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A MODIFIED MODEL OF RESIDUAL STRENGTH PREDICTION FOR METAL PLATES WITH THROUGH-THICKNESS CRACKS

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A new model, accurate and easy-to-apply, has been proposed to predict the residual strength for metal plates intact or with different damage degrees. In this model, we introduced the damage degree factor (DDF) to quantify the initial damage condition of a plate. The middle crack tension (M(T)) tests and multiple site damage (MSD) tension tests were performed on plate specimens in aluminum alloy LY12-CZ and 2524-T3, respectively. For various damage degrees, the predicted results of this new model showed an improved correlation with test results compared to the net section yield criterion, K-apparent criterion and Duong's method.

Keywords: residual strength, damage degree factor, crack, multiple site damage

1. Introduction

The strength of a structure can be significantly degraded by the presence of cracks. During the whole lifetime, structures may suffer from many kinds of damage, especially cracks, which will reduce their loading capacity and even collapse the whole structure. The damage tolerance design, therefore, is widely used in aerospace industry to manage the crack propagation through the application of fracture mechanics (Riddick, 1984). The load or force that a damaged object or material can still withstand without failing is called the residual strength, which is related to material toughness, fracture size, geometry and orientation. To find an effective method to predict residual strength is an approach to prevent materials and structures from catastrophes caused by damage.

Since Griffith proposed the concept of energy release rate based on energy balance and established one of the basic equations in 1921 – Griffith criterion (Griffith, 1921), fracture mechanics was established as a branch of solid mechanics. In a subsequent study, Irwin (1948) partitioned the energy into two categories: the elastic stain energy and the dissipated energy (surface energy and plastic dissipation). He gave a modified version of Griffith's energy criterion and made it applicable to deal with the fracture of ductile materials. Later in 1957, Irwin further proposed to use the stress intensity factor to represent the stress singularity at the crack tip, thus the phenomenon of low stress brittle fracture was successfully explained. Then, a number of methods to predict residual strength of cracked structures were proposed. For example, fracture toughness criteria, including various crack driving force parameters, such as the elastic energy release rate G, the stress intensity factor K, the J-integral, the crack-tip opening displacement (CTOD) and the crack-tip opening angle (CTOA), etc. were examined (Irwin, 1957; Zhu, 2011; Zhu and Joyce, 2012). Besides, in engineering applications, we have the net section yield criterion (Cherry *et al.*, 1997), Feddersen's engineering method (Feddersen, 1971), finite element method (Li and Siegmund, 2002; Scheider and Brocks, 2008; Zerbst *et al.*, 2009). Moreover, some other criteria including the ligament yield criterion (Jeong and Brewer, 1995; Swift, 1993), average displacement criterion (Jeong and Brewer, 1995) and average stress criterion (Jeong and Brewer, 1995; Young *et al.*, 1998) were developed.

Guz and Dyshel (2004) investigated the effect of mechanical properties of plates, geometrical parameters of plates and cracks on predicting the critical stresses (residual stresses) corresponding to local loss of stability of plates with the crack in tension. They also introduced an equation to predict the critical stress for plates with straight and curved cracks by empirical results obtained by means of mechanical and geometrical parameters of plates and cracks. Duong *et al.* (2001) proposed an energy-based method for predicting the strength of MSD plates. Based on the suggestions of Broberg (1971) and Cotterell and Reddel (1977), the total work of fracture can be expressed by the essential work performed in the end region and the non-essential work in the screening plastic region. Through the established failure line, the predicted values can be obtained by forcing the crack link-up to occur only at the load level which yields the parameters satisfying the equation of failure line.

Several institutions, including the Federal Aviation Administration (FAA), the National Aeronautics and Space Administration (NASA), sponsored programs that developed and assessed methodologies and fracture criteria suitable for predicting the residual strength of structural elements with MSD. Most of the criteria mentioned above are presented in the list. Therein, CTOA and the ligament yield criterion are widely used because of their simplicity of application and extensive correlation with test data for both simple laboratory specimens and complex structures, i.e. stiffened and splice plates (Wang *et al.*, 1996; Newman *et al.*, 1993; Young *et al.*, 1998). Most of these classical criteria are established on the basis of elastic-plastic fracture parameters for a single middle crack. While these criteria extended to intact plates and MSD plates, their predictability are yet to be confirmed. Guz's method (Guz and Dyshel, 2004) and Duong's method (Duong *et al.*, 2001) are good in predicting the residual strength of plates with single straight or curved cracks and MSD, respectively. However, the process of determining the proportionality factor for the former and establishing the failure lines and link-up load curves for the latter is relatively complicated. In general, there is still a potential improvement in applicability and accuracy of current methods. Thus, a universal model, which is available for various damage conditions (i.e. intact, with single middle crack or with MSD) and easy-to-apply, is expected to be built up.

