INCLUSION OF THE D-OPTIMALITY IN MULTISINE MANOEUVRE DESIGN FOR AIRCRAFT PARAMETER ESTIMATION

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This paper is concerned with the designing of simultaneous flight control deflections for aircraft system identification. The elevator, ailerons and rudder are excited with harmonically related multisine signals. The optimal deflections are designed when there is no information about the stability and control derivatives and when this information is available. The inclusion of the system dynamics in the inputs design phase is done with the D-optimality criterion. Both sets of optimal flight surface deflections are used as excitations of a nonlinear aircraft model which is identified through the maximum likelihood estimation method. Parameters accuracy for those maneuvers (designed with and without a-priori knowledge) is presented and compared.

Keywords: inputs design, system identification, flight dynamics

1. Introduction

To obtain precise information about aircraft dynamics, aerodynamic stability and control derivatives have to be determined. This can be done by various methods like wind tunnel tests (Hoe *et al.*, 2012), Computational Fluid Dynamics – CFD (Mader and Martins, 2011) and System Identification – Sys-ID (Jameson and Cooke, 2012; Jategaonkar, 2006; Lichota and Ohme, 2014). Among this group, the gathering of aerodynamic databases from flight test data is the most reliable as it is based on experiments made on real objects. On the other hand, as multiple flight tests are performed in the Sys-ID, it is of high cost and time consuming. CFD methods are the least expensive in this group, however final results of aerodynamic derivatives (regardless of the object) have to be compared with experiments (Rogowski and Maroński, 2015).

The Sys-ID approach consists of four main steps that are: designing and performing manoeuvres, measuring the data, modelling the object and estimating unknown parameters. Those steps are strictly connected, and if the registered data will be inaccurate or there will be not enough information about the aircraft dynamics in the output signals, reliable estimation will not be possible. As modern sensors allow one to obtain very high accuracy of the data and measurement techniques are well developed a strong emphasis should be put on designing inputs used for exciting the optimal aircraft response.

The research concerning excitations that were to maximize the information content in the measured data was extensively investigated in the seventies and eighties. Those studies showed that sine-sweep (linear or logarithmical) or typical multi-step inputs (pulse, doublet, 3-2-1-1) can be used to develop a good mathematical model of an aircraft if only one flight control surface is deflected through a manoeuvre. It has been shown recently that simultaneous multi-step excitations could be used for this purpose as well (Lichota and Ohme, 2014), but this requires *a priori* knowledge of the aerodynamic derivatives.

If the flight controls can be deflected at the same time, the use of harmonically related multisine signals is possible as well. This approach does not require the initial information about

the system and allows one to obtain estimates with the same quality (in terms of parameters accuracy) as the design with simultaneous multi-step inputs. If the inclusion of the *a priori* knowledge in the multisine inputs design phase would increase the quality of estimation, it would mean that multisine signals are more adequate for aircraft Sys-ID purposes than simultaneous multi-step excitations.

The incorporation of initial knowledge of aerodynamic parameters can be done by introducing the D-optimality criterion instead of the Relative Peak Factor in the multisine optimization phase. In the present study, a linear aircraft model is used for this purpose as it is less computationally demanding than the nonlinear representation. The designed set of simultaneous multisine excitations is used as inputs for a nonlinear aircraft model that has been created in Matlab. On the basis of recorded signals the unknown parameters of the object are estimated by applying the maximum likelihood principle.

A similar procedure is applied to multisine signals that are designed without *a priori* knowledge of the system dynamics. Evaluated ailerons, elevator and rudder deflections are used as excitations for the nonlinear aircraft model. The response of the object is recorded and then the unknown parameters of the system are identified. The results are compared with the estimates obtained from the manoeuvre in which initial values of the aerodynamic derivatives are available in the inputs design phase.

2.   Multisine input signals

The multisine input is an excitation that is composed of summed harmonic sinusoids with individual amplitudes A_k and phase shifts ϕ_k (Morelli, 2012)

$$\delta = \sum_{k=1}^{M} A_k \sin(2\pi f_k t + \phi_k) \tag{2.1}$$

where $k = 1, \ldots, M$ stands for the number of the harmonic, f_k is the frequency of the k-th component and δ is the input signal.

An important feature of the multisine signals is that they can be designed as mutually orthogonal in the time and frequency domain (Morelli, 2003). This means that simultaneous ailerons, elevator and rudder deflections can be independent. In order to achieve this aim, it is required to assign different harmonics to each flight control (e.g. $2f_0$, $5f_0$, $8f_0$,... to ailerons, $3f_0$, $6f_0$, $9f_0$,... to elevator, $4f_0$, $7f_0$, $10f_0$,... to rudder). This assignment provides orthogonality in the frequency domain as distinct spectral lines form the frequency content of each input. In the time domain, the mutual orthogonality is achieved due to orthogonality properties of the sine function (Morelli, 2012). It is practicable to omit the first harmonic in this assignment in order to optimize the cost function effectively.

