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THE COMPUTATIONAL MODEL OF THE LOAD DISTRIBUTION BETWEEN ELEMENTS IN A PLANETARY ROLLER SCREW

Jan Ryś, Filip Lisowski
Cracow University of Technology, Institute of Machine Design, Cracow, Poland
e-mail: szymon@mech.pk.edu.pl; flisow@mech.pk.edu.pl

The paper presents a computational model of the load distribution between elements in a planetary roller screw. The idea of the model is to consider the deformations of rolling elements as deformations of rectangular volumes subjected to shear stresses. The contact deformations of threads and the deformations of screw and nut cores are taken into account by a properly chosen shear modulus. This modulus is computed on the basis of stiffness determined using the finite element method. Based on the relation between displacements and forces differential equations are obtained. The solution to these equations is a hyperbolic function that illustrates the load distribution on the threads between cooperating elements. The following considerations are carried out to assess the suitability of the analytical model for the preliminary design and analysis of this type of transmissions.

Keywords: planetary roller screw, computational model, load distribution

1. Introduction

The planetary roller screw (PRS) is a low-friction mechanical actuator that uses threaded rollers to transfer the load between the screw and the nut (Fig. 1). The main elements of such a structure are: main screw (1) cooperating with rollers (4) connected by planetary toothed conjunction (5,6) with nut (3).

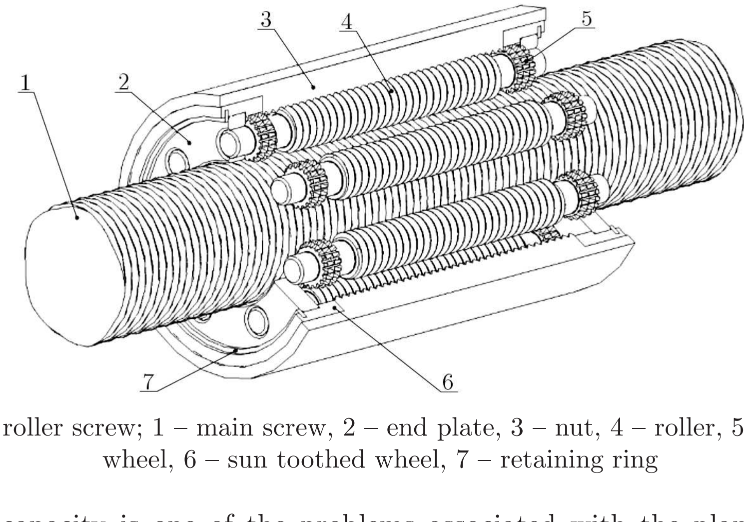

Fig. 1. Planetary roller screw; 1 – main screw, 2 – end plate, 3 – nut, 4 – roller, 5 – satellite toothed wheel, 6 – sun toothed wheel, 7 – retaining ring

The carrying capacity is one of the problems associated with the planetary roller screw. The related problem is to determine the load distribution between cooperating elements. Load characteristics provide information about the most loaded parts of threads. Smoothing of the load characteristics results in a better load distribution on threads and therefore can have an effect on an increase in the carrying capacity. The goal of the paper is to assess the suitability of the analytical model for the preliminary design of planetary roller screws, which is based on the comparison of analytical and numerical results obtained for models based on the replacement of the cooperation of helical surfaces of threads by rectangular volumes subjected to shear stresses. Two cases for which internal forces in the screw and the nut are consistent (tension--tension, compression-compression) or opposite (tension-compression, compression-tension) are considered.

Previous publications on planetary roller screws considered researches on the capabilities and limitations (Hojjat and Mahdi Agheli, 2009), kinematics (Jones, Velinsky, 2012; Velinsky *et al.*, 2009; Sokolov *et al.*, 2005), calculation method for an elastic element operating at a large displacement with a distributed load (Tselishchev, Zharov, 2008), axial stiffness and load distribution (Chen, 2009), analysis of contact deformation and stress distribution based on a three dimensional model (Ma *et al.*, 2011), contact analysis where series of balls were used to replace rounded profiles of rollers and stress distribution of the roller thread (Zhang *et al.*, 2012), and a new study on the parameter relationships (Ma *et al.*, 2012). In the above works the analysis of deformations and load distribution were based on the Hertzian contact theory. They considered the axial contact deformation of the cooperating threads, whereas deformations resulting from shearing of the screw and nut cores should be also taken into account to determine continuous characteristics of the load distribution for an arbitrary number of rollers. Methods described in the publications are rather complicated and are not suitable for the preliminary design, which should be a basis for the further analysis. At the stage of a preliminary design, it is significant to estimate the carrying capacity, and then in the next stage, which is detailed design, perform more complex analysis based on the Hertzian contact theory and take into consideration the complex state of deformation of planetary roller screw components. Another problem related to the planetary roller screw is a design of planetary gear transmission with non-interference in internal engagement. This issue requires consideration of changes in geometry caused by correction coefficients. The procedure of modification coefficients in planetary gear transmissions was developed by Ryś (2012).

