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NUMERICAL SIMULATIONS OF THE WHEEL-RAIL TRACTION FORCES USING THE ELECTROMECHANICAL MODEL OF AN ELECTRIC LOCOMOTIVE

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This paper presents a methodology for modelling the movements of an electrical rail vehicle on any track. Vehicle movement is the result of the drive system of a vehicle transferring electromechanical torque generated by the electric motor onto the wheels of the vehicle, while the appropriate driving force is a result of forces resulting from contact between the wheel surface and the rail. The following applicable models were prepared: electromechanical (for electric motor connections that constitute rail vehicle drive) of the vehicle with the drive system and of the contact wheel-rail. These models were implemented in Matlab/Simulink. The presented simulation tool for rail vehicles has been used to test a variety of cases, demonstrating capabilities of the proposed method in a wide range of situations. These cases involved analysis of dynamics of the start-up of the EU07 rail vehicle in a straight line. The results obtained for the analysed rail are valuable information that makes it possible to determine both the loading of the locomotive drive system and to estimate the vehicle travelling speed.

Keywords: rail vehicle, electromechanical model

1. Introduction

The analysis of the existent modeling and simulating applications for complex electromechanical systems in the context of durability and reliability leads to a conclusion that searching for a new algorithm supported by numerical tools has a purpose. Electric railway vehicles constitute a separate group. Modern vehicles of this type make up a large group of electromechanical machines which require a specific and interdisciplinary approach to modeling. Dynamic phenomena occurring in the above may be described by division of these systems into two groups: electromechanical and mechanical systems. The laws of movement governing such systems may be presented in a form of differential algebraic, that is electromechanical equations. The simulation research on railway vehicles movement requires preparing both, models of the vehicles and models of the coupled electric motors which constitute the drive, while taking the phenomena which occur at the contact of the rail and wheel into consideration.

Currently, complex testing of dynamics of the wheel-rail mechanical system (Kisilowski, 1991) may be conducted using computer numerical simulations (Duda, 2011; Grzesikiewicz and Seńko, 2009). The numerical simulations of motion of railroad vehicles require implementation of a mathematical model of the vehicle, which would describe the vehicle correctly in the software. It is very convenient to apply a methodology based on multibody dynamics in creation of such models (Ambrósio *et al.*, 2012; Kortum and Sharp, 1993).

The solution to the problem of the wheel-rail contact requires an accurate description of contact kinematics by calculating microslips, that is the normalized relative speeds at the point of contact. Scientific literature provides broad descriptions of the wheel-rail contact models of various simplification levels, based on the rolling contact theory (Kalker, 1982, 1991).

Plots of time-histories for dynamic values in these systems depend, to a large extent, on the characteristics and power of the motor driving the machine. The analyses of the impact of the electric motor on the mechanical system can be performed by modelling the drive system as an electro-mechanical system (Mężyk, 2001; Świtoński *et al.*, 2001). This is the way to determine the function of electric motor torque $M_e(t)$ that depends on the construction features of the mechanical system and the drive motor.

Investigating the dynamic phenomena of rail vehicle drive systems while disregarding the vehicle itself leads to oversimplification. The load of the drive system is very complex. On one hand, there are forces coming from the electric motor constituting the drive of the vehicle, and being also, in some locomotive models, electrically coupled with the drives of remaining axles; on the other hand, there are forces coming from the interaction between the wheel and the rail. These forces are determined by the travel of the vehicle itself, while its motion results from the application of just these forces. It is undoubtedly the coupling between dynamics of the drive system with dynamics of the vehicle, which makes it necessary to take into account the real seating of the rail vehicle drive system in the body constituting the vehicle chassis.

Whereas models of railway vehicles, wheel-rail contact or motors – both, direct current commutator and asynchronous that constitute drives of electric locomotives, are long present in the scientific literature, complex models which consider mutual coupling of the electrical and mechanical part are an insufficiently described scientific area.

This paper presents the methods that can be used to analyze the dynamic phenomena in rail vehicle drive systems including their real seating. The tests are performed by providing a detailed description of electromagnetic phenomena in drive motors and contact phenomena at the interface between the wheel and the rail. The forces determined by using these models represent the load of a rail vehicle drive system.