Frequencies of the consecutive components in the multisine signals are evenly spaced and based on the excitation time T: $f_k = k/T$. This also limits the minimum available frequency which must satisfy the condition: $f_{min} \geqslant 2/T$. The maximum available frequency is limited by the frequency range of interest in the investigated case.

Multisine input signals have wide-frequency band and the amplitudes of the different harmonics A_k are chosen to achieve desired power spectrum. If there is no need to put an emphasis on specific frequencies, a uniform power spectrum should be used. For simultaneous aileron, elevator and rudder deflections and flat power spectrum, the amplitudes A_k related to the j-th control are given by

$$A_{j,k} = \frac{A_j}{\sqrt{M_j}} \tag{2.2}$$

where A_j is the amplitude of the j-th flight control and M_j is the number of the harmonic components assigned to the j-th flight surface.

For each flight control, input energy maximization is done through proper selection of consecutive harmonics phase angles $\phi_{j,k}$. The phase angles should maximize the excitation effectiveness without unnecessary increase in each signal value as this can cause that aircraft will go too far from the trim conditions and will hinder the Sys-ID process. For this purpose, the Relative Peak Factor (RPF) can be used as it expresses the input amplitude range divided by the excitation energy. The RPF is also scaled so that for a single sinusoid it is equal to 1. For the j-th flight surface deflection δ_j, the RPF is given by

$$RPF(\delta_j) = \frac{\max \delta_j - \min \delta_j}{2\sqrt{2}\mathrm{rms}(\delta_j)} \tag{2.3}$$

RPF minimization is equivalent to input effectiveness maximization and can be achieved with a simplex optimization algorithm. In our study, the Schroeder phase angles are used as initial values of the phase angles ϕ_k for the j-th flight control (Schroeder, 1970)

$$\phi_{j,k} = \phi_{j,k-1} + 2\pi(f_{j,k-1} - f_{j,k})t_{j,k-1} \qquad t_{j,k} = T\sum_{l=1}^{k} p_{j,l} \tag{2.4}$$

where $f_{j,k}$ is the k-th harmonic frequency assigned to the j-th input and t_k is time epoch. For the flat power spectrum and the j-th flight control, the power of the k-th component is $p_{j,k} = 1/M_j$.

As deflections should start and end with zero amplitude, it is necessary to find a constant time offset for the components of each excitation. This is equivalent to sliding the inputs along the time axis until the zero crossing occurs at $t = 0$.

3. D-optimal criterion

The design process of multisine input signals does not require knowledge of the system dynamics in terms of stability and control derivatives. However, it is considered that inclusion of some information about aerodynamic parameters in the design phase could increase the quality of the aircraft response in terms of the Sys-ID. This requires introducing estimator based on the minimum mean squared error (Kay, 1993)

$$\mathrm{MSE}(\widehat{\boldsymbol{\Theta}}) = \mathrm{E}[(\widehat{\boldsymbol{\Theta}} - \boldsymbol{\Theta})^2] = \mathrm{cov}\,(\widehat{\boldsymbol{\Theta}}) + \mathrm{bias}^2(\boldsymbol{\Theta}, \widehat{\boldsymbol{\Theta}}) \tag{3.1}$$

where $\boldsymbol{\Theta}$ are the parameters of the model and $\widehat{\boldsymbol{\Theta}}$ are their estimates.

When the estimator is designed to be unbiased, the minimum mean squared error consists only of the covariance term. This part can be determined on the basis of Cramér-Rao inequality which states that the covariance of the unbiased estimator is at least as high as the inverse of the Fisher Information Matrix $\mathcal{F}$

$$\mathrm{cov}\,(\widehat{\boldsymbol{\Theta}}) \geqslant \mathcal{F}^{-1} \tag{3.2}$$

The Fisher Information Matrix which is a measure of the amount of information that observable variables carry about the unknown system parameters is defined as

$$\mathcal{F} = E\left\{\left[\frac{\partial \ln \mathcal{L}(\boldsymbol{\Theta}|\mathbf{z})}{\partial \boldsymbol{\Theta}}\right]\left[\frac{\partial \ln \mathcal{L}(\boldsymbol{\Theta}|\mathbf{z})}{\partial \boldsymbol{\Theta}}\right]^{\mathrm{T}}\right\} \tag{3.3}$$

where the likelihood function $\mathcal{L}(\boldsymbol{\Theta}|\mathbf{z})$ is equal to $p(\mathbf{z}|\boldsymbol{\Theta})$ – conditional probability that the measurement vector $\mathbf{z}$ is observed for the model parameters $\boldsymbol{\Theta}$.