2. Damage degree factor model

2.1. Damage degree factor

Considering through-thickness mode I crack of length $2a$ in an infinite plate, shown in Fig. 1, the plate is subjected to a biaxial stress σ at infinity. The general form of stress can be simplified by a tensor representation (Paris and Sih, 1965; Liu *et al.*, 2015) as

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \tag{2.1}$$

where σ_{ij} are the Cauchy stresses, representing the stress σ_x, σ_y and τ_{xy}; K_I is the stress intensity factor, the subscript I stands for mode I crack; r is the distance from the crack tip; θ is the angle with respect to the plane of the crack; $f_{ij}(\theta)$ is the function that depends on the crack geometry and loading conditions. It is constant in an infinite plate.

In mode I crack growth direction ($\theta = 0$), $f(\theta) = 1$. The intensity factor K_I could be expressed as $K_I = \sigma\sqrt{\pi a}$. The stress in the crack propagation direction is

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \qquad \theta = 0 \tag{2.2}$$

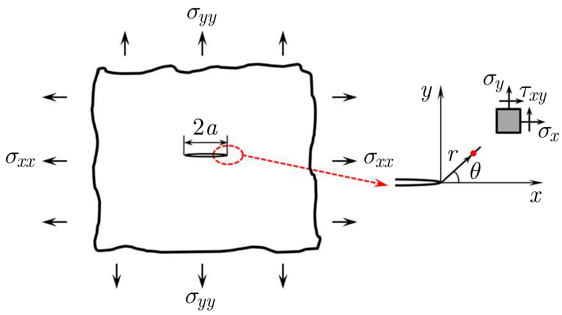

Fig. 1. Mode I crack in an infinite plate

In general, the stress field at the crack tip is determinate when the stress intensity factor K_I is known. Eq. (2.2) gives an elastic solution which could be infinite at the crack tip. However, in most engineering materials, the region around the crack tips reaches the yield stress of the material and the plastic zone is surrounded by the elastic zone. On the assumption of small scale yielding, while the plastic zone is small enough, the stress intensity factor K_I is still likely to determine the stress field around the plastic zone. The linear elastic theory could still be used to estimate the size of the plastic zone (Broek, 2012). In front of the crack, we can obtain

$$r_p = \frac{1}{2\pi}\Big(\frac{K_I}{\sigma_{ys}}\Big)^2 \tag{2.3}$$

where r_p indicates the size of the plastic zone at the crack tip (McClintock and Irwin, 1965). We suppose that the stress intensity factor K_I reaches its critical value K_C, when the normal stress σ_y reaches the ultimate tensile stress (UTS) σ_b. The plastic zone also increases to its critical size, so that the crack grows and the material behind the crack tip unloads. We define the size of the corresponding critical plastic zone as R_p, which is illustrated in Fig. 2. R_p can be derived from the formula of the critical state $\sigma_b = K_C/\sqrt{2\pi r}$, that is

$$R_p = \frac{1}{2\pi}\Big(\frac{K_C}{\sigma_b}\Big)^2 \tag{2.4}$$

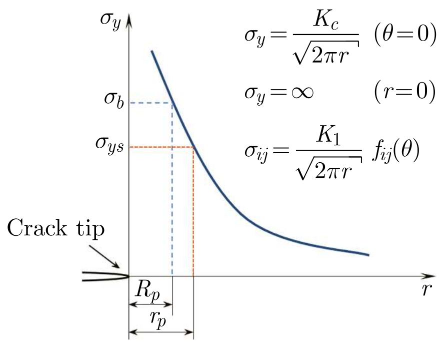

Fig. 2. Normal stress at the crack tip and illustration of r_p and R_p

As a reflection of material properties, R_p can be used as a parameter to measure the damage tolerance of the material: the ratio of toughness to strength. Because of the plastic deformation at the crack tips, the effective crack size would be larger than the linear solution (Irwin, 1960). The length of the equivalent crack could be expressed as $2(a + R_p)$. For an intact plate, $2a \to 0$, it could be regarded as an equivalent crack of $2R_p$ long when it is loaded. Thus, R_p can be used to evaluate the damage resistance of materials. Here is a new definition

$$\xi = \frac{a}{R_p} \tag{2.5}$$

where ξ denotes the DDF, as Eq. (2.5) shows. It is defined as the ratio between the half-length of crack a and R_p, and used for describing the damage conditions of plates. The geometrical parameters of plates and cracks, as well as the mechanical properties of materials are taken into account in the DDF model. In terms of ξ, the damage degree of a plate is known.

