag times. This model is appropriate for thermal analysis of metals (Tzou, 1995, 1997) and is used to analysis heat transfer in some relative applications (Afrin *et al.*, 2014; Ghazanfarian and Abbassi, 2009, 2012; Ghazanfarian and Shomali, 2012; Han *et al.*, 2006; Liu and Chen, 2010).

Considering a finite speed for thermal propagation and phonon-electron interactions in metals, the dual-phase-lag heat conduction model is

$$q + \tau_q \frac{\partial q}{\partial t} = -k \left( \nabla T + \tau_T \frac{\partial}{\partial t} \nabla T \right) \quad (1.1)$$

where  $T$  is temperature,  $q$  is heat flux,  $k$  is thermal conductivity,  $\tau_q$  shows the time lag of heat flux and  $\tau_T$  is the time lag of the temperature gradient with constant magnitudes about picosecond and a common specific ratio of  $\tau_T/\tau_q = 100$  for metals (Chandrasekharaiah, 1998).

The dual-phase-lag heat conduction equation is obtained using the above constitutive relation and energy equation as

$$\tau_q \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \left( \nabla^2 T + \tau_T \frac{\partial}{\partial t} \nabla^2 T \right) \quad (1.2)$$

where  $\alpha = k/(\rho C)$  is thermal diffusivity,  $\rho$  is density and  $C$  is specific heat of the material. This equation transforms to the C-V Non-Fourier heat conduction model when  $\tau_T = 0$  and reduces to the classical Fourier heat conduction if  $\tau_T = \tau_q = 0$ .

Some engineering problems in cylindrical coordinates are solved by a non-Fourier model. Torabi and Saedodin (2011) studied the temperature distribution in cylindrical coordinates numerically and analytically and reported that for some cases in hyperbolic heat conduction, up to 40% error occurred in the numerical method. Mishra and Sahai (2012) implemented the lattice Boltzmann method for analysis of hyperbolic heat conduction in spherical and cylindrical coordinates.

Gheitaghy and Talaee (2013) solved the hyperbolic and parabolic heat conduction by an electrical simulation method. Talaee *et al.* (2014) presented an exact analytical solution for non-Fourier thermal stress in a cylindrical shell considering C-V relation under a periodic boundary condition. Saedodin and Barforoush (2017) solved the hyperbolic heat conduction equation in a typical cylinder under special heat flux and found that an increase of the Vernotte number led to delay in sense of thermal wave in a specific point.

Also the DPL method is used for analysis of thermal phenomena by some researchers. Akbarzadeh and Chen (2012) investigated the transient heat conduction in a functionally graded cylindrical panel based on DPL theory and reported the ability of this model for analysis of parabolic and hyperbolic heat conduction. In addition, they found that an increase of the phase lag led to a decrease of the speed of thermal wave along with growth in amplitude of the transient temperature. Torabi and Zhang (2014) investigated the temperature distribution of a cylindrical geometry subjected to two types of continuously and exponential pulsed heat flux boundary conditions by semi-analytical and numerical methods.

The inner surface of gun barrel is exposed to high heat load due to hot species of combustion products that must be calculated in design of an automatic gun. This phenomenon will be more critical and can threaten the shooter by self-ignition or melting due to temperature rise in the automatic gun in the continuous fire mode. In addition, change in the gun caliber due to high temperature can reduce the shooting accuracy. Therefore, for proper thermal design of the gun barrel, some research studies are done to determine the temperature of the barrel at different conditions.

The classical Fourier law is used as a practical tool to analyze most heat transfer phenomena such as thermal analysis of gun barrels. Chen *et al.* (2007) presented an inverse method for estimation of heat flux in a multi-layer gun barrel at continuous firing conditions which showed that the bore surface material can be melted due to heat flux of shoots. Wu *et al.* (2008) studied the heat transfer in a 155 mm mid wall cooled compound gun barrel theoretically and numerically and demonstrated ineffectiveness of natural air cooling and effectiveness of forced mid wall cooling for decreasing temperature of the barrel. Chen and Liu (2008) estimated two-dimensional heat flux in a gun barrel by using finite element schemes and an inverse estimation method, and showed the ability of that method for estimation of time varying heat flux and temperature distribution in the gun chamber. Lee *et al.* (2009) estimated the time varying heat flux and thermal stresses in a machine gun by an inverse algorithm based on the conjugated gradient method. Mishra *et al.* (2010) computed the heat transfer to the gun barrel for nine shots by the finite element method and validated the model with experimental measurements. In addition, they found that exponentially decaying heat flux was an accurate model for temperature variation of the gun barrel. Hill and Conner (2012) presented a numerical model of transient heat transfer in machine gun barrels which defined the temperature profile through thickness of the barrel by the finite difference method.