If the likelihood function is twice differentiable with respect to the model parameters, it can be shown that

$$\boldsymbol{\mathcal{F}} = -\mathrm{E}\left[\frac{\partial^2 \ln \mathcal{L}(\boldsymbol{\Theta}|\mathbf{z})}{\partial \boldsymbol{\Theta}\boldsymbol{\Theta}^{\mathrm{T}}}\right] \tag{3.4}$$

Multivariate normal distribution is usually chosen to evaluate the probability density function at a certain time point t_k. Due to variables, independence the probability can be defined for all time points in the manoeuvre (Jategaonkar, 2006)

$$p(\mathbf{z}|\boldsymbol{\Theta}) = \frac{1}{\sqrt{(2\pi)^n|\mathbf{R}|}^N} \exp\left(-\frac{1}{2}\sum_{k=1}^{N}[\mathbf{z}(t_k) - \mathbf{y}(t_k)]^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}(t_k) - \mathbf{y}(t_k)]\right) \tag{3.5}$$

where n is the number of model outputs y, $k = 1, \ldots, N$ is the index of time samples and $\mathbf{R}$ is the measurement noise covariance matrix.

The described approach with neglecting small terms leads strictly to the Fisher Information Matrix sensitivity form

$$\boldsymbol{\mathcal{F}} \approx \sum_{k=1}^{N}\left[\frac{\partial \mathbf{y}(t_k)}{\partial \boldsymbol{\Theta}}\right]^{\mathrm{T}} \mathbf{R}^{-1} \left[\frac{\partial \mathbf{y}(t_k)}{\partial \boldsymbol{\Theta}}\right] \tag{3.6}$$

where the output signals gradients $\partial \mathbf{y}(t_k)/\partial \boldsymbol{\Theta}$ can be obtained through forward difference formula. Introducing central differences does not increase the accuracy significantly and raises the computational time, therefore, it is not used in the evaluations.

The columns of the Fisher Information Matrix represent contributions of the model parameters, so if the columns are independent, the determinant reaches its maximum value. On the contrary, if the columns are linearly dependent, the determinant will accept the minimum value. Therefore, the optimality criterion (D-optimality) is expressed by maximization of the Fisher Information Matrix determinant what means that the estimation error ellipsoid volume is minimized.

Inclusion of the D-optimality criterion in the multisine design phase can be used in order to introduce the initial information about stability and control derivatives. This task can be achieved by incorporating the linear aircraft model and minimization of the Fisher Information Matrix inverse (Parameter Error Covariance Matrix $\boldsymbol{\mathcal{P}}$) determinant. The linear representation has been selected because it well describes the object, and the computational time for single evaluation is relatively short.

4.   Linear model

The aircraft dynamic equations of motion are derived in a vehicle carried coordinate system $Oxyz$ and linearised in accordance with Etkin (1972). The origin of the $Oxyz$ reference frame is located at the center of gravity. The Ox axis coincides with the longitudinal axis of the airplane. The Oy axis is normal to the aircraft symmetry plane and is pointing in the direction of the right wing. The Oz axis is oriented downward, so it completes the right-handed coordinate system.

The $Oxyz$ system is related to the vehicle carried $Ox_g y_g z_g$ reference frame through rotation angles: Φ (roll angle), Θ (pitch angle), Ψ (yaw angle) which are used to describe the orientation of the object. The $Ox_g y_g z_g$ coordinate system remaines parallel to the earth fixed reference frame $O_1 x_1 y_1 z_1$, whose origin is located at an arbitrary point of the Earth with the Ox_1 axis pointed north, Oy_1 axis pointed east and the Oz_1 axis is directed toward center of the Earth. Relations between the described coordinate systems are shown in Fig. 1, and the transformations of linear
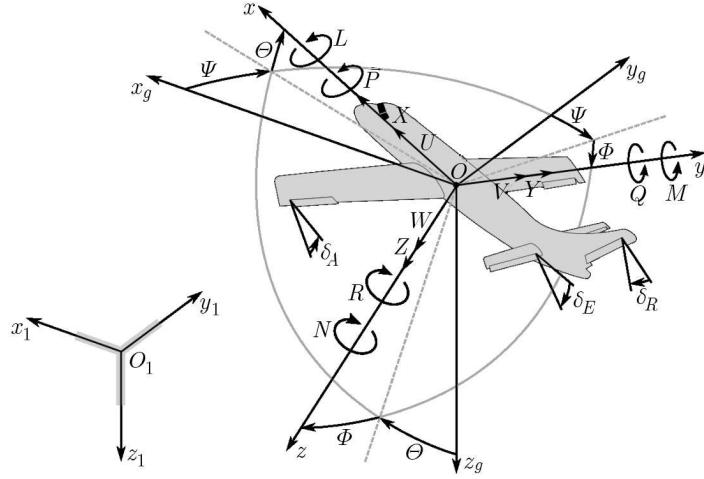
Fig. 1. Coordinate systems and motion variables

and angular quantities from the gravitational reference frame $Ox_g y_g z_g$ to the body coordinate system $Oxyz$ are given by the matrices

$$
\mathbf{\Lambda}_V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix} \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{bmatrix} \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
$$
\mathbf{\Lambda}_\Omega = \begin{bmatrix} 1 & 0 & -\sin\Theta \\ 0 & \cos\Phi & \sin\Phi\cos\Theta \\ 0 & -\sin\Phi & \cos\Phi\cos\Theta \end{bmatrix}
\tag{4.1}
$$