2.2. Residual strength reduction coefficient

By dint of the concept of damage variable of Kachanov (1958) and Lemaitre (1985), we can illustrate the expression of the residual strength reduction coefficient $\varDelta$ by way of analogy. For a damaged element at microscale, its damage variable can be expressed by the ratio of the damage area and the overall section area. And the effective stress is concerned with the effective resisting area. Considering the microstress concentrations in the vicinity of discontinuities and the interactions between closed defects, the effective resisting area will not strictly be equal to the difference between the overall section area and the damaged area. Similarly, at macroscale, strength of the plate is not proportionate to the effective resisting area. Through experimental research, Moukawsher *et al.* (1996) pointed out that both the net section yield criterion and K-apparent criterion always overestimate the residual strength in certain conditions as they do not consider the structural load carrying capacity loss caused by plastic deformation at the crack tip. Therefore, a modified coefficient should be worked out.

We set S as the overall cross-section area of the plate, S_D as the damaged area. σ_C indicates the residual strength obtained by calculation, and $\varDelta(\xi)$ represents the residual strength reduction coefficient. Then we have

$$\frac{\sigma_C}{\sigma_b} = \frac{S - S_D}{S}\varDelta(\xi) \tag{2.6}$$

In terms of the analysis on a series of DDFs corresponding to each critical ultimate strength (Li *et al.*, 2003), and considering the boundary conditions:

- when $\xi \to 0$, $\varDelta(\xi) = \varDelta(0) = 1$, the plate is intact, failure can be controlled by the UTS;
- when $\xi \to \infty$, $\varDelta(\xi) = \varDelta(\infty) = 0$, the plate completely loses its load carrying capacity.

The empirical function of the residual strength reduction coefficient for the though-thickness cracked plates could be given as

$$\varDelta(\xi) = \mathrm{e}^{-a\xi^b} \tag{2.7}$$

where a, b are defined as the material constants, given by experimental data.

2.3. Damage degree factor model

According to Eq. (2.6), Eq. (2.8) could be obtained via a simple mathematical formulation

$$\sigma_C = \sigma_b \frac{S - S_D}{S}\varDelta(\xi) \stackrel{\mathrm{def}}{=} \sigma_\xi \tag{2.8}$$

where σ_ξ is defined as the residual strength obtained via DDF model. For the material selected, the value of the critical fracture toughness K_C and the critical crack tip stress σ_b, which are bound up with the material properties, can be determined. Therefore, the residual strength of a certain plate can be easily obtained through Eq. (2.8). Notice here, K_C should be obtained by experiments or empirical equations (Zerbst *et al.*, 2009). The value of K_C is affected by thickness and width of the specimens (Yablonskii, 1980), its value has a great impact on the accuracy of the DDF model prediction.

Here is a new failure criterion, expressed as

$$\sigma \leqslant \sigma_b \frac{S - S_D}{S} \tag{2.9}$$

where σ represents the load applied on the plate. If σ satisfies inequality (2.9), the plate is secure. Otherwise, it will be damaged.

We introduce $[\sigma]$ as the stress from the maximum-normal-stress theory (Budynas and Nisbett, 2008), n as the safety factor. Then we have $\sigma \leqslant [\sigma] = \sigma_b/n$, the safety factor can be obtained by

$$n = \Big(\Delta(\xi)\frac{S - S_D}{S}\Big)^{-1} \tag{2.10}$$

3. Empirical analysis of the damage degree factor model

In this Section, middle crack tension tests and MSD tension tests were carried out to validate the DDF model. We take an aluminum flat plate for example as it is commonly used in aircraft industry. Table 1 lists 3 models, including the net section yield criterion, K-apparent criterion and the DDF model. The net section yield criterion, using yield stress and the ratio of net and gross cross-sectional areas as variables, predicts the residual strength by material properties and geometrical dimensions (Cherry *et al.*, 1997). In the fracture toughness K criterion, σ is determined by the material properties. And the rest part $\sqrt{\pi a}\sec(\pi a/W)$ is relevant to geometrical dimensions (Kirsch, 1989). These two yield criteria are commonly used in calculating the residual strength. In order to further improve the prediction accuracy, besides the factors relevant to material properties and dimensions, the DDF model also introduces the residual strength reduction coefficient, which is a term related to plastic deformation.