2. Modelling of the electromechanical system of an electric locomotive

To analyze dynamic phenomena of complex electromechanical systems, it is necessary to adopt a physical model of a real object under analysis, representing the most relevant features and phenomena involved, required for dynamical analysis to be performed. Such a model frequently constitutes a compromise between the accuracy of the object representation and the complexity of description of the involved phenomena affecting credibility of the obtained solution, simulation duration, or – in extreme cases – the possibility to obtain a solution. In the case of electromechanical systems, the powertrain components – both mechanical and electrical – are mutually coupled dynamic systems. By analyzing dynamic phenomena, particularly in transient states, it is necessary to use a model that makes it possible to provide the electromechanical feedback (Mężyk, 2001).

The electromagnetic and mechanical systems of the electric rail vehicle powertrain couple mutually through electromagnetic torque M_e and angular rotor velocity ω. For the rail vehicle system presented in Fig. 1, analysis of dynamics of the vehicle and its drive system must take into account the electromagnetic and mechanical elements.

Taking into account the unique character of the solution of the electric locomotive drive system (type of its electric motor, number of motors and the configuration of their connections), the paper presents an electromagnetic model of the locomotive chosen for further analysis. For this purpose, the electric locomotive EU07 has been chosen (Duda, 2011). In spite of being aware that this is a bit obsolete model, this one was chosen because of easy access to technical documentation making it possible to model this vehicle in a virtual space to determine basic features of the vehicle, including its weight and moment of inertia necessary to build a mechanical system model.

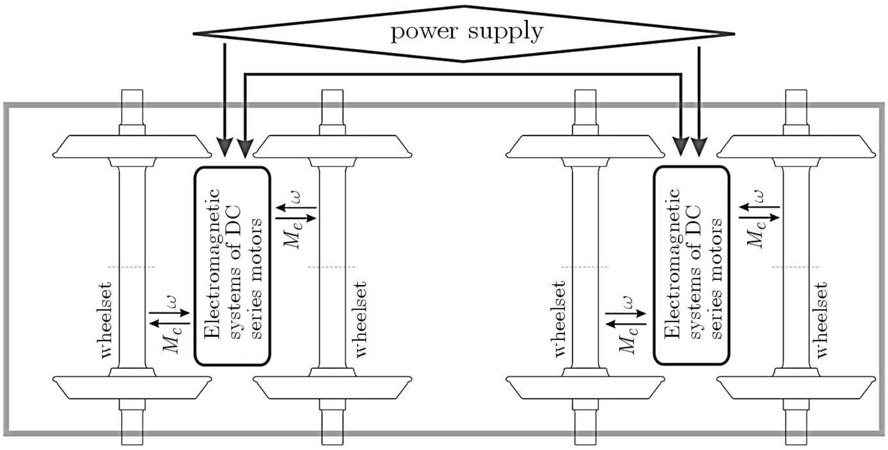

Fig. 1. The structure of the drive system

2.1. Developing a model of the non-linear motor connection

In their operation mode, the traction motors of the EU07 locomotive work in two configurations. During start-up four motors are connected in series; then, to increase the voltage, the motors are switched over to a parallel circuit, two motors per branch. A serial locomotive motor connection was analyzed.

The figure below (Fig. 2) presents a schematic diagram for the motor circuits connected in series.

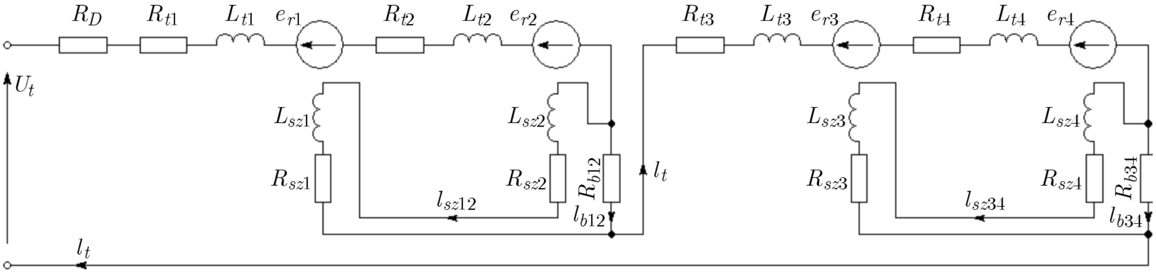

Fig. 2. Assumed substitute diagram of the electric motors connected in series

For the above assumed schematic diagram, the following voltage equations and physical relations have been formulated