The obtaining an exact temperature distribution is of importance due to the need of exact thermal design of the gun barrel. It can be seen from the literature that there is not any exact analytical model introduced for thermal analysis of the gun barrel. Here, an exact closed

form solution is introduced for the heat conduction equation in cylindrical coordinates under consecutive inner time dependent surface heat flux by both the Fourier and DPL models. The comparison of estimation is done for the solution, and parametrical study is done to show the ability of finding a solution for the best gun thermal design. The conditions of self-ignition and melting relative to the shooting frequency and the number of shoots can be determined exactly by this model. The closed form solution can be applied for verification of other numerical works in this area.

## 2. Mathematical model

The gun barrel model as an infinite hollow cylinder is shown in Fig. 1 with inner and outer radius of  $r_i$  and  $r_o$ .

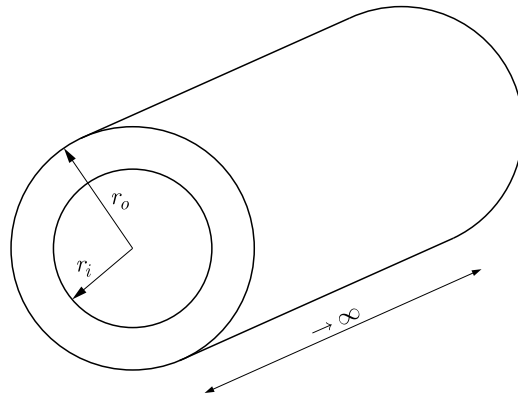


Fig. 1. Schematic of hollow cylindrical model

The one-dimensional DPL thermal wave model in cylindrical coordinate is considered as

$$\tau_q \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \tau_T \left( \frac{1}{r} \frac{\partial^2 T}{\partial t \partial r} + \frac{\partial}{\partial t} \frac{\partial^2 T}{\partial r^2} \right) \right] \quad (2.1)$$

The inner boundary condition of the barrel is considered as subject to an exponentially decaying heat flux as  $q(r_i, t) = q_0 f(t)$  for each shoot, where  $f(t)$  is defined as  $f(t) = \exp(-t/t_0)$  (Mishra *et al.*, 2010) as shown in Fig. 2 in which  $t_0$  is a constant that depends on pressure and muzzle velocity and projectile mass (Lawton, 2001). The two time-dependent function of the produced heat flux for Gun (2) (Seiler *et al.*, 2003) and Gun (3) (Dębski *et al.*, 2016) is shown in Fig. 2.

Thus, the time dependent boundary condition on the inner surface is

$$\tau_T \frac{\partial^2 T}{\partial t \partial r} + \frac{\partial T}{\partial r} = -\frac{q_0}{k} \left( f(t) + \tau_q \frac{df(t)}{dt} \right) \quad r = r_i \quad (2.2)$$

This boundary condition can be seen as a first order ordinary differential equation related to time, which can be solved to get

$$\left. \frac{\partial T}{\partial r} \right|_{r=r_i} = -\frac{q_0}{k} \frac{t_0 - \tau_q}{t_0 - \tau_T} \left( e^{-\frac{t}{t_0}} - e^{-\frac{t}{\tau_T}} \right) \quad (2.3)$$

The outer surface of the gun barrel is considered to be exposed to air, therefore, undergoes convection heat transfer, which can be defined as

$$\left. \frac{\partial T}{\partial r} \right|_{r=r_o} = h(T - T_\infty) \quad (2.4)$$

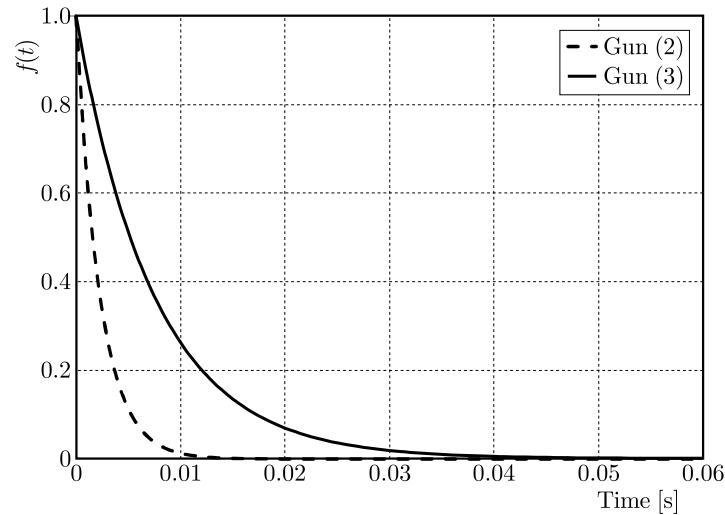


Fig. 2. The time-dependent function of heat flux applied to the inner surface of the gun barrel,  $f(t)$