Dynamic equations of motion are derived from Newton's second law of motion in $Oxyz$ and kinematic relationships, which leads to

$$
m(\dot{\mathbf{V}}_O + \mathbf{\Omega} \times \mathbf{V}_O) = \mathbf{F} \qquad \mathbf{I}\dot{\mathbf{\Omega}} + \mathbf{\Omega} \times (\mathbf{I}\mathbf{\Omega}) = \mathbf{M}_O \qquad \dot{\mathbf{\Phi}} = \mathbf{\Lambda}_\Omega^{-1}\mathbf{\Omega}
\tag{4.2}
$$

where $\mathbf{V}_O = [U \ V \ W]^\mathrm{T}$ is the velocity of the origin, $\mathbf{\Omega} = [P \ Q \ R]^\mathrm{T}$ is the angular velocity, $\dot{\mathbf{\Phi}} = [\Phi \ \Theta \ \Psi]^\mathrm{T}$ describes the aircraft orientation, m stands for mass, $\mathbf{I}$ for inertia matrix, $\mathbf{F} = [X \ Y \ Z]^\mathrm{T}$ and $\mathbf{M}_O = [L \ M \ N]^\mathrm{T}$ are forces and moments acting on the object. The dot symbol is used to denote derivatives with respect to time.

When the level flight is the equilibrium state, it is possible to linearize the equations of motion and describe the system as follows

$$
\begin{aligned}
\dot{u} &= X_u u + X_\alpha \alpha + (X_q - u_0\alpha_0)q + g\theta\cos\Theta_0 + X_{\delta_E}\delta_E \\
\dot{\beta} &= Y_\beta\beta + (Y_p + \alpha_0)p + (Y_r - 1)r + \frac{g}{|\mathbf{V}_0|}\cos\Theta_0 + Y_{\delta_A}\delta_A + Y_{\delta_R}\delta_R \\
\dot{\alpha} &= Z_u u + Z_\alpha\alpha + (Z_q + 1)q - \frac{g}{U_0}\sin\Theta_0 + Z_{\delta_E}\delta_E \\
\dot{p} &= L_\beta\beta + L_p p + L_r r + L_{\delta_A}\delta_A + L_{\delta_R}\delta_R \\
\dot{q} &= M_u u + M_\alpha\alpha + M_q q + M_{\delta_E}\delta_E \\
\dot{r} &= N_\beta\beta + N_p p + N_r r + N_{\delta_A}\delta_A + N_{\delta_R}\delta_R \\
\dot{\phi} &= p + r\tan\Theta_0 \\
\dot{\theta} &= q
\end{aligned}
\tag{4.3}
$$

where small letters are used to describe the perturbations of the flight state from the trim condition which is denoted by subscript 0 (e.g. $U = u_0 + u$). Symbol g is used for gravitational

acceleration, α is the angle of attack, β is the angle of sideslip and δ_A, δ_E, δ_R are ailerons, elevator and rudder deflections respectively. The signs of flight control deflections are defined with accordance to Hopkin (1970) – a positive flight surface deflection causes a negative aircraft response. This means that: left aileron up, elevator down and rudder to left are considered as positive. The symbols that are not defined yet are known as dimensional stability and control derivatives, e.g. M_α is the pitching moment derivative with respect to the angle of attack.

5.   D-optimal multisine inputs

Application of the D-optimality criterion for phase angles selection in multisine inputs design requires minimization of the Fisher Information Matrix inverse determinant (cost function)

$$\delta = \min_{\phi} \boldsymbol{\mathcal{F}}^{-1} \qquad\qquad (5.1)$$

The cost function has multiple local minima, therefore a genetic algorithm is selected for finding the optimal solution. The phase angles related with j-th flight surface and k-th harmonic $\phi_{j,k}$ are coded in a binary string that consists of fixed-length strings s. Base-2 floating point representation is used to determine phase angles for different aileron, elevator and rudder deflections

$$\phi_{j,k} = 2\pi \sum_s b_s 2^{-s} \qquad\qquad (5.2)$$

where $s = 1, \dots, 12$ is a part of the string that contains bit values $b_s \in \{0, 1\}$ used for coding a specific phase angle $\phi_{j,k}$.

Genetic algorithms mimics the process of natural selection and require multiple executions of four steps that are: population generation, selection mating, exchange of the information and mutation (Mitchel, 1999).

The initial population contains P possible solutions that are randomly drawn. The solutions are decoded into input signals and sorted in descending order in accordance with the cost function. Then different selection probability is assigned for each solution. The binary string that represents simultaneous excitations and produces the highest cost function has the smallest drawing probability $p_{min} = 1/\sum_{i=1}^{P} i$. The binary string that represents excitations with the smallest cost function has the highest drawing probability $p_{max} = P/\sum_{i=1}^{P} i$. The probabilities are assigned with the linear scale. In the next step, solutions are drawn in accordance to their probabilities and combined in pairs.