$$\frac{d}{dt}I_t = \frac{1}{\sum\limits_{i=1}^{4} L_{ti}}\left[U_t - \sum_{i=1}^{4} e_{ri} - I_t\left(\sum_{i=1}^{4} R_{ti} + R_D\right) - R_{b12}(I_t - I_{sz12}) - R_{b34}(I_t - I_{sz34})\right] \tag{2.1}$$

$$\frac{d}{dt}I_{sz12} = \frac{1}{\sum\limits_{i=1}^{2} L_{szi}}\left(R_{b12}(I_t - I_{sz12}) - I_{sz12}\sum_{i=1}^{2} R_{szi}\right) \tag{2.2}$$

$$e_{r1} = c_{E1}f(I_{sz12})\phi_{n1}\omega_1 \qquad e_{r2} = c_{E2}f(I_{sz12})\phi_{n2}\omega_2 \tag{2.3}$$

$$\frac{d}{dt}I_{sz34} = \frac{1}{\sum\limits_{i=3}^{4} L_{szi}}\left(R_{b34}(I_t - I_{sz34}) - I_{sz34}\sum_{i=3}^{4} R_{szi}\right) \tag{2.4}$$

$$e_{r3} = c_{E3}f(I_{sz34})\phi_{n3}\omega_3 \qquad e_{r4} = c_{E4}f(I_{sz34})\phi_{n4}\omega_4 \tag{2.5}$$

$$M_{e1} = I_t\frac{e_{r1}}{\omega_1} \qquad M_{e2} = I_t\frac{e_{r2}}{\omega_2} \qquad M_{e3} = I_t\frac{e_{r3}}{\omega_3} \qquad M_{e4} = I_t\frac{e_{r4}}{\omega_4} \tag{2.6}$$

$$
\begin{aligned}
&\frac{d}{dt}\omega_1 = \frac{1}{J_1}(M_{e1} - M_{ob1}) \qquad &&\frac{d}{dt}\omega_2 = \frac{1}{J_2}(M_{e2} - M_{ob2})\\
&\frac{d}{dt}\omega_3 = \frac{1}{J_3}(M_{e3} - M_{ob3}) \qquad &&\frac{d}{dt}\omega_4 = \frac{1}{J_4}(M_{e4} - M_{ob4})
\end{aligned}
\tag{2.7}
$$

where I_t is the traction motor armature current, I_{sz12}, I_{sz34} – currents conducted by excitation windings in motors S1, S2 and S3, S4, k_{Ei} – machine constants, ϕ_{ni} – streams at normal excitation, $f(I_{szi})$ – relative non-linear magnetizing characteristics, R_{ti} – armature resistances, R_{b12}, R_{b34} – shunt resistances for excitation winding, R_{szi} – serial circuit resistances, R_D – total of additional resistances, L_{szi}, L_t – inductance for excitation windings, ω_i – rotor angular velocities, T_{ei} – motor electromagnetic torques, e_{ri} – voltages induced in armature circuits.

The mathematical model of the motor given by equations (2.1)-(2.7) includes the additional resistance R_D and shunt resistance R_b of the excitation winding, whereas the non-linear characteristics of magnetization (Fig. 3) are expressed in equation (2.5).

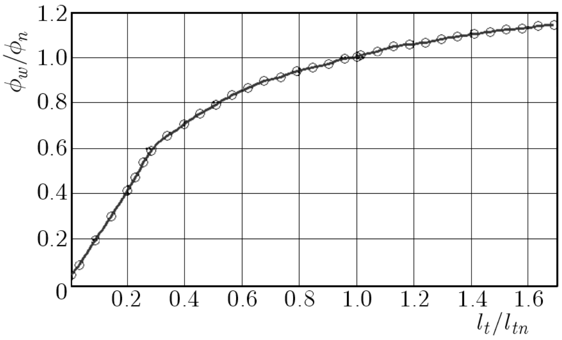

Fig. 3. Relative magnetization characteristic

The characteristic feature of a series motor is the fact that the armature current equal to the motor load current is at the same time its magnetizing current. To study dynamics of the drive system, a non-linear model of the analyzed motor has been developed resulting from its non-linear magnetization curve (Duda *et al.*, 2004) that represents the relation between the magnetic stream and excitation. This characteristics in relative units, is essentially equal to the characteristic idle operation of the machine at $n =$ const. Being in possession of magnetization characteristics and voltage characteristics induced in armature windings by the current $E = f(I_t)$ at $n =$ const and known machine resistances, it is possible to determine the mechanical characteristics of the series motor. Manufacturers usually provide the mechanical characteristics of their motors.