2. Computational model of the load distribution

The idea of the model is to consider deformations of cooperating elements as the deformations of rectangular volumes (Fig. 2a and 2b) subjected to shear stresses.

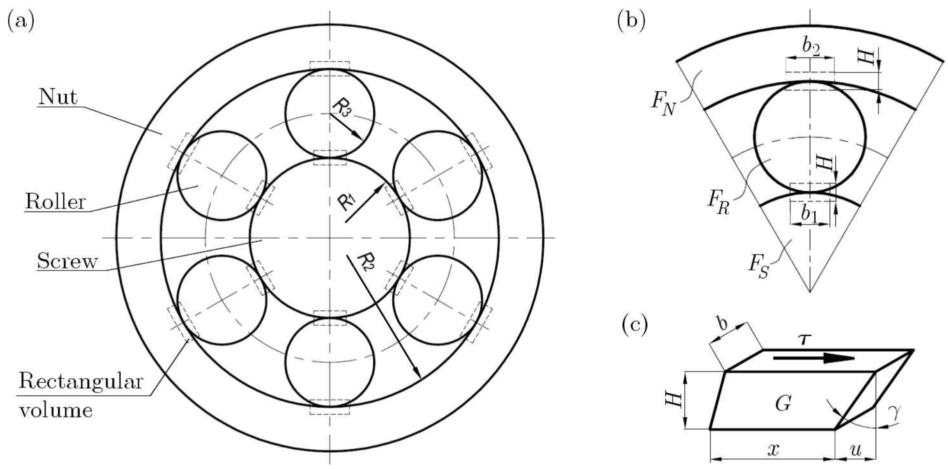

Fig. 2. (a) Model of planetary roller screw; (b) location and dimensions of the rectangular volumes; (c) load and displacement of the rectangular volume

Taking into account an elementary surface for the fine pitch distance of thread Pb, the shear stress τ at the surface of the rectangular volume is $\tau = Q/Pb$. The stiffness c and transverse displacement u are given by Eqs. (2.1), where P is the pitch of thread, b, H – width and height of the rectangular volume, γ – shear strain, Q – internal shearing force

$$c = \frac{Q}{u} \qquad u = H\gamma \tag{2.1}$$

The shear modulus G can be calculated separately for the rectangular volumes with the width b_1 and b_2 using Eqs. (2.2). The stiffness c_1 and c_2 are determined based on the additional finite element analysis of the detailed model including pair of cooperating elements within one coil of thread. The index 1 refers to the screw, 2 – to the nut

$$\frac{\tau_i}{\gamma_i} = \frac{c_i H}{P b_i} = G_i \qquad i = 1,2 \tag{2.2}$$

The balance of axial forces within the planetary roller screw for the case of the consistent load is shown in Fig. 3a and given by Eq. $(2.3)_1$, whereas for the opposite load is shown in Fig. 3b and given by Eq. $(2.3)_2$. Q denotes the axial load, N_1 – internal force in the screw, N_2 – internal force in the nut, N_3 – internal force in the rollers

$$Q = N_1 + N_2 + N_3 \qquad 0 = N_1 - N_2 - N_3 \tag{2.3}$$

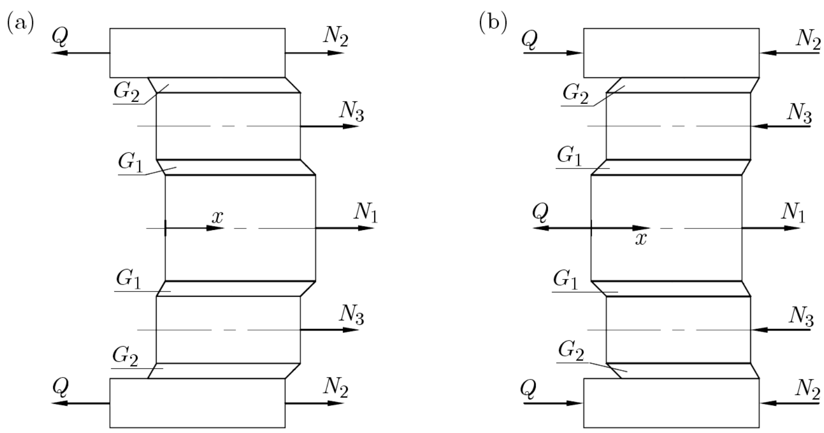

Fig. 3. Balance of forces – consistent load (a) and opposite load (b) of screw and nut