The initial conditions of this problem are

$$T(r, 0) = T_{\infty} \quad \left. \frac{\partial T}{\partial t} \right|_{t=0} = 0 \quad (2.5)$$

### 3. Analytical solution procedure

The method of separation of variables can not be applied at first due to the time-dependent boundary condition of the inner surface. Therefore, this kind of problem should be solved with a time-independent boundary condition, and then its time dependency should be applied to the solution by using Duhamel's integral. Thus the pure analytical solution procedure according to the superposition principle consists of four steps (Talaei *et al.*, 2014, 2016, 2018; Atefi and Talaei, 2011; Talaei and Sarafrazi, 2017; Talaei and Atefi, 2011; Talaei and Kabiri, 2017a,b):

- The steady equation with a non-homogeneous time-independent boundary condition  $[\theta_0(r)]$
- The transient equation with homogeneous and time-independent boundary conditions and the reformed initial condition  $[\theta_1(r, t)]$
- Summing the steady and transient solutions as a solution to the problem with time-independent boundary conditions
- Applying the time function of the boundary condition to the final solution using Duhamel's integral.

The above steps are explained in the following. According to the compliance of the results and the solution procedure, the variable  $\theta = T - T_{\infty}$  is used afterwards.

#### 3.1. Steady-state problem

The steady equation with the non-homogeneous boundary condition is

$$\frac{d^2\theta_0}{dr^2} + \frac{1}{r} \frac{d\theta_0}{dr} = 0 \quad (3.1)$$

and

$$\left. \frac{d\theta_0}{dr} \right|_{r_i} = -\frac{q_0}{k} \frac{t_0 - \tau_q}{t_0 - \tau_T} \quad \left. \frac{d\theta_0}{dr} \right|_{r_o} = -\frac{h}{k} \theta_0(r_o) \quad (3.2)$$

Solution to the steady problem can be simply obtained as

$$\theta_0(r) = \frac{q_0 r_i}{k} \frac{t_0 - \tau_q}{t_0 - \tau_T} \left( \frac{k}{hr_o} + \ln \frac{r_o}{r} \right) \quad (3.3)$$

### 3.2. Transient problem

The transient DPL heat conduction equation with homogeneous boundary conditions and the reformed initial condition is shown in Eqs. (3.4) to (3.6), respectively

$$\tau_q \frac{\partial^2 \theta_1}{\partial t^2} + \frac{\partial \theta_1}{\partial t} = \alpha \left( \frac{\partial^2 \theta_1}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_1}{\partial r} + \tau_T \frac{1}{r} \frac{\partial^2 \theta_1}{\partial t \partial r} + \tau_T \frac{\partial}{\partial t} \frac{\partial^2 \theta_1}{\partial r^2} \right) \quad (3.4)$$

$$\left. \frac{\partial \theta_1}{\partial r} \right|_{r_i} = 0 \quad \left. \frac{\partial \theta_1}{\partial r} \right|_{r_o} = 0 \quad (3.5)$$

$$\theta_1(r, 0) = -\theta_0(r) \quad \left. \frac{\partial \theta_1}{\partial t} \right|_{t=0} = 0 \quad (3.6)$$

The solution yields by separating the variables ( $\theta_1(r, t) = R(r)T(t)$ ) in Eqs. (3.4) to (3.6) as

$$\theta_1(r, t) = \sum_{m=1}^{\infty} (A_m e^{r_1 t} + B_m e^{r_2 t}) \Phi_m(r) \quad (3.7)$$

where

$$r_1, r_2 = \frac{-(1 + \alpha \lambda_m^2 \tau_T) \pm \sqrt{(1 + \alpha \lambda_m^2 \tau_T)^2 - 4 \tau_q \alpha \lambda_m^2}}{2 \tau_q}$$

$$\Phi_m(r) = J_0(\lambda_m r) - \frac{J_1(\lambda_m r_i)}{Y_1(\lambda_m r_i)} Y_0(\lambda_m r)$$

and  $J_0$ ,  $Y_0$  are the Bessel functions of the first and second kind, respectively, and  $\lambda = \lambda_m$  is the root of the following transcendental equation that is obtained by application of boundary conditions

$$Y_1(\lambda r_i) \left( \frac{h}{k} J_0(\lambda r_o) - \lambda J_1(\lambda r_o) \right) - J_1(\lambda r_i) \left( \frac{h}{k} Y_0(\lambda r_o) - \lambda Y_1(\lambda r_o) \right) = 0 \quad (3.8)$$