Exchanging information between the solutions in each mated pair (parents) is done by applying uniform crossover which results in creating two new solutions (offspring). Applying uniform crossover requires drawing a mask of flags which has the same length as coded solutions and is filled with zeros and ones. The first offspring is formed from fields of the first parent when the flag of the mask is 0 and from fields of the second parent when the flag of the mask is 1. The second offspring is created in a similar way – from fields of the first parent when the flag of the mask is 1 and from fields of the second parent when the flag of the mask is 0. The uniform crossover idea is shown in Fig. 2. Fields of the first parent have white background and of the second parent – gray. The described procedure is carried for all mated pairs and results in creating a set of new solutions whose size is equal to the initial population.

To introduce more diversity in the newly created population, a mutation operator is used. This action requires drawing a number for each field of each solution. When this number is higher than the threshold (0.95), the value of the field is changed from 0 to 1 or from 1 to 0. After performing mutation for all solutions, the initial population is replaced by the new one. In order to preserve the best simultaneous inputs, the parent that has the lowest cost function is also included in this population.
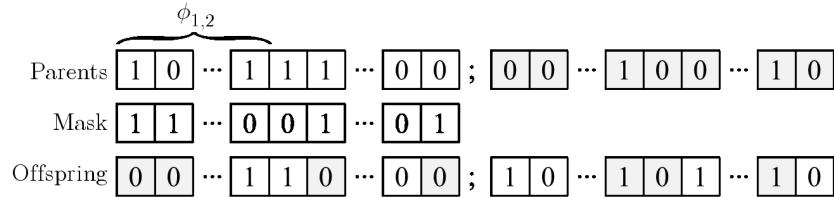
Fig. 2. Uniform crossover

The described steps: selection, crossing, mutation and population generation are performed until the global maximum is found.

6. Nonlinear model

Multisine input signals designed with and without *a priori* knowledge of stability and control derivatives are used as excitations of the nonlinear aircraft model (Lichota and Ohme, 2014; Raab, 2006). The dynamic equations of motion of the object are given by (4.2). The forces and moments that act on the aircraft consist of aerodynamic, gravitational and propulsion terms

$$\mathbf{F} = \mathbf{F}_a + \mathbf{F}_g + \mathbf{F}_t \qquad \mathbf{M}_O = \mathbf{M}_a + \sum_{i=1}^{4} \mathbf{r}_i \times \mathbf{F}_t \qquad (6.1)$$

where $\mathbf{r}$ is the thrust force arm and the indices a, g and t indices denote aerodynamic, gravitational and propulsion forces and moments.

The aerodynamic forces $\mathbf{F}_a = [X_a \ \ Y_a \ \ Z_a]$ and moments $\mathbf{M}_{aO} = [L_a \ \ M_a \ \ N_a]$ are obtained with the use of dimensionless force and moment coefficients (about quarter-chord point)

$$
\begin{aligned}
C_D &= C_{D0} + kC_L^2 \\
C_Y &= C_{Y0} + C_{Y\beta}\beta + (C_{Yp} + C_{Yp\alpha}\alpha)p^* + (C_{Yr} + C_{Yr\alpha}\alpha)r^* + C_{Y\delta_R}\delta_R \\
C_L &= C_{L0} + C_{L_{WB}\alpha}\alpha + \frac{S_H}{S}(C_{L_H\alpha_H}\alpha_H + C_{L_H\delta_E}\delta_E) \\
C_{l25} &= C_{l0} + C_{l\beta}\beta + (C_{lp} + C_{lp\alpha}\alpha)p^* + (C_{lr} + C_{lr\alpha}\alpha)r^* + C_{l\delta_A}\delta_A + C_{l\delta_R}\delta_R \\
C_{m25} &= C_{m0} + C_{m_{WB}q}q^* - \frac{x_{LH}}{\overline{c}}\frac{S_H}{S}(C_{L_H\alpha_H}\alpha_H + C_{L_H\delta_E}\delta_E) \\
C_{n25} &= C_{n0} + C_{n\beta}\beta + (C_{np} + C_{np\alpha}\alpha)p^* + (C_{nr} + C_{nr\alpha}\alpha)r^* + C_{n\delta_A}\delta_A + C_{n\delta_R}\delta_R
\end{aligned}
\qquad (6.2)
$$

where $p^* = pb/(2V_O)$, $q^* = q\overline{c}/(V_O)$ and $r^* = rb/(2V_O)$ are normalised angular rates, S is the wing area, S_H the horizontal tail area, x_{LH} is the horizontal tail arm and k is the drag polar coefficient. The indices WB and H in the equations for the lift force and pitching moment coefficients denote Wing-Body and Horizontal tail, respectively.

The angle of attack at the horizontal tail is given by

$$\alpha_H = \alpha - \frac{\partial\varepsilon}{\partial\alpha}\alpha\Big(t - \frac{x_{LH}}{V_O}\Big) + i_H + \alpha_{dyn} \qquad (6.3)$$

where ε is the downwash angle, i_H is the horizontal tail setting angle and $\alpha_{dyn} = qx_{LH}/V_O$ is the dynamic angle of attack.