The internal forces in the screw and in the nut per one roller can be obtained from Eqs. $(2.4)_1$, and their first derivatives from Eqs. $(2.4)_2$. After taking into account Eqs. (2.2) in Eqs. $(2.4)_1$, the shear stress of rectangular volumes (Eqs. $(2.4)_3$) is obtained. n_r denotes the number of rollers

$$\left.\begin{aligned} N_i' &= \frac{1}{n_r} N_i = b_i \int_0^x \tau_i(x)\,dx \\ \frac{dN_i'}{dx} &= b_i \tau_i \\ \gamma_i &= \frac{\tau_i}{G_i} = \frac{1}{G_i b_i n_r} \frac{dN_i}{dx} \end{aligned}\right\} \qquad i = 1,2 \tag{2.4}$$

2.1. Load distribution for the analytical model – consistent load of the screw and nut

On the assumption of ideal rigid roller cores, the balance of forces within the planetary roller screw takes the form: $Q = N_1 + N_2$. Increments of shear strain are obtained from Eqs. (2.5), where: E – Young's modulus, F_1, F_2 – cross-sectional areas of the screw and the nut cores

$$\frac{d\gamma_1}{dx} = \frac{1}{H}\Big(\frac{N_1}{EF_1} - \frac{N_2}{EF_2}\Big) \qquad \frac{d\gamma_2}{dx} = \frac{1}{H}\Big(\frac{N_2}{EF_2} - \frac{N_1}{EF_1}\Big) \tag{2.5}$$

Including Eqs. $(2.4)_2$ and balance of forces, the second derivative of the internal force in the screw is obtained as given by

$$\frac{d^2N_1}{dx^2} = \frac{G_1b_1n_r}{HEF_1}\Big[N_1\Big(1+\frac{F_1}{F_2}\Big) - \frac{F_1}{F_2}Q\Big] \tag{2.6}$$

The second-order differential equation describing the load distribution between the screw and the rollers as well as constants occurring in this equation are given by

$$N_1''(x) - \omega^2 N_1(x) = \frac{-\omega\beta Q}{1+\beta} \qquad \omega^2 = (1+\beta)\frac{G_1b_1n_r}{HEF_1} \qquad \beta = \frac{F_1}{F_2} \tag{2.7}$$

Solution to equation Eqs. (2.7) is a hyperbolic function expressed by

$$N_1(x) = \frac{\beta}{1+\beta}Q + C_1\sinh(\omega x) + C_2\cosh(\omega x) \tag{2.8}$$

Constants of integration C_1 and C_2 are determined with boundary conditions assumed for the case of the consistent load of the screw and the nut as follows

$$\begin{aligned} &1^\circ \quad x=0, \quad N_1=0 \qquad && C_1 = \frac{Q}{1+\beta}\frac{1+\beta\cosh(\omega l)}{\sinh(\omega l)} \\ &2^\circ \quad x=l, \quad N_1=Q \qquad && C_2 = -\frac{\beta}{1+\beta}Q \end{aligned} \tag{2.9}$$

The load distribution in the nut can be calculated using the equation of balance of forces. The distribution of shear stress can be evaluated by using

$$\tau_i = \frac{1}{b_in_r}\frac{dN_i(x)}{dx} \qquad i=1,2 \tag{2.10}$$

2.2. Load distribution for the analytical model – opposite load of the screw and nut

In the case of the opposite sense of internal force vectors in the screw and the nut, on the assumption of ideal rigid of roller cores, the balance of forces takes the form $N_1 - N_2 = 0$. The increments of shear strain are obtained by

$$\frac{d\gamma_1}{dx} = \frac{1}{H}\Big(\frac{N_1}{EF_1} + \frac{N_2}{EF_2}\Big) \qquad \frac{d\gamma_2}{dx} = \frac{1}{H}\Big(-\frac{N_2}{EF_2} + \frac{N_1}{EF_1}\Big) \tag{2.11}$$

Taking into account Eqs. $(2.4)_3$ and the equation of balance of forces in Eqs. (2.11), the second derivative of the internal force in the screw is obtained as given by

$$\frac{d^2N_1}{dx^2} = \frac{G_1bn_r}{HEF_1}N_1\Big(1+\frac{F_1}{F_2}\Big) \tag{2.12}$$