The magnetization characteristics are determined from the mechanical characteristics at the non-shunted excitation winding, i.e. from the curve with 100%. By knowing the resistance in the armature circuit, it is possible to determine the voltage drops in the armature circuit at various current values and a constant supply voltage.

Taking into account the determined characteristics for the calculated rated stream, it is possible to calculate relative magnetization characteristics presented in Fig. 3.

Assuming that the series excitation winding current will change during machine operation in the range of 0.4-1.7I_{tn}, regarding the characteristics presented in Fig. 3, it is possible to approximate a line, calculating the value of the excitation winding inductance coefficient.

2.2. Dynamical model of the rail vehicle

At present, a complex study of the mechanical system – the rail-vehicle can be provided by numerical simulations by using a computer as a tool. Numerical simulations of rail vehicles require implementation of a mathematical model of the vehicle in a computer program that would describe it with a reasonable precision. Methods based on the multibody dynamics constitute a very convenient approach for developing such models. Over the last few years we have been observing an immense evolution in the modelling technique based on the multibody systems, ranging from a simple program to high specialty codes making it possible to analyze kinematics and dynamics of any mechanical system, sometimes coupled with models of electric, hydraulic or pneumatic motors. With the development of advanced numerical methods, more and more effective and reliable numerical programs allowing for formulating and solving highly complex problems are created.

As a result of analyzing the technical documentation of the rail vehicle, a physical model has been created in form of a multi-rigid-body system mutually coupled with proper kinematic pairs and elastic and damping components. Subsequently, its interpretation in the SimMechanics application was obtained. The necessary parameters describing the model, including masses, moments of inertia and dimensions determining the position of kinematic pairs were obtained from a 3D model created by using the Autodesk Inventor software. The remaining parameters, i.e. stiffness and damping suspension components for the first and second unsprung mass reduction, were found in the documentation submitted by the Zakłady Naprawcze Lokomotyw Elektrycznych (Electric Locomotive Repair Plant) in Gliwice, Poland.

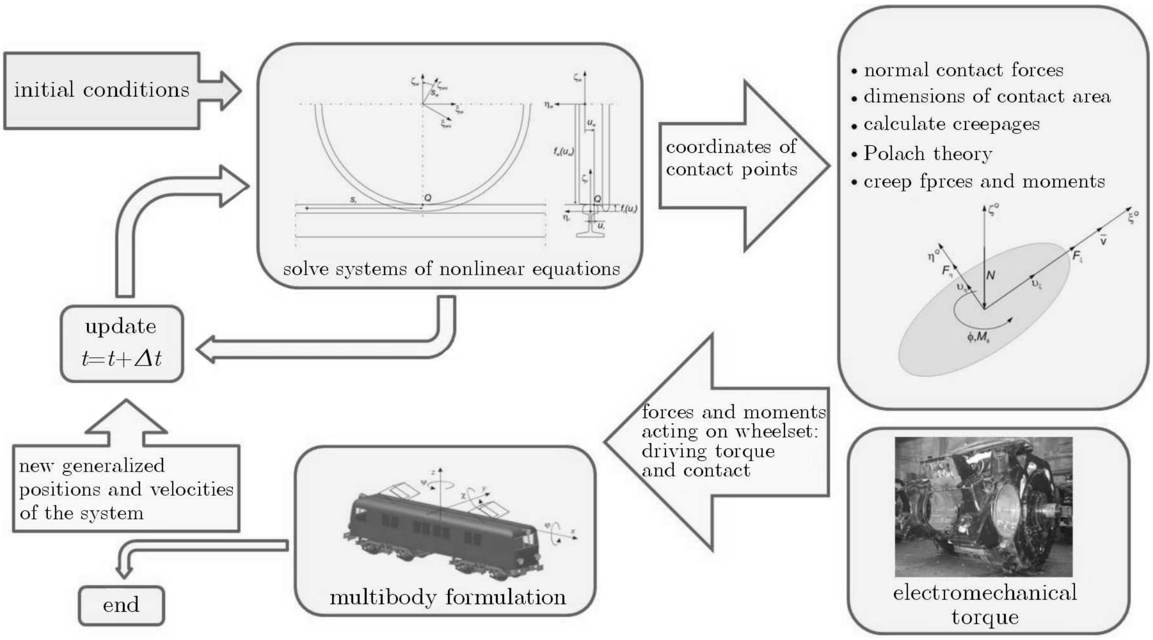

Fig. 4. Calculation algorithm for analysis of dynamics of the rail vehicle travelling on any railway track

The study of dynamics of electric rail vehicles requires creation of three intercoupled models: a vehicle including the drive system, rail and the wheel-rail interface. At the first stage of the rail vehicle modelling process during its run on the railway track, the subsystem models are built separately. Then, the models are interconnected to make a complete system. This method was implemented in a proprietary software created in the Matlab environment. The calculation algorithm for analysis of the rail vehicle travelling on any railway track is presented in form of a schematic diagram in Fig. 4.

