# DYNAMIC RESPONSE OF A SPUR GEAR SYSTEM WITH UNCERTAIN PARAMETERS

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In this paper, we propose a new approach for taking into account uncertainties based on the projection on polynomial chaos. The new approach is used to determine the dynamic response of a spur gear system with uncertainty associated to gear system parameters. The simulation results are obtained by the polynomial chaos approach for dynamic analysis under uncertainty. The proposed approach is an efficient probabilistic tool for uncertainty propagation. It has been found to be an interesting alternative to the parametric studies. The polynomial chaos results are compared with Monte Carlo simulations.

*Keywords:* uncertainty, spur gear system, polynomial chaos approach, gear parameters

## 1. Introduction

The gearing is the best solution to transmit rotational motion and torque, which offers numerous advantages (Dalpiaz *et al.*, 1996) including mechanical reliability. Furthermore, its mechanical efficiency is of the order of 0.96 to 0.99. But today, several applications require gear transmissions to be more and more reliable, light and having long service life. This requires control of the acoustic broadcast and vibratory behavior of such gearings (Begg *et al.*, 2000).

Several parametric studies (coefficient of friction, assembly of defects, manufacturing defects, eccentricity default,...) have shown great sensitivity of the dynamic behavior of gear systems. However, these parameters assume strong dispersions (Guerine *et al.* 2015a,b). Therefore, it becomes necessary to take into account the uncertainties to ensure the robustness of the analysis. Also, there are several studies on reliability in vibration structures taking into account these uncertainties (Abo Al-kheer *et al.*, 2011; El Hami *et al.*, 1993, 2009; El Hami and Radi, 1996).

Several methods are proposed in the literature. Monte Carlo (MC) simulation is a well-known technique in this field (Fishman, 1996). It can give the entire probability density function of any system variable, but it is often too costly since a great number of samples are required for reasonable accuracy. Parallel simulation (Papadrakakis and Papadopoulos, 1999) and proper orthogonal decomposition (Lindsley and Beran, 2005) are some solutions proposed to circumvent the computational difficulties of the MC method.

Polynomial chaos expansion (PCE) is presented in the literature as a more efficient probabilistic tool for uncertainty propagation. It was first introduced by Wiener (1938) and pioneered

by Ghanem and Spanos (1991) who used Hermite orthogonal polynomials to model stochastic processes with Gaussian random variables. The exponential convergence of such an expansion was shown by Cameron and Martin (1947) and then was generalized to various continuous and discrete distributions using orthogonal polynomials from the so called Askey-scheme (Askey and Wilson, 1985; Xiu and Karniadakis, 2003).

Polynomial chaos (PC) gives a mathematical framework to separate stochastic components of a system response from deterministic ones. The stochastic Galerkin method (Babuska *et al.*, 2004; Le Maître *et al.*, 2001), collocation and regression methods (Isukapalli *et al.*, 1998a,b; Crestaux *et al.* 2009) are used to compute deterministic components called stochastic modes in an intrusive and non-intrusive manner while random components are concentrated in the polynomial basis used. Non-intrusive procedures prove to be more advantageous for stochastic dynamic systems since they need no modifications of the system model, contrary to the intrusive method. In the latter, Galerkin techniques are used to generate a set of deterministic coupled equations from a stochastic system model, and then a suitable algorithm is adapted to obtain stochastic modes.

The capabilities of polynomial chaos have been tested in numerous applications, such as treating uncertainties in environmental and biological problems (Isukapalli *et al.*, 1998a,b), in multibody dynamic systems (Sandu *et al.*, 2006a,b), in solving ordinary and partial differential equations (Williams, 2006; Xiu and Karniadakis, 2002), in component mode synthesis techniques (El Hami and Radi, 1996, Sarsri *et al.*, 2011) and parameter estimation (Saad *et al.*, 2007; Blanchard *et al.*, 2009; Smith *et al.*, 2007).

The main originality of the present paper is that the uncertainty of gear system parameters in the dynamic behavior study of the one stage gear system is taken into account. The main objective is to investigate the capabilities of the new method to determine the dynamic response of a spur gear system subject to uncertain gear parameters. So, an eight degree of freedom system modelling the dynamic behavior of a spur gear system is considered. The modelling of a one-stage spur gear system is presented in Section 2. In the next Section, the theoretical basis of the polynomial chaos is presented. In Section 4, the equations of motion for the eight degrees of freedom are presented. Numerical results are presented in Section 5. Finally in Section 6, to conclude, some comments are made based on the study carried out in this paper.

## 2. Modelling of a one-stage gear system

The global dynamic model of a one-stage gear system is shown in Fig. 1. This model is composed of two blocks ($j = 1, 2$). Each block ($j$) is supported by a flexible bearing whose bending stiffness is $k_j^x$ and the traction-compression stiffness is $k_j^y$.