Table 1. The prediction models of residual strength

Symbol	Model	Formula
σ_N	Net section yield criterion (Cherry *et al.*, 1997)	$\sigma_N = \sigma_{ys}\frac{S-S_D}{S}$
σ_K	K-apparent criterion (Kirsch, 1989)	$\sigma_K = K/\sqrt{\pi a \sec\frac{\pi a}{W}}$
σ_ξ	DDF model	$\sigma_\xi = \sigma_b\frac{S-S_D}{S}\Delta(\xi)$

3.1. Middle crack tension tests

The M(T) specimens were made of aluminum alloy LY12-CZ, 6 mm thick. Figure 3 illustrates geometry of the specimens. The necessary mechanical parameters could be obtained by experiments, shown in Table 2 (Li *et al.*, 2003). The test results were obtained from experiments on 27 pieces of the M(T) specimens. Thus, the material constants a and b can be determined. The function of the residual strength reduction coefficient can be expressed as $\Delta(\xi) = \exp(-0.27\xi^{0.27})$.

Table 2. The mechanical parameters of M(T) specimens in LY12-CZ

σ_{ys} [MPa]	σ_b [MPa]	K_C [MPa$\sqrt{\text{m}}$]	K_{app} [MPa$\sqrt{\text{m}}$]
323.4	450.8	93	73.2

Table 3 presents the test results of the 3 different criteria. And in Fig. 4a, the scatter diagram which reflects their predictive abilities by the offset degree deviating from the 45° reference line was built up. In this figure, the perfect agreement would be for all points to fall in the 45° line.

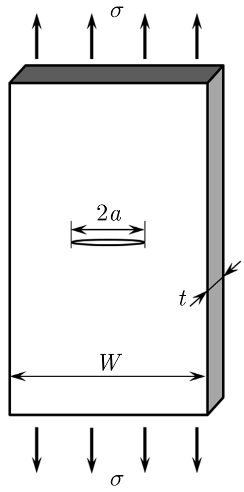

Fig. 3. Geometry of M(T) specimen

Table 3. Results of the middle crack tension test and 3 prediction models

Relative crack length $2a/W$	Prediction models			Test average [MPa]
	σ_N [MPa]	σ_K [MPa]	σ_ξ [MPa]	
0.000	323.4	–	450.8	466.2
0.100	291.1	818.4	290.3	287.9
0.118	285.2	561.9	294.7	291.1
0.124	283.3	409.6	321.4	314.5
0.150	274.9	332.6	274.2	271.8
0.233	248.1	203.9	226.9	223.2
0.248	243.2	335.2	264.3	269.4
0.350	210.2	213.4	192.4	194.5
0.500	161.7	126.9	134.4	132.1