The presented algorithm used to develop the computer program for analyzing the rail vehicle travel dynamics on any railway track can be expressed in a few steps:

- Assuming the initial conditions for generalized coordinates $\mathbf{q}(t^0)$ and generalized velocities $\dot{\mathbf{q}}(t^0)$ as well as determining the initial surface parameter values $s_r(t^0)$, $u_r(t^0)$, $s_w(t^0)$ and $u_w(t^0)$ related to a specific wheel-rail pair;
- Solving the system of non-linear equations to obtain the surface parameters that determine the contact point coordinates related to the specific wheel-rail pair;

- Calculating normal forces during the contact that are generated as a result of the wheel-rail interaction and depend on the contact surface size;
- Calculating creepages, tangent micro slides and spin moments generated as a result of the wheel-rail interaction;
- Determining drive torques for each axle of the wheelset separately;
- Adding the forces and torques occurring in the contact related to each wheel as well as adding drive torques to the vector of external forces acting on the system. Using the multibody system formalism to obtain a solution, a new system of generalized positions and velocities for the subsequent time step $t + \Delta t$;
- Updating the system for another moment by adopting the initial data from the previous step to determine non-generalized surfaces related to each wheel-rail pair;
- Continuing the whole process for a new time step until final the time of the analysis is obtained.

Detailed relationships used for the calculation carried out in points of the presented algorithm can be found in Lankarani and Nikravesh (1990), Polach (1999), Pombo and Ambrósio (2003), Shabana *et al.* (2008).

The examinations have been performed using adequate models of:

- the vehicle based on the multibody system formalism in the Matlab/SimMechanics software,
- discrete dynamic models of the electromechanical drive system developed by using the Matlab/SimMechanics software and the Matlab application script,
- electric motor connections constituting the rail vehicle drive system developed by using the Matlab/Simulink software,
- the wheel-rail contact used to determine the support and guide forces developed by using the proprietary script package from the Matlab software.

3. Verification of the assumptions made while modelling the systems

The verification of the elaborated model of the drive system of the electric locomotive was carried out by comparison of the results obtained from measurements taken during the travel of the real locomotive EU07 over the Katowice-Gliwice route with the passenger rolling stock and the results obtained from numerical simulation. During the travel of the train, values of the current in the main circuit with the resistance control (Fig. 6) of the locomotive were recorded by means of a digital camera (Fig. 5). The indications of the instruments were read during the start-up of the rolling stock.

In the simulations of the start-up of the locomotive, the mass of 224 t of the whole draft of carriages pulled by the locomotive was taken into consideration. This mass was reduced uniformly to particular axles of the locomotive. Figure 6 shows comparison of the curves obtained from the measurements (dashed line) with the results of computer simulation (solid line) for the resistance control carried into effect in accordance with Fig. 7.

4. Numerical simulations for of motion of a rail vehicle

As a result of the carried out numerical calculations, the curves of drive system parameters, displacements, vehicle velocity as well as forces and contact torques for two resistance controls were obtained. To compare the curves, the results were listed in two columns (Fig. 7).

Fig. 5. A view of the operator's control desk

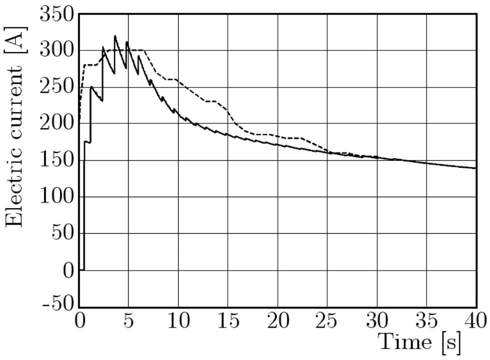

Fig. 6. Course of variations in the main circuit current depending on the start-up time