The second-order differential equation describing the load distribution between the screw and rollers is formulated by Eq. (2.13). The constants in the equation were previously defined by Eqs. (2.7)

$$N_1''(x) + \omega^2 N_1(x) = 0 \tag{2.13}$$

The solution to Eq. (2.13) is a hyperbolic function expressed by

$$N_1(x) = C_1\sinh(\omega x) + C_2\cosh(\omega x) \tag{2.14}$$

The constants of integration C_1 and C_2 are determined with boundary conditions assumed for the case of the opposite load of the screw and the nut

$$\begin{aligned} &1^\circ \quad x=0, \quad N_1=Q \qquad && C_2 = Q \\ &2^\circ \quad x=l, \quad N_1=0 \qquad && C_1 = -Q\frac{\cosh(\omega l)}{\sinh(\omega l)} \end{aligned} \tag{2.15}$$

The load distribution in the nut can be calculated based on the forces balance equation. The distribution of shear stress can be evaluated by using Eqs. (2.10).

3. Finite element model

In order to verify the results obtained for the analytical model, comparative numerical calculations were carried out. A finite element model including a 1/8 of the planetary roller screw simplified geometry was built by applying ANSYS software. The model included cores of the screw, nut and roller as well as rectangular volumes. A bonded contact was defined between the cooperating elements. Symmetric boundary conditions were added on the side walls and the axial load was applied as shown in Figs. 4a and 4b. The shear modulus of rectangular volumes was chosen individually for each rectangular volume based on the additional numerical calculation.

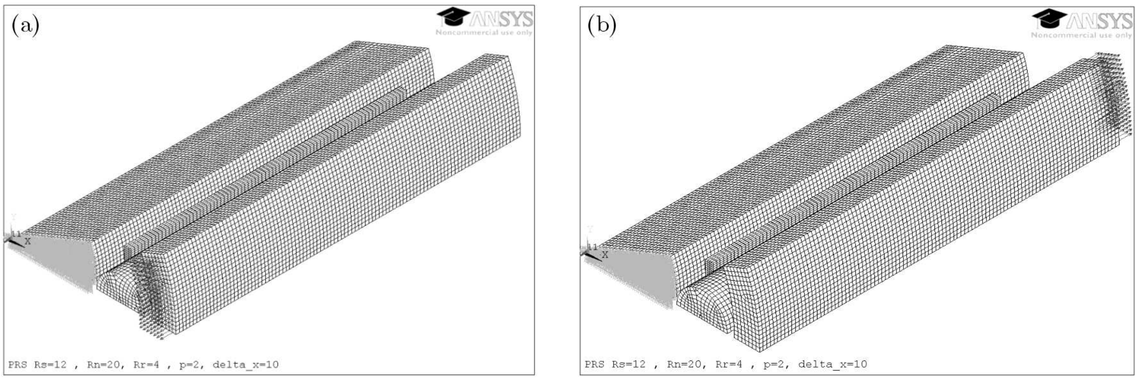

Fig. 4. FEM model of PRS – consistent load (a) and opposite load (b) of screw and nut

4. Comparison of analytical and numerical results

Characteristics of the shear stress distribution and the load distribution in the axial direction obtained from analytical and numerical models were compared for a representative example. For this example, the following main parameters of the planetary roller screw have been assumed: rolling radius of the screw $R_1 = 12\,\mathrm{mm}$, rolling radius of the nut $R_2 = 20\,\mathrm{mm}$, rolling radius of the roller $R_3 = 8\,\mathrm{mm}$, pitch of the thread $P = 2\,\mathrm{mm}$, linear displacement of the planetary roller screw per one rotation of the screw $\Delta x = 10\,\mathrm{mm}$.

Comparisons for the case of consistent load of the screw and the nut are shown in Figs. 5a and 5b, whereas results for the case of opposite load are presented in the Figs. 6a and 6b. Both the shear stress distribution and the load distribution in the axial direction show greater variation in values in the centre section for the analytical solution. This is due to the accepted assumption of ideal rigid cores of rollers and taking into account only shear stress. However, in most of the loaded regions of cooperating elements, the analytical model gives a higher stress concentration factor except for the load distribution in the screw for the consisted load. The stress concentration factor was calculated as the ratio of the maximum stress value to the average value for each model. The stress concentration factors are compared in Table 1.