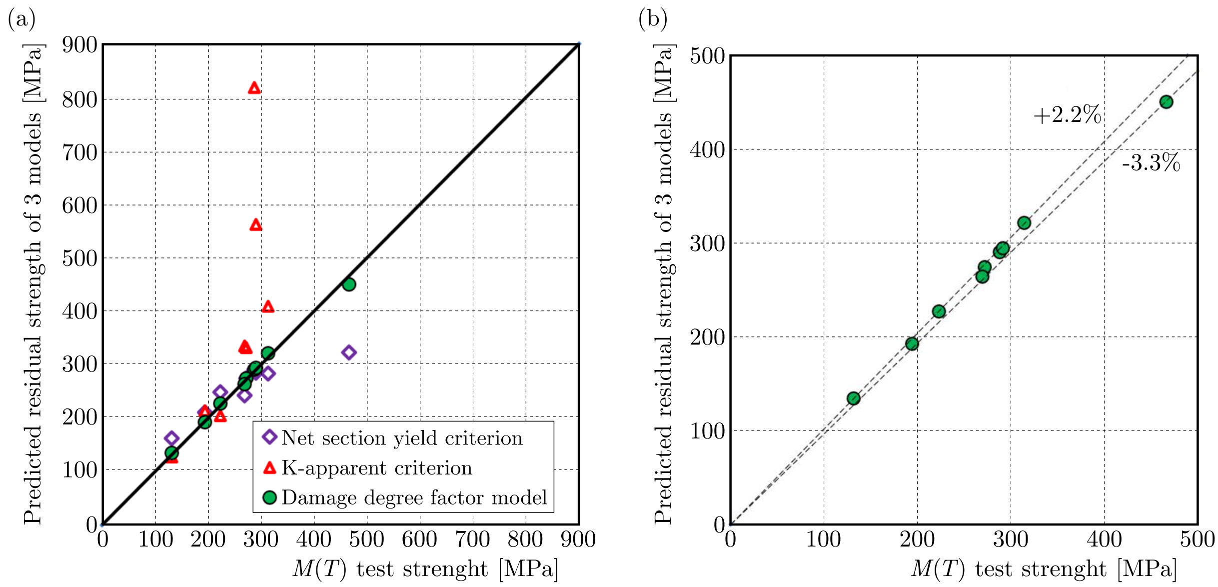

Fig. 4. (a) Empirical analysis results compared to other models of middle crack tension tests.
(b) Predictions of DDF model vs. test results of middle crack tension tests

Comparing with the test results, it is obvious that the fracture toughness K-apparent criterion predictions overestimate the strength of plates. The K-apparent criterion is relatively accurate in predicting the failure strengths in a condition of relatively low strengths, while it meets relative large deviations with small relative crack length $2a/W$. Based on the theory of linear elastic analysis, the K-apparent criterion does not take account of the loss of load capacity caused by plastic deformation at the crack tip (Cherry *et al.*, 1997). It only applies to predicting the residual strength of the plates with relative high $2a/W$. The net section yield criterion gives acceptable results when the relative crack length $2a/W$ is relative large. But for intact plates ($a = 0$), there is a relative big deviation. Theoretically, the calculated residual strength for an intact plate should be equal to its UTS. It can be observed here, for the intact plate ($2a/W = 0$), the predicted result of the DDF model corresponds closely to the theoretical one, with an error of 3.3%.

As shown in Table 4, the maximum error and average error of the DDF model are only −3.3% and 1.6%, respectively, significantly less than that of the K-apparent criterion and the net section yield criterion. The DDF model is the most accurate among the 3 criteria. Figure 4b shows the agreement between the DDF model predicted results and the middle crack tension tests results. The agreement between its predicted results and the test results is within 3.3%.

Table 4. Maximum error and average error between prediction models and middle cracked tension test results

Prediction model	Maximum error [%]	Average error [%]
Net section yield criterion	−30.6	10.7
K-apparent criterion	+184	41.8
DDF model	−3.3	1.6

3.2. Multiple site damage tests

The MSD tension tests were implemented on 15 unstiffened plates made of aluminum alloy 2524-T3, of 1.6 mm thick. These plates consisted of a central lead crack connecting several holes and small radial cracks emanating from the other holes. A schematic diagram of MSD specimens is shown in Fig. 5. The mechanical parameters of MSD specimens are listed in Table 5.

Table 5. The mechanical parameters of MSD specimens in 2524-T3

σ_{ys} [MPa]	σ_b [MPa]	K_C [MPa$\sqrt{\text{m}}$]	K_{app} [MPa$\sqrt{\text{m}}$]
318.13	449.55	163.29	104.91

Similarly, the function of the residual strength reduction coefficient for 2524-T3 aluminum plate, 1.6 mm thick, (Fan *et al.*, 2015) can be expressed as $\Delta(\xi) = \exp(-0.3\xi^{0.24})$.

As the central lead crack is longer than the other cracks, its stress intensity factor is the maximum. We take the central lead crack as the equivalent crack. That is to say, the half-length of the crack a in Eq. (2.5) equals to the half-length of central lead crack a_2 in Fig. 5. The other cracks will be considered along with the residual cross sectional area.

For specimen No. 1 and No. 2, the three cracks a_1, a_2, a_3 connect together as the leading cracks. For specimen No. 11 and No. 14, a_1 and a_2, a_2 and a_3, connect as the leading crack, respectively. The diameters of the side holes, a_0 and a_4 are 6 mm.