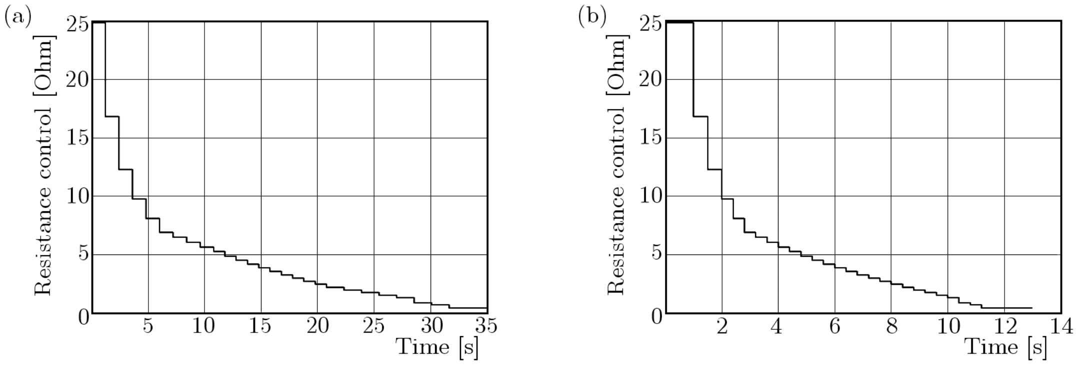

Fig. 7. Resistance control

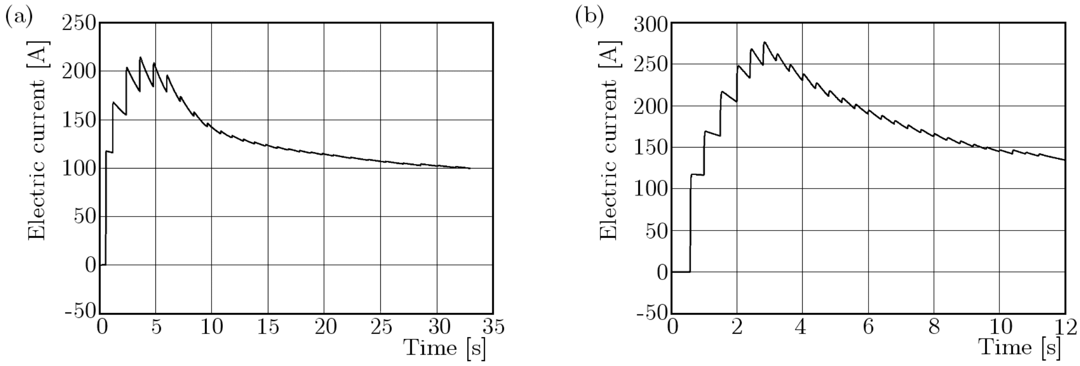

Fig. 8. Course of changes in the electric current of the stator in the auxiliary circuit of motor connections in a serial system as a function of time

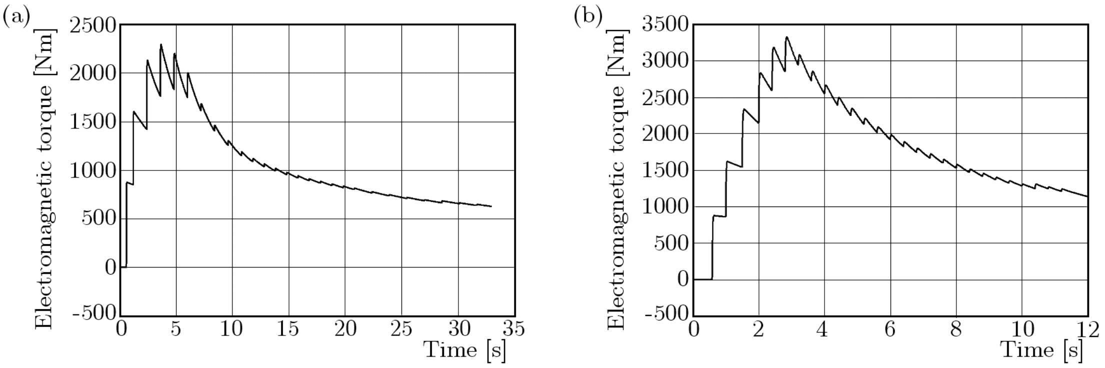

Fig. 9. Course of variations in the electromagnetic torque on the first motor as a function of time