Wheels (11) and (22) characterize respectively the motor side and the receiving side. The shafts ($j$) admit some torsional stiffness $k_j^\theta$.

Angular displacements of each wheel are noticed by $\theta_{(i,j)}$ with the indices $j = 1$ to 2 designating the number of the block, and $i = 1$ to 2 designate the two wheels of each block. Moreover, linear displacements of the bearing denoted by $x_j$ and $y_j$ are measured in the plane which is orthogonal to the wheels axis of rotation.

In this study, we model the gear mesh stiffness variation $k(t)$ by a square wave form (Fig. 2). The gear mesh stiffness variation can be decomposed in two elements: an average component denoted by $kc$, and a time variant one noted by $kv(t)$ (Walha *et al.*, 2009).

The extreme values of the mesh stiffness are defined by

$$k_{min} = -\frac{kc}{2\varepsilon^\alpha} \qquad k_{max} = -k_{min}\frac{2-\varepsilon^\alpha}{\varepsilon^\alpha - 1} \qquad (2.1)$$
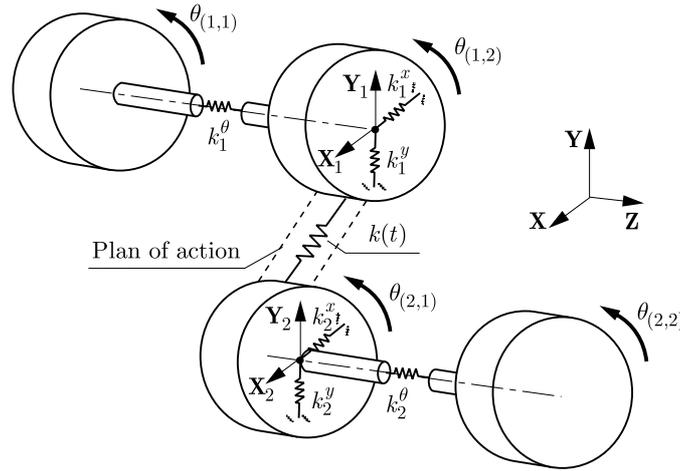
Fig. 1. Global dynamic model of the one stage gear system
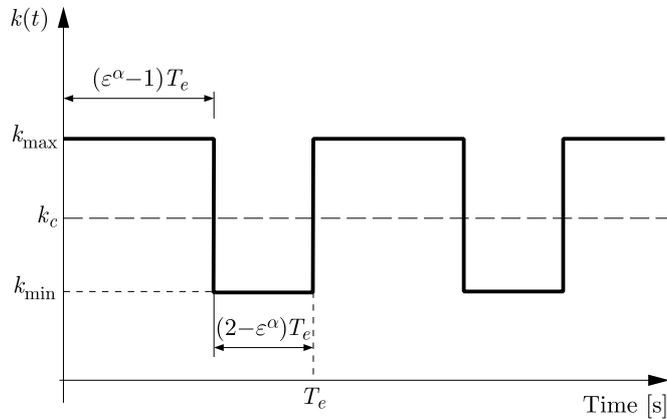


Fig. 2. Modelling of the mesh stiffness variation

where $\varepsilon^\alpha$ and $Te$ represent respectively the contact ratio and mesh period corresponding to the two gearmesh contacts.

## 3. Polynomial chaos approach

In this Section, we propose a new method based on the projection on polynomial chaos. This method consists in projecting stochastic desired solutions on a basis of orthogonal polynomials in which variables are Gaussian orthonormal. The properties of the base polynomial are used to generate a linear system of equations by means of projection. The resolution of this system leads to an expansion of the solution on the polynomial basis, which can be used to calculate the moments of the random solution. With this method, we can easily calculate the dynamic response of a mechanical system.

Let us consider a multi-degrees of freedom linear system with mass and stiffness matrices $\mathbf{M}_T$ and $\mathbf{K}_T$, respectively. The equations of motion describing forced vibration of a linear system are

$$\mathbf{M}_T \ddot{\mathbf{u}}_T(t) + \mathbf{K}_T \mathbf{u}_T(t) = \mathbf{f}_T(t) \tag{3.1}$$

where $\mathbf{u}_T$ is the nodal displacement vector and $\mathbf{f}_T$ is the external excitation.