Table 1. Stress concentration factor in the most loaded regions of cooperating elements

	Consistent load of screw ant nut		Opposite load of screw and nut	
	Analytical	FEM	Analytical	FEM
Screw	1.30	1.03	1.84	1.06
Nut	1.27	1.31	1.66	1.31

Comparing the analytical and numerical results, it can be concluded that the load distribution is not particularly different, especially with regard to the screw. Some differences occur in the load distribution in the nut, which in fact undergoes deformations which are not necessarily

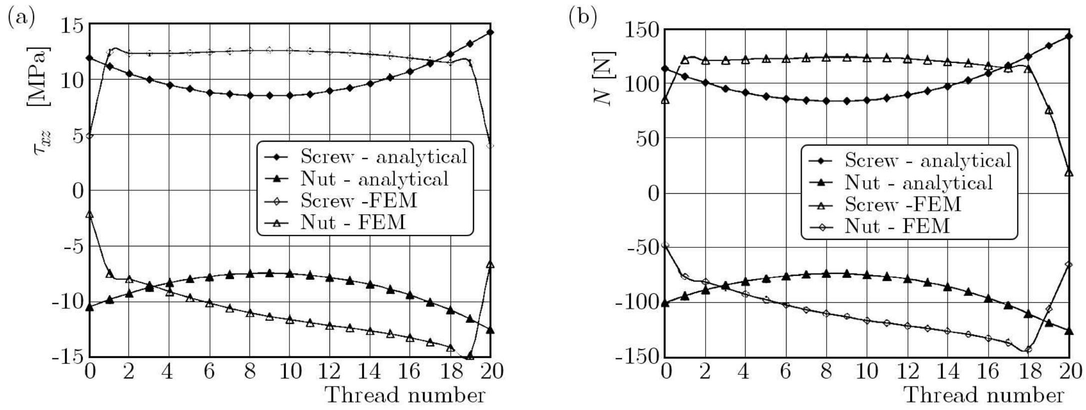

Fig. 5. Share stress distribution (a) and load distribution (b) – consistent load of the screw and nut

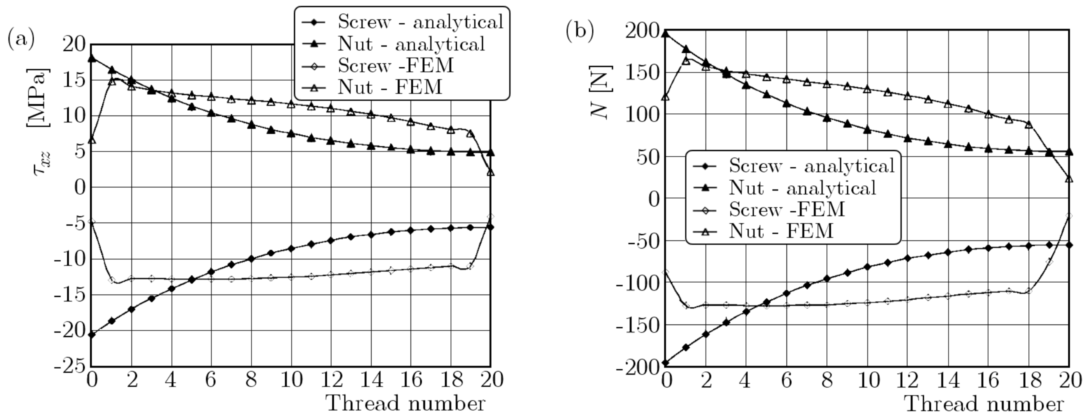

Fig. 6. Share stress distribution (a) and load distribution (b) – opposite load of the screw and nut

axisymmetric. It is also important to note the differences in average values of the load, which are higher for the finite element model. These differences are respectively 11% and 20% for the bolt and the nut for the case of the consistent load, whereas for the opposite load 16% and 22%. The percentage differences were calculated relative to the results of the analytical solution, which were based on clearly defined assumptions and therefore can be considered as the basic solution with respect to the numerical solution, which depends on the accepted conditions such as mesh density. The reason for changes in the average values of the load is a complex state of deformation in the numerical calculations.

5. Conclusions

The results obtained for the computational model of the load distribution betweenthe cooperating elements in the planetary roller screw were compared with the numerical results. It was shown that the stress concentration for the analytical solution in comparison to the numerical solution is higher with one exception. That indicates that the proposed model gives results in favour of certainty of calculations. The proposed analytical model can be used to determine the load distribution between the cooperating elements in the planetary roller screw for an arbitrary number of rollers without the need of detailed numerical calculations. The presented method can be the basis for preliminary analysis of carrying capacity and a conceivable modification of the rollers thread. Based on the conducted comparisons, it can be concluded that the analytical method fulfils the requirements for use in the preliminary design of planetary roller screws.

Furthermore, based on the results, it can be concluded that the most preferred method for supporting the screw and the nut is when the internal forces in those elements are consistent. The resulting stress concentration on the threads of cooperating elements is less than for the case of opposite loads.

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