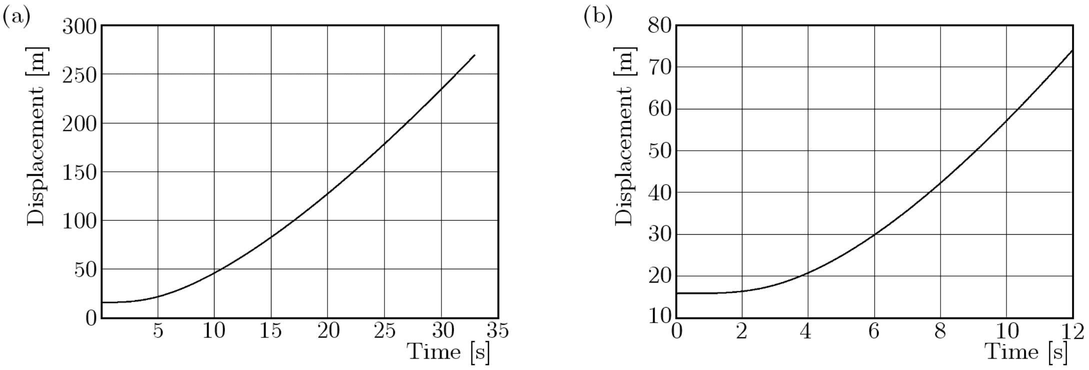

Fig. 10. Displacement of the center of mass of the first wheelset as a function of time

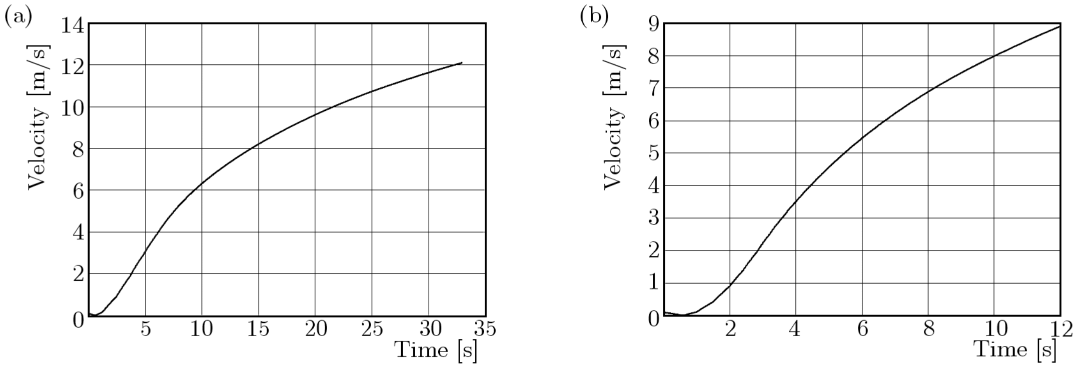

Fig. 11. Velocity of the center of mass of the first wheelset as a function of time

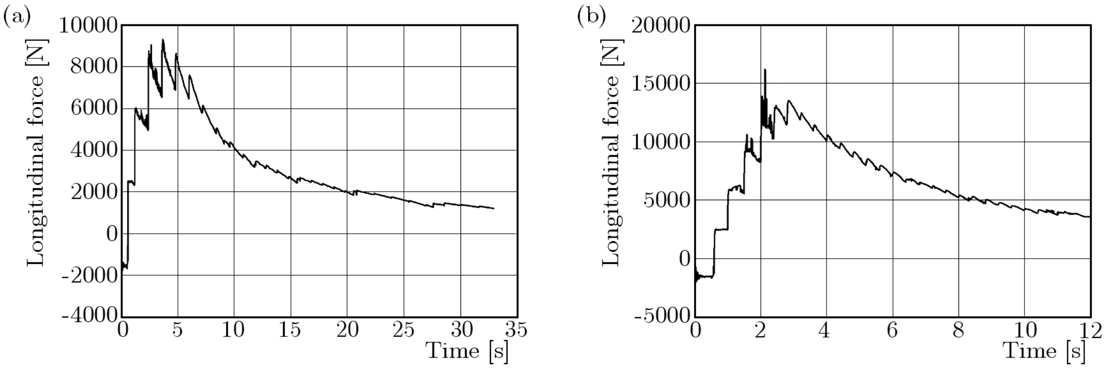

Fig. 12. Longitudinal force at the point of contact of the rail with the left wheel of the first wheelset as a function of time