The chaotic polynomials $\psi_m$ corresponding to the multidimensional Hermite polynomials obtained by formula (3.2)

$$\psi_m(\alpha_1,\ldots,\alpha_P) = (-1)^P e^{\left(\frac{1}{2}\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{\alpha}\right)} \frac{\partial^P e^{\left(\frac{1}{2}\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{\alpha}\right)}}{\partial\alpha_1\cdots\partial\alpha_P} \tag{3.2}$$

where $\boldsymbol{\alpha}$ is the vector grouping the random variables

$$\boldsymbol{\alpha}^{\mathrm{T}} = [\alpha_1,\ldots,\alpha_P] \tag{3.3}$$

The random mass and stiffness matrices $\mathbf{M}_T$ and $\mathbf{K}_T$ of the mechanical system can be written as

$$\mathbf{M}_T = [\mathbf{M}_T]_0 + \widetilde{\mathbf{M}}_T \qquad \mathbf{K}_T = [\mathbf{K}_T]_0 + \widetilde{\mathbf{K}}_T \tag{3.4}$$

where $[\mathbf{M}_T]_0$ and $[\mathbf{K}_T]_0$ are deterministic matrices, $\widetilde{\mathbf{M}}_T$ and $\widetilde{\mathbf{K}}_T$ correspond to the random part of the mass and stiffness matrices. $\widetilde{\mathbf{M}}_T$ and $\widetilde{\mathbf{K}}_T$ are rewritten from an expression of the Karhunen-Loeve type (Ghanem and Spanos, 1991) in the following form

$$\widetilde{\mathbf{M}}_T = \sum_{p=1}^{P}[\mathbf{M}_T]_p\alpha_p \qquad \widetilde{\mathbf{K}}_T = \sum_{p=1}^{P}[\mathbf{K}_T]_p\alpha_p \tag{3.5}$$

where $\alpha_p$ is an independent Gaussian random variable which may correspond to the first polynomial $\psi_p$, while the matrices $[\mathbf{M}_T]_p$ and $[\mathbf{K}_T]_p$ are deterministic.

We pose $\alpha_0 = 1$, we can write then

$$\mathbf{M}_T = \sum_{p=0}^{P}[\mathbf{M}_T]_p\alpha_p \qquad \mathbf{K}_T = \sum_{p=0}^{P}[\mathbf{K}_T]_p\alpha_p \tag{3.6}$$

In the same way, we can write for $\mathbf{f}_T$

$$\mathbf{f}_T = \sum_{p=0}^{P}\{\mathbf{f}_T\}_p\alpha_p \tag{3.7}$$

The dynamic response is obtained by solving the following equation, knowing that the initial conditions are predefined

$$\mathbf{K}_{eq}\mathbf{u}_T(t+\Delta t) = \mathbf{F}_{eq} \tag{3.8}$$

where

$$\begin{aligned} \mathbf{K}_{eq} &= \mathbf{K}_T + a_0\mathbf{M}_T \\ \mathbf{F}_{eq} &= \mathbf{f}_T(t+\Delta t) + \mathbf{M}_T[a_0\mathbf{u}_T(t) + a_1\dot{\mathbf{u}}_T(t) + a_2\ddot{\mathbf{u}}_T(t)] \end{aligned} \tag{3.9}$$

where

$$a_0 = \frac{1}{A\Delta t^2} \qquad a_1 = \frac{B}{A\Delta t} \qquad a_2 = \frac{1}{A\Delta t} \tag{3.10}$$

$A$ and $B$ are the parameters of the Newmark method. $\mathbf{u}_T(t+\Delta t)$ is decomposed on polynomials to $P$ Gaussian random orthnormal variables, where $P$ is the number of random variables

$$\mathbf{u}_T(t+\Delta t) = \sum_{n=0}^{N}\{\mathbf{u}_T(t+\Delta t)\}_n\,\psi_n(\{\boldsymbol{\alpha}_i\}_{i=1}^{P}) \tag{3.11}$$

where $N$ is the polynomial chaos order. It is the degree of polynomial chaos. $\mathbf{K}_{eq}$ and $\mathbf{F}_{eq}$ are written in the following form

$$\mathbf{K}_{eq} = \sum_{p=0}^{P}[\mathbf{K}_T]_p\alpha_p + a_0\sum_{p=0}^{P}[\mathbf{M}_T]_p\alpha_p = \sum_{p=0}^{P}[\mathbf{K}_{eq2}]_p\alpha_p$$

$$\mathbf{F}_{eq} = \sum_{p=0}^{P}\{\mathbf{f}_T(t+\Delta t)\}_p\alpha_p + \sum_{p=0}^{P}[\mathbf{M}_T]_p\alpha_p\Big(a_0\{\mathbf{u}_T(t)\}_0 + a_1\{\dot{\mathbf{u}}_T(t)\}_0 + a_2\{\ddot{\mathbf{u}}_T(t)\}_0\Big) \quad (3.12)$$

$$= \sum_{p=0}^{P}\{\mathbf{F}_{eq2}\}_p\alpha_p$$

Substituting Eqs. (3.11) and (3.12) into Eq. (3.8) and forcing the residual to be orthogonal to the space spanned by the polynomial chaos $\psi_m$, yields the following system of linear equations

$$\sum_{p=0}^{P}\sum_{n=0}^{N}[\mathbf{K}_{eq2}]_p\{\mathbf{u}_T\}_n\langle\alpha_p\ \psi_n\ \psi_m\rangle = \sum_{p=0}^{P}\{\mathbf{F}_{eq2}\}_p\langle\alpha_p\ \psi_m\rangle \qquad m = 0,1,\ldots,N \qquad (3.13)$$

where $\langle\cdot\ \cdot\rangle$ denotes the inner product defined by the expectation operator.