Applying the initial conditions in Eq. (3.6), two equations are defined for coefficients  $A_m$  and  $B_m$  as

$$A_m + B_m = - \frac{\int_{r_i}^{r_o} \theta_0(r) \Phi_m(r) r dr}{\int_{r_i}^{r_o} \Phi_m^2(r) r dr} = I \quad A_m r_1 + B_m r_2 = 0 \quad (3.9)$$

So

$$A_m = I \frac{r_2}{r_2 - r_1} \quad B_m = -I \frac{r_1}{r_2 - r_1} \quad (3.10)$$

in which  $I$  is

$$\begin{aligned} I = & - \left\{ \frac{q_0 r_i}{k} \frac{t_0 - \tau_q}{t_0 - \tau_T} \left[ \frac{kr J_1(\lambda_m r)}{hr_o \lambda_m} - \frac{J_0(\lambda_m r) - r \lambda_m J_1(\lambda_m r) \ln \frac{r_o}{r}}{\lambda_m^2} \right. \right. \\ & \left. \left. - \frac{J_1(\lambda_m r_i)}{Y_1(\lambda_m r_i)} \left( \frac{kr Y_1(\lambda_m r)}{hr_o \lambda_m} - \frac{Y_0(\lambda_m r) - r \lambda_m Y_1(\lambda_m r) \ln \frac{r_o}{r}}{\lambda_m^2} \right) \right] \right\} / \left\{ \frac{r^2}{2} [J_0^2(\lambda_m r) \right. \\ & \left. + J_1^2(\lambda_m r) + \left( \frac{J_1(\lambda_m r_i)}{Y_1(\lambda_m r_i)} \right)^2 [Y_0^2(\lambda_m r) + Y_1^2(\lambda_m r)] - 2 \frac{J_1(\lambda_m r_i)}{Y_1(\lambda_m r_i)} [J_0(\lambda_m r) Y_0(\lambda_m r) \right. \right. \\ & \left. \left. + J_1(\lambda_m r) Y_1(\lambda_m r)] \right] \right\} \end{aligned} \quad (3.11)$$

### 3.3. Superposition of the temperature field

According to the superposition principle, solution of the problem with a time-independent boundary condition is defined as

$$\theta(r, t) = \theta_0(r) + \theta_1(r, t) = \frac{q_0 r_i}{k} \frac{t_0 - \tau_q}{t_0 - \tau_T} \left( \frac{k}{hr_o} + \ln \frac{r_o}{r} \right) + \sum_{m=1}^{\infty} A_m G(t) \Phi_m(r) \quad (3.12)$$

where

$$G(t) = e^{r_1 t} - \frac{r_1}{r_2} e^{r_2 t}$$

### 3.4. Application of Duhamel's integral

In this step, the complete solution is defined by applying the function  $F(t)$  on the solution of the problem with the time-independent boundary condition  $\theta(r, t)$ . The Duhamel relation can be written as (Talaei and Atefi, 2011)

$$\bar{\theta}(r, t) = F(0)\theta(r, t) + \int_0^t \theta(r, t - \tau) \frac{\partial F(\tau)}{\partial \tau} d\tau \quad (3.13)$$

Here  $F(t) = e^{-t/t_0} - e^{-t/\tau_T}$  is the time-dependent part of the boundary condition, and  $F(0) = 0$ . Thus, the complete solution can be determined as

$$\bar{\theta}(r, t) = \int_0^t \left\{ \left[ \frac{q_0 r_i}{k} \frac{t_0 - \tau_q}{t_0 - \tau_T} \left( \frac{k}{hr_o} + \ln \frac{r_o}{r} \right) + \sum_{m=1}^{\infty} A_m G(t - \tau) \Phi_m(r) \right] \left( \frac{e^{-\frac{\tau}{\tau_T}}}{\tau_T} - \frac{e^{-\frac{\tau}{t_0}}}{t_0} \right) d\tau \right\} \quad (3.14)$$