The force and moment coefficients are defined in the aerodynamic coordinate system $Ox_ey_ez_e$. The Ox_e axis has the direction of the flow, the Oy_e axis is directed towards the right wing and

the Oz_e axis is directed upwards. As the equations of motion are derived in the body fixed coordinate system $Oxyz$, the following transformation is required

$$C_x = -C_D \cos\alpha + C_L \sin\alpha \qquad C_y = C_Y \qquad C_z = -C_D \sin\alpha - C_L \cos\alpha$$
$$C_l = C_{l25}\cos\alpha - C_{n25}\sin\alpha \qquad C_m = C_{m25} \qquad C_n = C_{l25}\sin\alpha + C_{n25}\cos\alpha \tag{6.4}$$

This allows one to obtain components of the aerodynamic forces and moments as

$$X_a = C_x \overline{q} S \qquad Y_a = C_y \overline{q} S \qquad Z_a = C_z \overline{q} S$$
$$L_a = C_l \overline{q} S b \qquad M_a = C_m \overline{q} S \overline{c} \qquad N_a = C_n \overline{q} S b \tag{6.5}$$

The thrust force is a function of the throttle position, altitude and speed: $\mathbf{T}(\delta_{th}, h, V_O)$. It is stored in a 3D array and interpolated for specific flight conditions. The gravitational force $\mathbf{F}_g = m\mathbf{g}$ is defined in the $Ox_g y_g z_g$ coordinate system, therefore transformation to $Oxyz$ is required.

7. System Identification

The classical and most widely used definition of System Identification was given by Zadeh: "*Identification is the determination, on the basis on input and output, of a system within a specified class of systems, to which the system under test is equivalent*" (Zadeh, 1962). If the response of the dynamical system and input signals are recorded and the structure of the model is known then the Sys-ID is equal to adjusting the model parameters $\boldsymbol{\Theta}$ so that the model outputs $\mathbf{y}$ match the measured aircraft response $\mathbf{z}$ when the inputs $\mathbf{u}$ are the same. The process can be done either offline or online in real time (Hendzel and Trojnacki, 2014).

The structure of the model we used in the Sys-ID was nonlinear and the same as for generating the aircraft response what allowed us to eliminate modelling errors in the study. The adjusting of model parameters was done in the time domain when all data was collected (offline). To find the set of unknown model parameters we used the Output Error Method whose aim was to minimize the error between measured system outputs and estimated response.

When the Maximum Likelihood Estimation principle is selected for error minimization the task is equivalent to finding the parameters vector that maximize the conditional probability $p(\mathbf{z}|\boldsymbol{\Theta})$

$$\widehat{\boldsymbol{\Theta}} = \arg\max_{\boldsymbol{\Theta}} p(\mathbf{z}|\boldsymbol{\Theta}) \tag{7.1}$$

where the hat symbol denotes the estimates, and the conditional probability $p(\mathbf{z}|\boldsymbol{\Theta})$ is given by (3.5).

As it is frequently done, we replace the probability maximization task by an easier action – minimization of a negative log likelihood function (Jategaonkar, 2006)

$$\mathcal{L}(\boldsymbol{\Theta}|\mathbf{z}) = \frac{1}{2}\sum_{k=1}^{N}[\mathbf{z}(t_k) - \mathbf{y}(t_k)]^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}(t_k) - \mathbf{y}(t_k)] + \frac{nN}{2}\ln(2\pi) + \frac{N}{2}\ln(\det(\mathbf{R})) \tag{7.2}$$

The unknown measurement covariance matrix $\mathbf{R}$ is estimated as

$$\widehat{\mathbf{R}} = \frac{1}{N}\sum_{k=1}^{N}[\mathbf{z}(t_k) - \mathbf{y}(t_k)][\mathbf{z}(t_k) - \mathbf{y}(t_k)]^{\mathrm{T}} \tag{7.3}$$

Substitution of the measurement covariance matrix estimate $\widehat{\mathbf{R}}$ into the negative log likelihood function and rejection of the constant terms allows one to obtain the cost function

$$J(\boldsymbol{\Theta}) = \det(\mathbf{R}) \tag{7.4}$$

The Gauss-Newton algorithm has been used for cost function minimization.

The accuracy of each estimated parameter is determined on the basis of the Parameter Error Covariance Matrix $\mathcal{P}$

$$\sigma(\Theta_i) = \sqrt{\mathcal{P}_{i,i}} \tag{7.5}$$

where σ is the standard deviation and $\mathcal{P}_{i,i}$ is the element of the Parameter Error Covariance Matrix $\mathcal{P} = \mathcal{F}^{-1}$.

8. Results

In the study, we investigate two designs with simultaneous multisine inputs: when there is no *a priori* knowledge of the system dynamics and when this information is available. The frequency range of interest upper bound is set to $f_{max} = 2\,\text{Hz}$ and the lower is determined on the basis of the excitation length $T = 20\,\text{s}$. A total of 39 harmonics is evaluated and assigned to the elevator $(f_2, f_5, \ldots)$, aileron $(f_3, f_6, \ldots)$ and rudder deflections $(f_4, f_7, \ldots)$. The maximum values of all inputs are set to $1\,\text{deg}$. In the multisine design without *a priori* knowledge, the phase angles ϕ_k are optimized through Relative Peak Factor minimization. When additional information about the system is available, the D-optimality criterion is used for signal optimization. In this case, the cost function extreme is found by a genetic algorithm.