This algebraic equation can be rewritten in a more compact matrix form as

$$\begin{bmatrix} \mathbf{D}^{00} & & \cdots & & \mathbf{D}^{0N} \\ & \ddots & & & \\ \vdots & & \mathbf{D}^{ij} & & \vdots \\ & & & \ddots & \\ \mathbf{D}^{N0} & & \cdots & & \mathbf{D}^{NN} \end{bmatrix} \begin{Bmatrix} \{\mathbf{u}_T(t+\Delta t)\}_0 \\ \vdots \\ \{\mathbf{u}_T(t+\Delta t)\}_j \\ \vdots \\ \{\mathbf{u}_T(t+\Delta t)\}_N \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{f}^{(0)}\} \\ \vdots \\ \{\mathbf{f}^{(j)}\} \\ \vdots \\ \{\mathbf{f}^{(N)}\} \end{Bmatrix} \qquad (3.14)$$

where

$$\mathbf{D}^{(ij)} = \sum_{p=0}^{P}[\mathbf{K}_{eq2}]_p\langle\alpha_p\ \psi_i\ \psi_j\rangle \qquad\qquad \mathbf{f}^{(j)} = \sum_{p=0}^{P}\{\mathbf{F}_{eq2}\}_p\langle\alpha_p\ \psi_j\rangle \qquad (3.15)$$

Due to the orthogonality of the polynomials, most terms $\langle\alpha_p\ \psi_n\ \psi_m\rangle$ are zero. Indeed, we have

$$\langle\psi_i\ \psi_j\rangle_{i\neq j} = 0 \qquad (3.16)$$

After resolution of algebraic system (3.14), the mean values and the variances of the dynamic response are given by the following relationships

$$E[\mathbf{u}_T] = \{\mathbf{u}_T(t+\Delta t)\}_0 \qquad \mathrm{Var}[\mathbf{u}_T] = \sum_{n=1}^{N}\Big(\{\mathbf{u}_T(t+\Delta t)\}_n\Big)^2\psi_j^2 \qquad (3.17)$$

## 4. Equations of motion

The equations of motion describing the dynamic behavior of our system (Fig. 1) are obtained by applying Lagrange formulation, and are given by

$$
\begin{aligned}
&m\ddot{x}_1 + k_1^x x_1 + \sin(\alpha)k(t)\langle L^\delta\rangle \mathbf{Q} = 0\\
&m\ddot{y}_1 + k_1^y y_1 + \cos(\alpha)k(t)\langle L^\delta\rangle \mathbf{Q} = 0\\
&m\ddot{x}_2 + k_2^x x_2 - \sin(\alpha)k(t)\langle L^\delta\rangle \mathbf{Q} = 0\\
&m\ddot{y}_2 + k_2^y y_2 - \cos(\alpha)k(t)\langle L^\delta\rangle \mathbf{Q} = 0\\
&I\ddot{\theta}_{(1,1)} + k_1^\theta(\theta_{(1,1)} - \theta_{(1,2)}) = Cm\\
&I\ddot{\theta}_{(1,2)} - k_1^\theta(\theta_{(1,1)} - \theta_{(1,2)}) + r_{(1,2)}^b k(t)\langle L^\delta\rangle \mathbf{Q} = 0\\
&I\ddot{\theta}_{(2,1)} - k_2^\theta(\theta_{(2,1)} - \theta_{(2,2)}) - r_{(2,1)}^b k(t)\langle L^\delta\rangle \mathbf{Q} = 0\\
&I\ddot{\theta}_{(2,2)} + k_2^\theta(\theta_{(2,1)} - \theta_{(2,2)}) = 0
\end{aligned}
\tag{4.1}
$$

where $\langle L^\delta\rangle$ is defined by

$$
\langle L^\delta\rangle = [\sin(\alpha) \quad -\sin(\alpha) \quad \cos(\alpha) \quad -\cos(\alpha) \quad 0 \quad r_{(1,2)}^b \quad -r_{(2,1)}^b \quad 0]
\tag{4.2}
$$

(4.9) $r_{(1,2)}^b$, $r_{(2,1)}^b$ represent the base radius of gears, $\alpha$ is the pressure angle. $\mathbf{Q}(t)$ is the vector of generalized coordinates

$$
\mathbf{Q}(t) = [x_1 \quad y_1 \quad x_2 \quad y_2 \quad \theta_{(1,1)} \quad \theta_{(1,2)} \quad \theta_{(2,1)} \quad \theta_{(2,2)}]^{\mathrm{T}}
\tag{4.3}
$$

## 5. Numerical simulation

The technological and dimensional features of the one-stage gear system are summarized in Table 1.