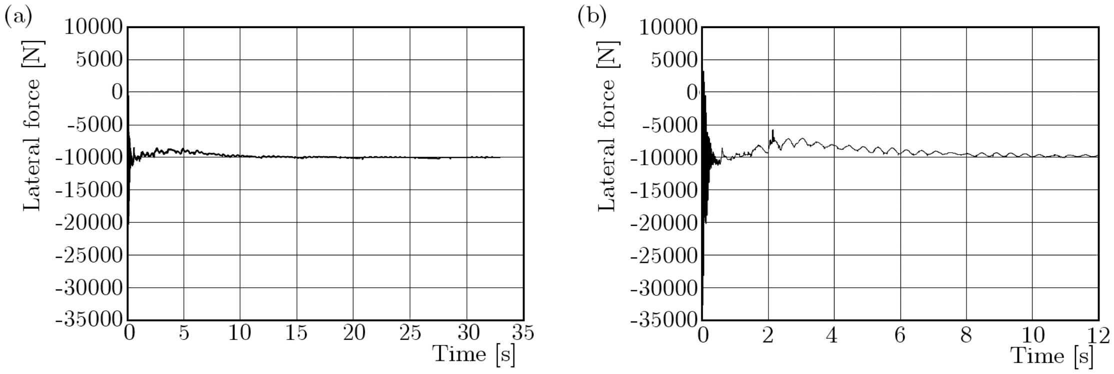

Fig. 13. Lateral force at the point of contact of the rail with the left wheel of the first wheelset as a function of time

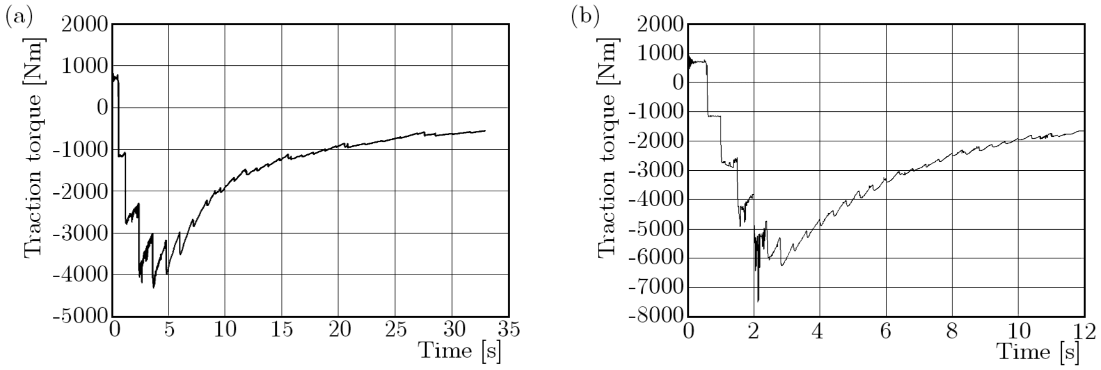

Fig. 14. Traction torque at the point of contact of the rail with the left wheel of the first wheelset as a function of time

Because of the similarity of the results obtained for the left and right wheel, the boundary conditions are almost symmetrical – the movement occurs centrally along the railway track axis at a velocity lower than critical, only those for the left wheel are presented.

The simulation performed shows that reduction of the operation time of additional resistors results in an increment in the electromechanical torque generated by motors in their main circuits (Fig. 7), which, in turn, yields higher traction forces (Fig. 14).

5. Final conclusions

The tool for simulating motion of rail vehicles has been used to test various cases, showing the potential of the presented method in a variety of situations. These cases included analysis of the EU07 rail vehicle dynamics during its travel on a straight railway track at various resistance controls.

The created vehicle model, in form of an electromechanical system including dedicated programs for determining the vehicle support and guide forces, provides the possibility of obtaining the following results:

- Curves of kinematic parameter changes at any selected point of model components (displacements, velocities, accelerations), interaction forces in particular kinematic pairs and at the wheel-rail interface.
- The point of contact between the wheel and rail (separately for the rolling base and the flange), sizes of the contact ellipse at the wheel-rail surface as well as the place of their occurrence.

The results of presented computer simulations allow for coming to the conclusion that the developed algorithms and computer programs are general in their character and can be used to determine structural features of electromechanical systems with similar design forms.

The presented models enable testing of dynamic phenomena occurring in power transmission systems, especially in transient states such as start-up or change of loading conditions. The models serve for determination of optimal traction parameters for locomotives by, e.g., selection of the gear ratio of the power transmission system.

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