Both designed sets of excitations are used as input signals for a nonlinear aircraft model. In both cases, the response of the model is recorded. The gathered data is used to perform a Sys-ID for a model with unknown aerodynamic parameters. The results of the estimation are shown in Fig. 3, Fig. 4 and in Table 1.
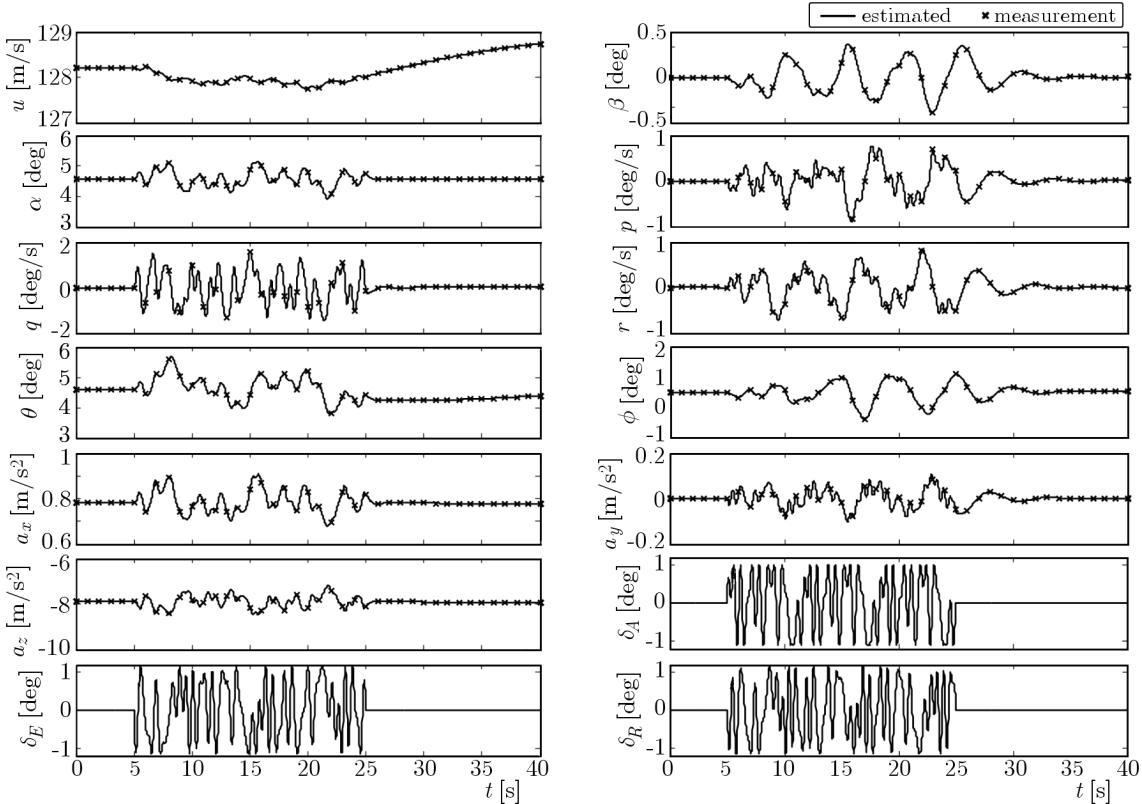
Fig. 3. Measured and estimated signals for multisine inputs
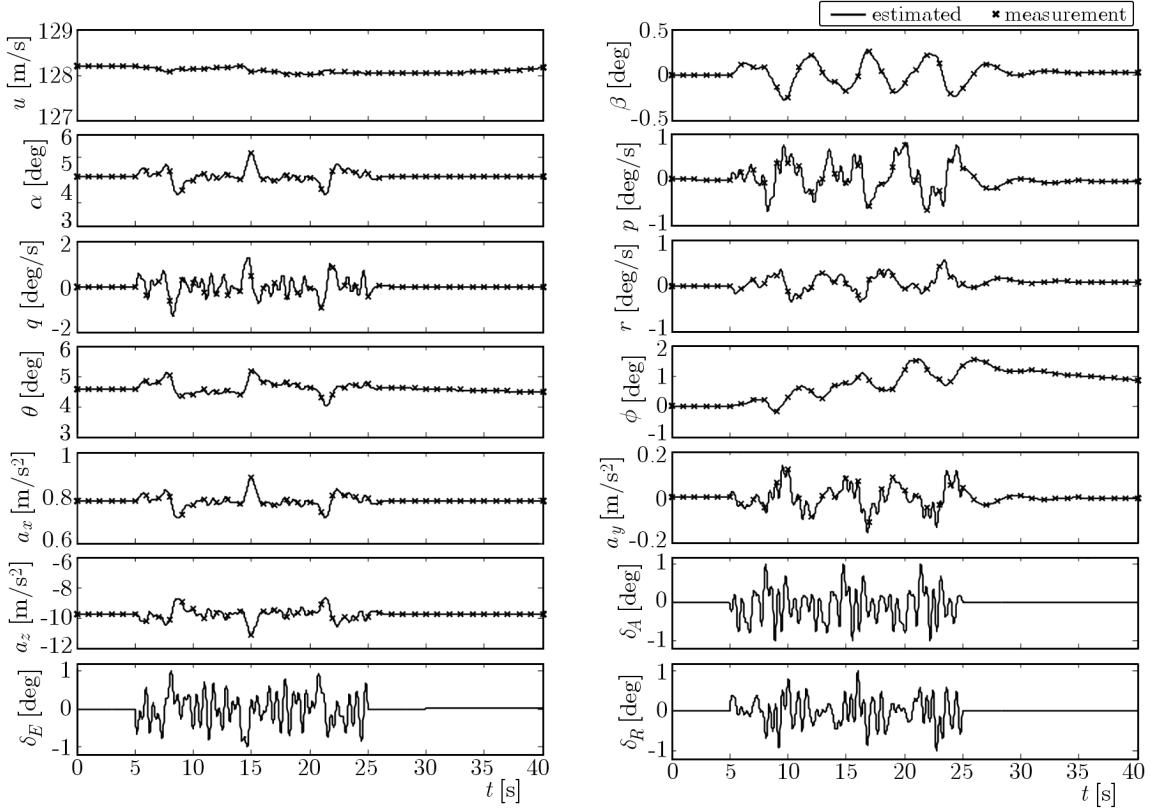
Fig. 4. Measured and estimated signals for the D-optimal multisine inputs