**Table 1.** System parameters

| Material: 42CrMo4 | $\rho = 7860\,\mathrm{Kg/m^3}$ |
|---|---|
| Motor torque | $Cm = 200\,\mathrm{N\,m}$ |
| Bearing stiffness | $k_j^x = 10^7\,\mathrm{N/m}, \quad k_j^y = 10^7\,\mathrm{N/m}$ |
| Torsional stiffness of shaft | $k_j^\theta = 10^5\,\mathrm{N\,m/rad}$ |
| Number of teeth | $Z(12) = 40, \quad Z(21) = 50$ |
| Module of teeth | $\mathrm{module} = 4.10^{-3}\,\mathrm{m}$ |
| Contact ratio | $\varepsilon^\alpha = 1.7341$ |
| The pressure angle | $\alpha = 20°$ |

In this Section, numerical results are presented for the new formulations derived in Section 3. The polynomial chaos results are compared with Monte Carlo findings for 100000 simulations.

The mass $m$ and the moment of inertia $I$ of the gears are supposed to be random variables and defined as follows

$$
m = m_0 + \sigma_m \xi \qquad I = I_0 + \sigma_I \xi
$$

where $\xi$ is the zero mean value Gaussian random variable, $m_0$ and $I_0$ are the mean values and $\sigma_m$ and $\sigma_I$ are the associated standard deviations.

The mean value and standard deviation of the dynamic component of the linear displacement of the first bearing in the two directions $x$ and $y$ have been calculated by the polynomial chaos

approach. The obtained results are compared with those given from Monte Carlo simulations for 100000 operations. The results are plotted in Figs. 3a, 4a, 5a and 6a for $\sigma_m = \sigma_I = 2\%$ and in Figs. 3b, 4b, 5b and 6b for $\sigma_m = \sigma_I = 5\%$.

These figures show that the obtained solutions oscillate around the Monte Carlo simulations which are the reference solution. It can be seen that for a small standard deviation $\sigma_m = \sigma_I = 2\%$, the polynomial chaos solutions in the second order polynomial provide a very good accuracy as compared with the Monte Carlo results. When the standard deviation increases, the error increases.
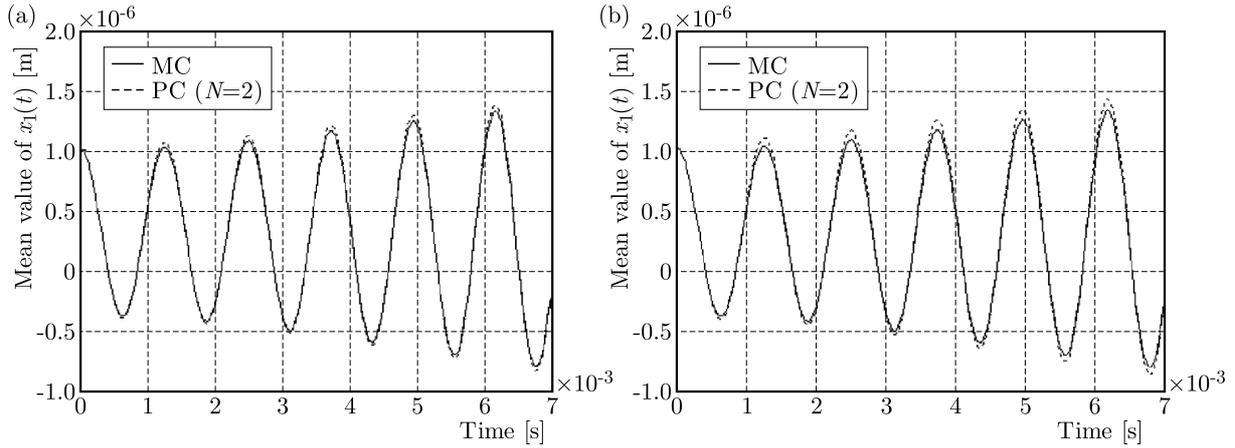