We listed the predicted results by the net section yield criterion, K-apparent criterion and by DDF model in Table 6, and the schema of test results and predicted results are shown in Fig. 6a. The results obtained by the net section criterion and the DDF model, show the same trend with the test results. The net section yield criterion gives conservative predicted values.

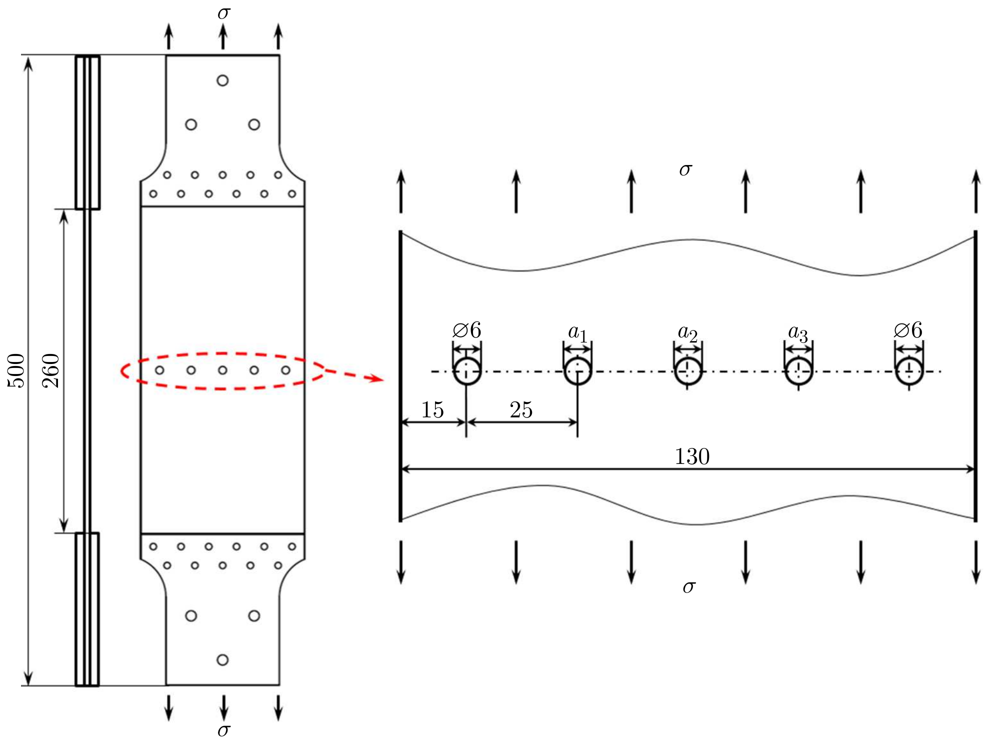

Fig. 5. Configuration of the MSD specimen

Table 6. Specimen parameters and residual strengths of 3 prediction models and MSD tension tests

Speci-mens No.	Hole size			Relative crack length $2a/W$	Prediction models			Test [MPa]
	a_1 [mm]	a_2 [mm]	a_3 [mm]		σ_N [MPa]	σ_K [MPa]	σ_ξ [MPa]	
1	–	61.04	–	0.94	139.4	182.2	140.8	164.3
2	–	60.37	–	0.93	141.0	183.1	142.6	160.1
3	8.38	13.73	8.38	0.21	214.2	405.5	244.2	263.3
4	8.16	13.44	8.96	0.21	214.0	411.4	244.3	258.9
5	9.00	12.92	7.97	0.20	215.6	415.4	246.8	247.3
6	9.37	15.86	10.34	0.24	201.7	367.8	227.8	238.1
7	10.38	16.63	7.59	0.26	204.1	362.7	229.8	243.4
8	9.80	21.00	8.54	0.32	192.5	322.2	213.2	218.0
9	12.01	17.61	9.20	0.27	193.8	343.0	217.3	226.4
10	10.37	18.1	9.94	0.28	194.8	3 46.6	218.0	223.8
11	44.81	–	15.11	0.69	142.1	198.63	147.9	149.7
12	11.4	22.63	11.58	0.35	177.1	258.9	195.1	205.2
13	11.12	23.52	10.82	0.36	177.5	282.7	195.0	197.9
14	10.23	41.52	–	0.64	162.1	211.3	169.9	202.4
15	9.37	26.03	11.79	0.40	173.3	246.7	188.8	206.2