Doing time integration of the above solution, the final closed form solution of the temperature distribution in the gun barrel due to a single shoot is reduced to

$$\begin{aligned} \bar{\theta}(r, t) = & \theta_0(r) \left( e^{-\frac{t}{t_0}} - e^{-\frac{t}{\tau_T}} \right) + \sum_{m=1}^{\infty} \Phi_m(r) \left[ A_m \left( \frac{e^{r_1 t} - e^{-\frac{t}{\tau_T}}}{r_1 \tau_T + 1} - \frac{e^{r_1 t} - e^{-\frac{t}{t_0}}}{r_1 t_0 + 1} \right) \right. \\ & \left. + B_m \left( \frac{e^{r_2 t} - e^{-\frac{t}{\tau_T}}}{r_2 \tau_T + 1} - \frac{e^{r_2 t} - e^{-\frac{t}{t_0}}}{r_2 t_0 + 1} \right) \right] \end{aligned} \quad (3.15)$$

For consecutive shooting,  $f(t)$  can be a series of single shoots as introduced (Talaei *et al.*, 2018) in Eq. (3.16)

$$f(t) = \sum_{n=1}^s e^{-\frac{t-(n-1)u}{t_0}} \quad (3.16)$$

where  $u$  is the time interval between the consecutive shoots and  $s$  is the number of shoots which is determined as the Bracket function of  $[(t/u) + 1]$ .

Finally, this solution for the consecutive shoots is given as

$$\begin{aligned} \bar{\theta}(r, t) = & \theta_0(r) \sum_{n=1}^s \left( e^{-\frac{t-(n-1)u}{t_0}} - e^{-\frac{t-(n-1)u}{\tau_T}} \right) \\ & + \sum_{n=1}^s \sum_{m=1}^{\infty} \Phi_m(r) \left[ A_m \left( \frac{e^{r_1(t-(n-1)u)} - e^{-\frac{t-(n-1)u}{\tau_T}}}{r_1 \tau_T + 1} - \frac{e^{r_1(t-(n-1)u)} - e^{-\frac{t-(n-1)u}{t_0}}}{r_1 t_0 + 1} \right) \right. \\ & \left. + B_m \left( \frac{e^{r_2(t-(n-1)u)} - e^{-\frac{t-(n-1)u}{\tau_T}}}{r_2 \tau_T + 1} - \frac{e^{r_2(t-(n-1)u)} - e^{-\frac{t-(n-1)u}{t_0}}}{r_2 t_0 + 1} \right) \right] \end{aligned} \quad (3.17)$$

The solution of the problem for one shoot in the Fourier model, doing the above procedure, is

$$\bar{\theta}(r, t) = \theta_0(r)e^{-\frac{t}{t_0}} + \sum_{m=1}^{\infty} C_m \Phi_m \left( e^{-\alpha \lambda_m^2 t} + \frac{e^{-\alpha \lambda_m^2 t} - e^{-\frac{t}{t_0}}}{\alpha t_0 \lambda_m^2 - 1} \right) \quad (3.18)$$

And the solution for the consecutive shoots in the Fourier (parabolic equation) model is

$$\begin{aligned} \bar{\theta}(r, t) = & \theta_0(r) \sum_{n=1}^s e^{-\frac{t-(n-1)u}{t_0}} \\ & + \sum_{n=1}^s \sum_{m=1}^{\infty} C_m \Phi_m \left( e^{-\alpha \lambda_m^2 (t-(n-1)u)} + \frac{e^{-\alpha \lambda_m^2 (t-(n-1)u)} - e^{-\frac{t-(n-1)u}{t_0}}}{\alpha t_0 \lambda_m^2 - 1} \right) \end{aligned} \quad (3.19)$$

#### 4. Results and discussion

In order to demonstrate the accuracy of the introduced model, the temperature variation during a single shoot is analyzed by both the Fourier and DPL models and compared with experimental data as shown in Fig. 3. The simulation properties are adopted for Gun (1) as discussed in literature in Table 1. As shown in Fig. 4, the dimensionless temperature of both approaches for one shoot have good agreement with the experiment (Lawton, 2001).

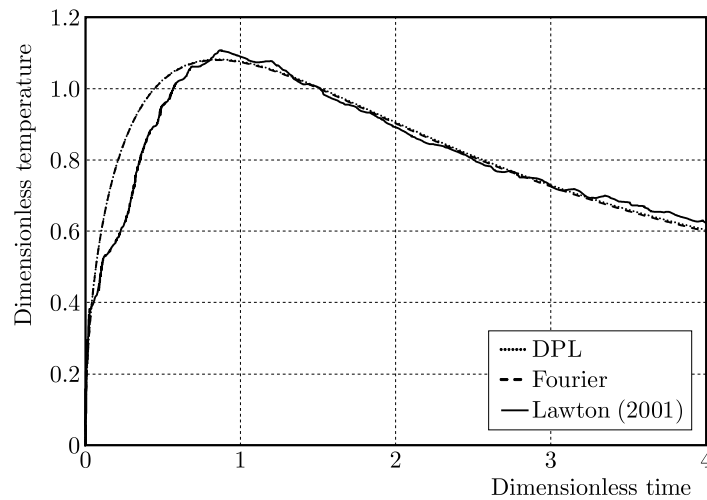


Fig. 3. Dimensionless temperature of the inner surface for a single shoot for Gun (1)