Fig. 3. Mean value of $x_1(t)$ for (a) $\sigma_m = \sigma_I = 2\%$ and (b) $\sigma_m = \sigma_I = 5\%$
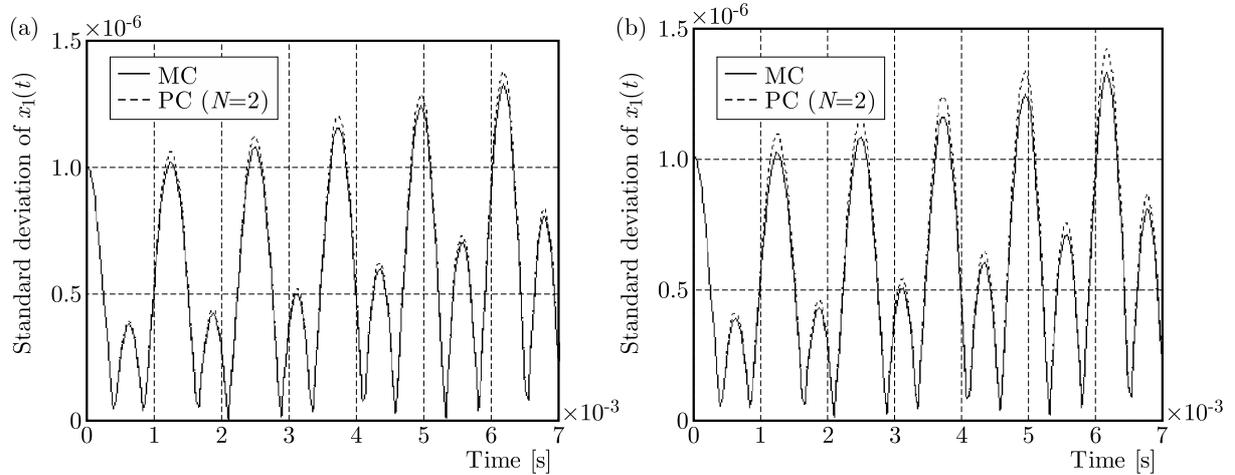


Fig. 4. Standard deviation of $x_1(t)$ for (a) $\sigma_m = \sigma_I = 2\%$ and (b) $\sigma_m = \sigma_I = 5\%$

The mean values of the dynamic component of the angular displacements $\theta_{(1,1)}$ and $\theta_{(2,1)}$ are presented in Figs. 7a and 7b for $\sigma_m = \sigma_I = 5\%$. The polynomial chaos results are compared with the Monte Carlo simulations for 100000 operations. The dynamic response of the angular displacement as predicted by polynomial chaos calculations matches exactly with that of the Monte Carlo analysis. $N = 8$ has been used for the PC model, and it is seen to be enough to capture the dynamic response of the angular displacement of our system.