However, the deviation is relative large when the relative crack is short. The net section yield criterion predicts failure based on the amount of the material available to carry the load (Cherry *et al.*, 1997). It is simple and gives relatively reliable predicted values. The K-apparent criterion always oversestimates the residual strength of plates. Its predicted results show a big difference with the test results, particularly with lower relative crack length. The DDF model always gives acceptable predicted results with various damage degrees. It could be observed from Table 7 that the average error of the DDF model is 6.7%, far less than that of the other two models. In addition, Fig. 6b also illustrates the agreement between the DDF model and the test results. Comparing with the approach of Duong *et al.* (2001) (error −10%-10%), the prediction of DDF model is conservative (error −16.1%-0%). Thus, the DDF model is moderate and the most accurate among these models.

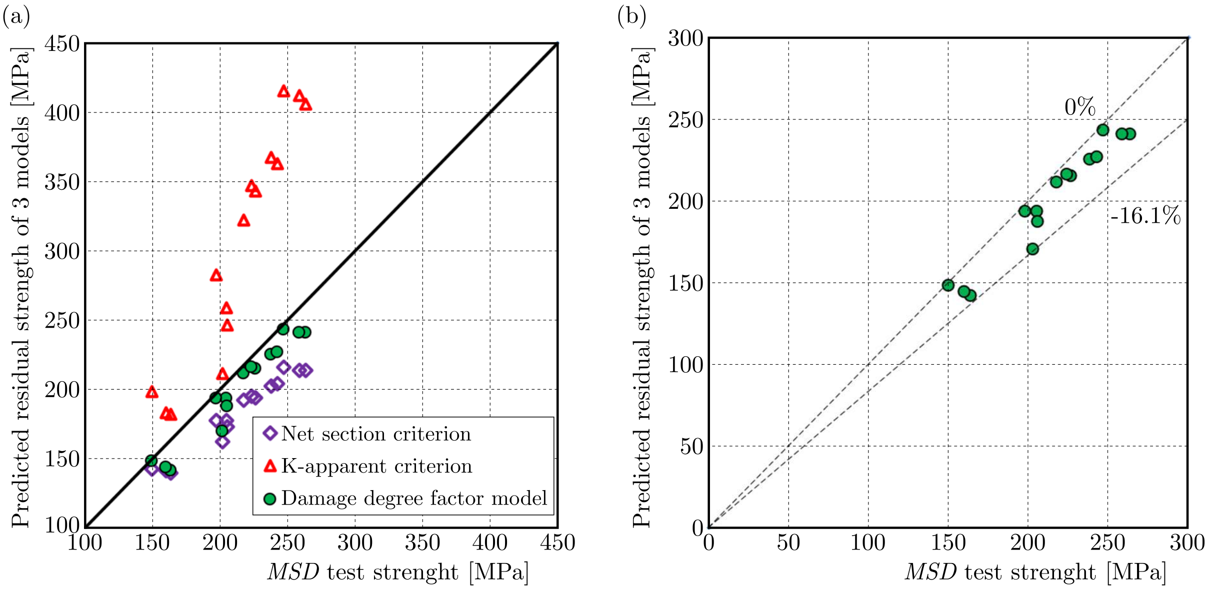

Fig. 6. (a) Empirical analysis results compared to other models of MSD tension tests, (b) Predictions of DDF model vs. test results of MSD tension tests

Table 7. Maximum error and average error between prediction models and MSD tension test results

Prediction model	Maximum error [%]	Average error [%]
Net section yield criterion	−19.9	14.1
K-apparent criterion	+68	39.3
DDF model	−16.1	6.7

Thus far, the preliminary results have demonstrated the feasibility and accuracy of the DDF model in predicting the residual strength for flat plates with different damage degrees (i.e. intact, with a single middle crack or with MSD).

4. Conclusions

In this investigation, the DDF model is proposed to predict the residual strength of aluminum plates over a wide range of damage degrees. To validate the model, middle crack tension tests on aluminum alloy LY12-CZ and MSD tension tests on aluminum alloy LY2524-T3 are conducted. Comparing with the prediction results of the net section yield criterion and *K*-apparent criterion, DDF model shows a better agreement with the test results, i.e. results of intact plates (error 3.3%), results of plates with the middle crack (error within 3.3% and average error 1.6%) and results of MSD plates (error −16.1%-0% and average error 6.7%). And compared with the methods of Duong *et al.* (2001) and Guz and Dyshel (2004), the DDF model is easier to apply.

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