At the next step, the ability of the solution is shown by a comparison with the experiment (Seiler *et al.*, 2003) in the state of consecutive multi shoots for Gun (2) as shown in Fig. 4. The temperature profile is plotted applying real parameters of the guns as shown in Table 1.

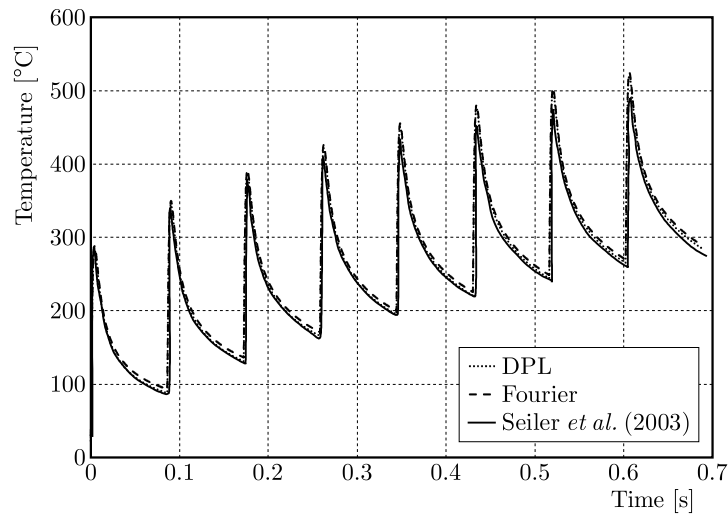
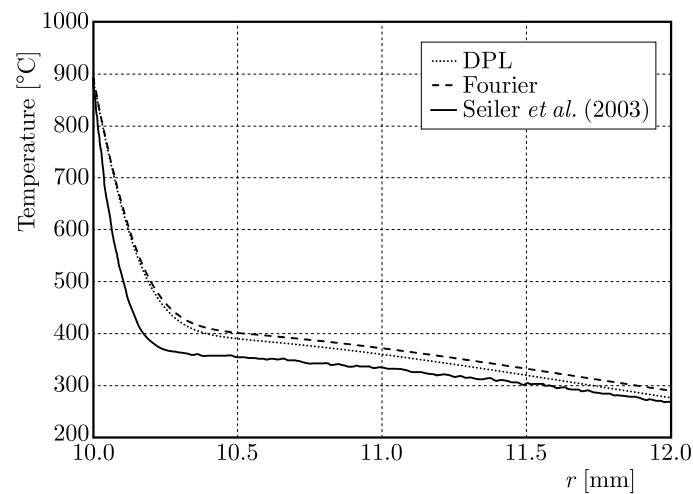
The ability of the solution for estimation of temperature distribution is shown in Fig. 4 compared with experimental data.

The temperature distribution in the radial direction of the barrel is shown in Fig. 5. It can be seen that the DPL model has a better estimation than the Fourier one.

The time interval between consecutive shoots or frequency of shooting has an important role in the magnitude of temperature in the barrel. Here, two different time intervals of 0.3 s and 0.15 s are adopted for parametrical analysis of temperature profiles using the DPL model. In the other words, the temperature variation at the inner and outer surfaces of the gun barrel are calculated in a specific time for different shooting rates of 200 rpm and 400 rpm.