The mean value and standard deviation of the dynamic component of the linear displacement of the second bearing in the two directions $x$ and $y$ are presented in Figs. 8 and 9 for $\sigma_m = \sigma_I = 10\%$.
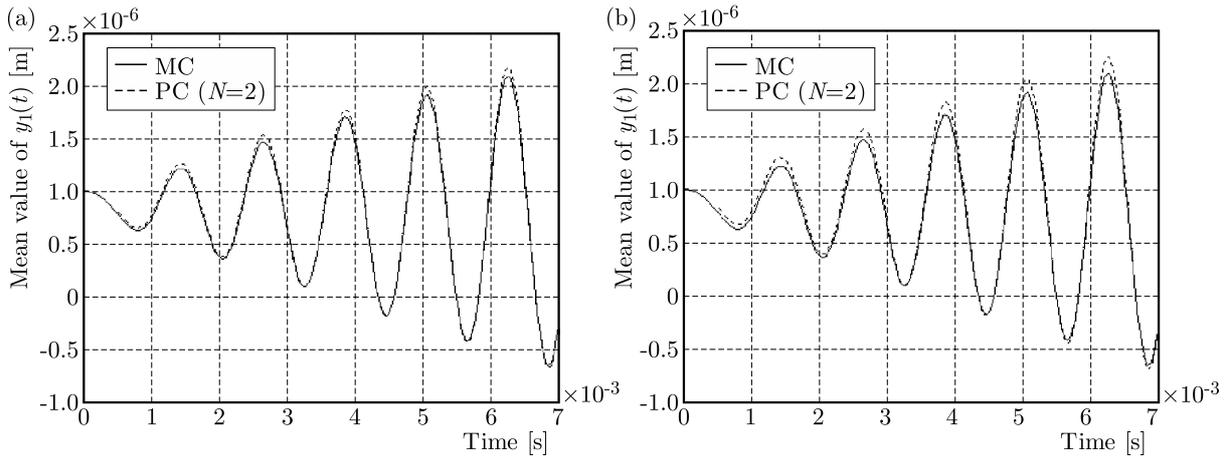
Fig. 5. Mean value of $y_1(t)$ for (a) $\sigma_m = \sigma_I = 2\%$ and (b) $\sigma_m = \sigma_I = 5\%$
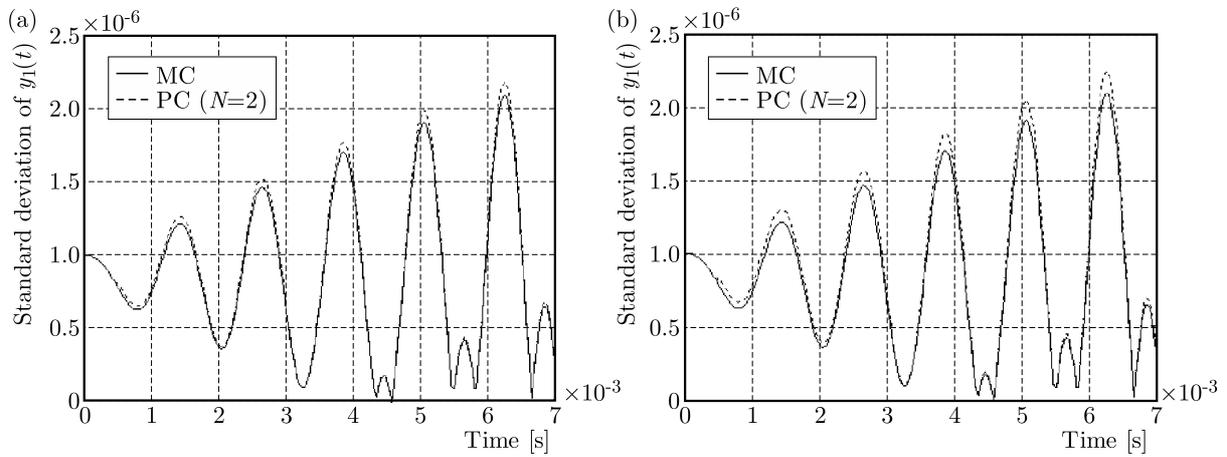


Fig. 6. Standard deviation of $y_1(t)$ for (a) $\sigma_m = \sigma_I = 2\%$ and (b) $\sigma_m = \sigma_I = 5\%$
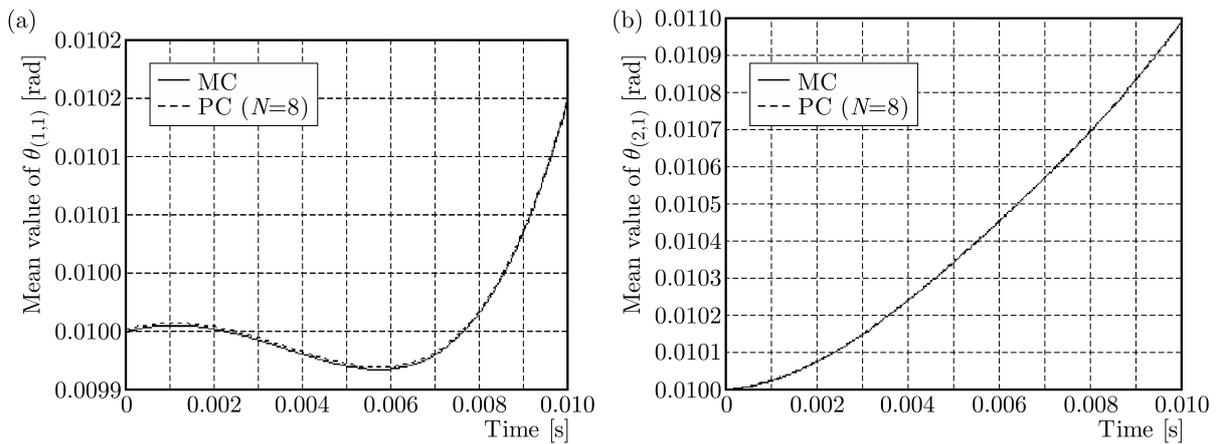


Fig. 7. Mean value of (a) $\theta_{(1,1)}(t)$ and (b) $\theta_{(2,1)}(t)$ for $\sigma_m = \sigma_I = 5\%$