Table 1. Standard deviations

Parameter	Multisine	D-optimal multisine	Parameter	Multisine	D-optimal multisine
k	2.5440E-004	3.5897E-004	C_{l0}	2.2346E-009	7.2915E-010
C_{L0}	3.6488E-005	3.9050E-005	$C_{l\beta}$	2.6671E-006	9.9505E-007
$C_{L_{WB}\alpha}$	2.6817E-004	3.1839E-004	C_{lp}	1.3783E-004	2.1207E-004
$C_{L_H\alpha_H}$	1.1588E-003	1.3114E-003	$C_{lp\alpha}$	1.6153E-003	2.6471E-003
$C_{L_H\delta_E}$	6.8113E-005	8.6226E-005	C_{lr}	2.3736E-004	2.6195E-004
$\partial\varepsilon/\partial\alpha$	1.7339E-004	1.9543E-004	$C_{lr\alpha}$	3.0391E-003	3.2459E-003
C_{D0}	1.0919E-004	1.6787E-004	$C_{l\delta_A}$	1.2369E-006	4.8513E-007
C_{m0}	2.1883E-004	1.8874E-004	$C_{l\delta_R}$	9.6532E-007	4.0260E-007
$C_{m_{WB}q}$	1.0397E-002	1.1892E-002	C_{n0}	2.1906E-009	7.2876E-010
C_{Y0}	2.2318E-008	8.4724E-009	$C_{n\beta}$	2.5441E-006	1.6348E-006
$C_{Y\beta}$	1.3871E-005	5.2384E-006	C_{np}	1.1095E-004	2.1430E-004
C_{Yp}	1.0030E-003	1.6169E-003	$C_{np\alpha}$	1.3218E-003	2.6642E-003
$C_{Yp\alpha}$	1.2198E-002	2.0359E-002	C_{nr}	1.9973E-004	2.3221E-004
C_{Yr}	1.6164E-003	2.0590E-003	$C_{nr\alpha}$	2.5561E-003	2.8803E-003
$C_{Yr\alpha}$	2.0288E-002	2.5716E-002	$C_{n\delta_A}$	1.5451E-006	8.3159E-007
$C_{Y\delta_R}$	4.8513E-006	2.6505E-006	$C_{n\delta_R}$	1.2249E-006	7.7315E-007

In Figs. 3 and 4, solid lines are used to denote the estimated aircraft response and the input signals. The cross points denote the recorded values of flight parameters. For presenting the measurements, a 10-point data reduction is used. From Figs. 3 and 4 it can be seen that the obtained set of estimates allows one to obtain a good visual match for flight parameters whether

the D-optimality criterion is used or not. However, as more than one set of model parameters can produce a response that visually fits well the data, another quality of estimation indicator should be used – standard deviations.

Table 1 indicates that for both analysed designs the evaluated standard deviations are small (of the order of 1.0E-002 or lower) for all parameters what means that the estimation is successful. The standard deviations obtained for model parameters when the D-optimality is used in the design phase are of the same order as when the criterion is not used in the optimization. Therefore we consider that the inclusion of the D-optimality criterion for phase angles optimization in the simultaneous multisine inputs design does not increase significantly the quality of the Sys-ID manoeuvre. Moreover, as the Parameter Error Covariance Matrix determinant minimization is more computationally demanding that Relative Peak Factor optimization, we found the inclusion of the D-optimality criterion impractical.

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