**Table 1.** Applied parameters for solving the problem

| Parameters            | Magnitude              |                                       |                                       | Dimension                       |
|-----------------------|------------------------|---------------------------------------|---------------------------------------|---------------------------------|
|                       | Gun (1) (Lawton, 2001) | Gun (2) (Seiler <i>et al.</i> , 2003) | Gun (3) (Dębski <i>et al.</i> , 2016) |                                 |
| $\alpha$              | $1.11 \cdot 10^{-5}$   | $1 \cdot 10^{-5}$                     | $0.69 \cdot 10^{-5}$                  | $\text{m}^2/\text{s}$           |
| $k$                   | 40                     | 36                                    | 30                                    | $\text{w}/\text{mk}$            |
| $\tau_T$              | 59                     | 59                                    | 59                                    | ps                              |
| $\tau_q$              | 0.59                   | 0.59                                  | 0.59                                  | ps                              |
| $t_0$                 | 4.74                   | 2.3                                   | 7.48                                  | ms                              |
| $q_0$                 | 192.7                  | 272.2                                 | 262.3                                 | $\text{Mw}/\text{m}^2$          |
| $h_\infty$            | 30                     | 30                                    | 30                                    | $\text{w}/(\text{m}^2\text{k})$ |
| $T_\infty$            | 27                     | 27                                    | 27                                    | $^\circ\text{C}$                |
| $2r_i$                | 155                    | 20                                    | 35                                    | mm                              |
| Nominal shooting rate | –                      | 300                                   | 550                                   | rounds/min                      |
| Permeated feed box    | –                      | 20                                    | –                                     | –                               |

Fig. 4. Temperature of the specified point of the barrel in  $150 \mu\text{m}$  from the inner surface for Gun (2) in the consecutive shooting modeFig. 5. Temperature profile at the given time  $t = 1205.5 \text{ ms}$  at the 15-th shoot



The time-history temperature variation of the barrel is analyzed for Gun (2) and (3), and the results are shown below for two types of guns in literature as described in Table 1. The temperature history of the inner and outer surface of the gun barrel in two shooting rates of 200 rpm and 400 rpm (round per minute) is depicted in Fig. 6, and the temperature profile in barrel thickness is shown in Fig. 7.

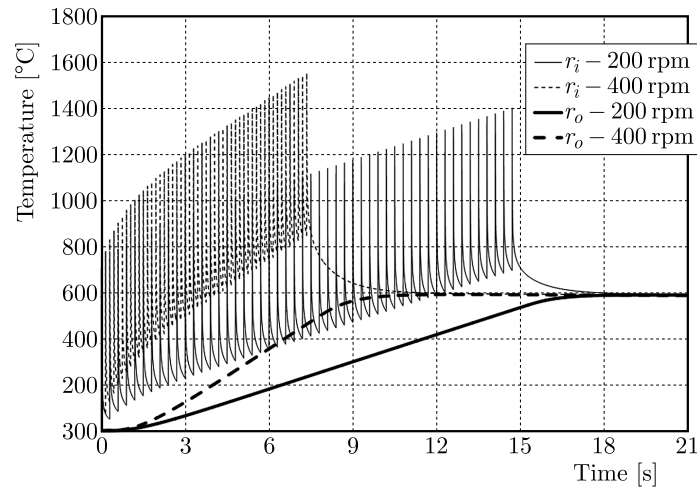


Fig. 6. Temperature variation of the inner and outer surfaces of Gun (2) for consecutive shoots

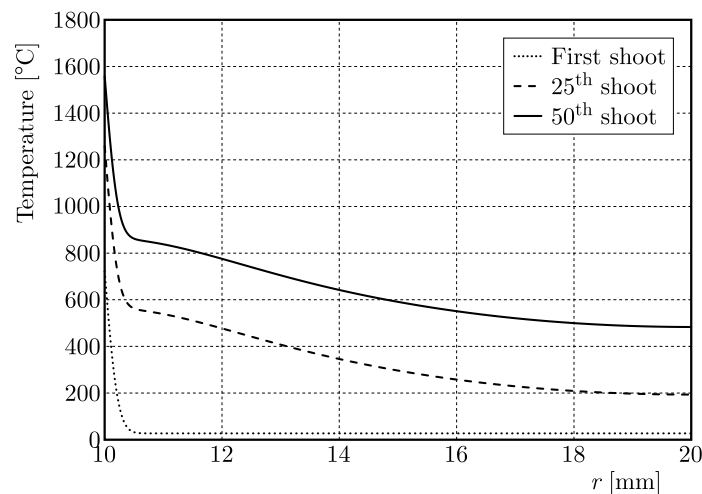


Fig. 7. Temperature distribution in barrel thickness of Gun (2) in 400 rpm

As shown in Fig. 7, the rate of temperature rise increases due to the increase of the shooting rate in Gun (2). For example, after about 8 s or 50 consecutive shoots, the temperature of the inner surface crosses  $1560^{\circ}\text{C}$ , at the rate of 400 rpm, for which the outer surface temperature is about  $500^{\circ}\text{C}$ . Since the melting point of steel is about  $1700^{\circ}\text{C}$ , it can be said that the shooting can be continued with the rate of 400 rpm up to 50 shoots. It can be seen in Fig. 7 that the shooting rate or feed box can be increased from the nominal rate of 300 rpm or 20 for this type of gun, respectively.