The polynomial chaos results are compared with the Monte Carlo results for 100000 simulations. It is evident from these figures that the case $N = 2$ clearly has not enough chaos terms to represent the output. As $N$ increases, the results seem to become better, and for $N = 8$, the dynamic response of the linear displacement of the second bearing with polynomial chaos values almost exactly match with the Monte Carlo simulation results.
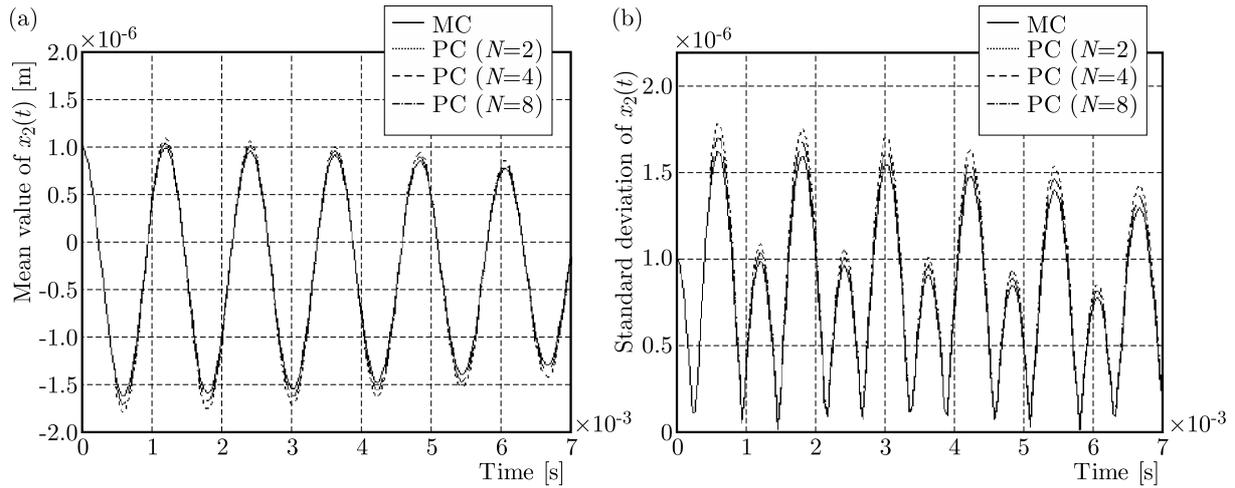
Fig. 8. (a) Mean value of $x_2(t)$ for $\sigma_m = \sigma_I = 10\%$, (b) standard deviation of $x_2(t)$ for $\sigma_m = \sigma_I = 10\%$
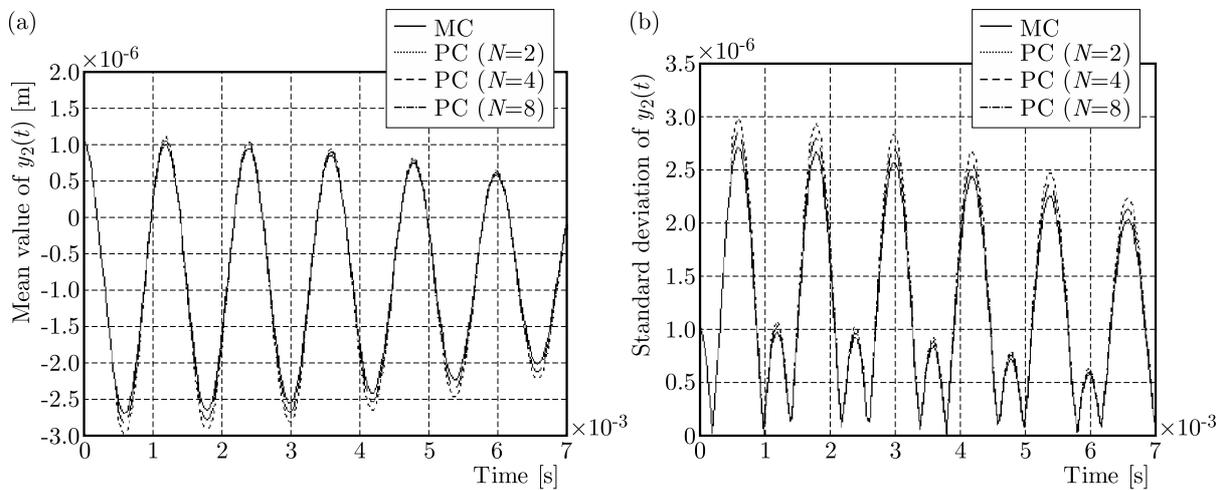


Fig. 9. (a) Mean value of $y_2(t)$ for $\sigma_m = \sigma_I = 10\%$, (b) standard deviation of $y_2(t)$ for $\sigma_m = \sigma_I = 10\%$

## 6. Conclusion

This paper has presents an approach to a one-stage gear system based on the polynomial chaos method while explicitly considering the uncertainty in parameters of the gear. The main results of the present study show that the polynomial chaos method may be an efficient tool to take into account the dispersions of the gear parameters. An interesting perspective is to apply this method to a system with higher degrees of freedom like an epicyclic gear system. Further work in this context is in progress.

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