The temperature behavior is calculated for bigger Gun (3) with higher caliber, as given in Table 1. The results are shown in Figs. 8 and 9.

This gun has a higher caliber and then higher produced heat load on the inner surface compared with Gun (2). Usually, the barrel of large caliber guns can be coated by a hot working material with high melting temperature such as Tantalum (Underwood *et al.*, 2007) which has about  $3000^{\circ}\text{C}$  melting temperature (Malter and Langmuir, 1939). As shown in Fig. 8, the tem-

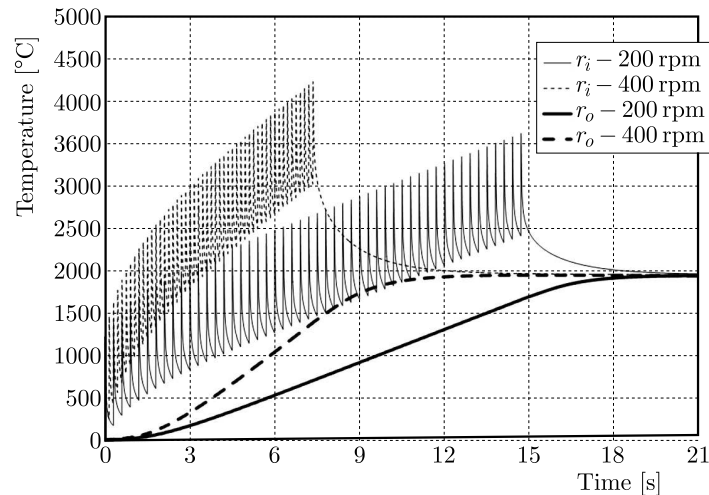


Fig. 8. Temperature variation of the inner and outer surfaces of Gun (3) for consecutive shoots

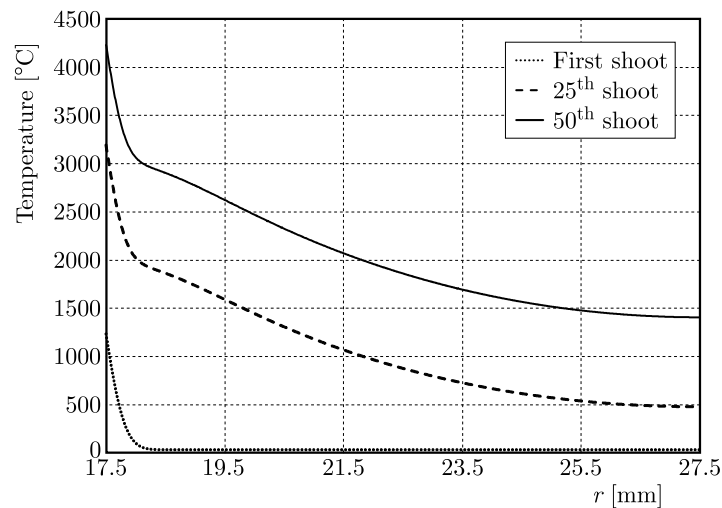


Fig. 9. Temperature distribution in barrel thickness of Gun (3) for 400 rpm

perature of the inner and outer surfaces is increased due to an increase in the shooting rate, and after about 3 s (21 consecutive shoots) the inner surface temperature reaches  $3000^{\circ}\text{C}$  at the rate of 400 rpm, in which the outer surface temperature is about  $350^{\circ}\text{C}$ . But at the rate of 200 rpm, the inner surface temperature reaches about  $2500^{\circ}\text{C}$  after about 6 s (21 consecutive shoots). Therefore, considering the temperature profile in Fig. 9, it can be said that if this gun is used in the state of 400 rpm, some phenomena such as barrel melting and corrosion, decrease of shooting accuracy and self-ignition are probable. It can be said that under consecutive shoots, the thermal conditions are more critical for guns with larger calibers, and unexpected incidents such as barrel melting are so probable. Finally, the ability of the introduced analytical solution for estimation of the temperature profile in the gun barrel is stated as shown above.

## 5. Conclusion

The one-dimensional dual-phase-lag heat conduction in a gun barrel is determined by an analytical solution. The pure analytical solution was found based on the method of separation of variables and Duhamel's integral. The effect of different shooting rates in consecutive shooting and different gun calibers on the magnitude and distribution of temperature has been studied

in this work. The results show that the decreasing time interval leads to higher temperatures in the gun barrel which may lead to barrel melting and threatens the safety of the shooter. The obtained results can be used for analysis and production of safe gun barrels to prevent from melting, self-ignition and decreasing the shooting accuracy.

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