JOURNAL OF THEORETICAL AND APPLIED MECHANICS
55, 1, pp. 117-127, Warsaw 2017
DOI: 10.15632/jtam-pl.55.1.117

RADIATIVE FALKNER-SKAN FLOW OF WALTER-B FLUID WITH PRESCRIBED SURFACE HEAT FLUX

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This article addresses the Falkner-Skan flow of an incompressible Walter-B fluid. Fluid flow is caused by a stretching wedge with thermal radiation and prescribed surface heat flux. Appropriate transformations are used to obtain the system of nonlinear ordinary differential equations. Convergent series solutions are obtained by the homotopy analysis method. Influence of pertinent parameters on the velocity, temperature and Nusselt number are investigated. It is observed that by increasing the viscoelastic parameter, the fluid velocity decreases. There is an enhancement of the heat transfer rate for the viscoelastic parameter and power law index. It is also found that the Prandtl number and radiation parameter decrease the heat transfer rate.

Keywords: Walter-B fluid, Falkner-Skan flow, prescribed surface heat flux, thermal radiation

1. Introduction

Non-Newtonian materials in view of its complex constitutive expression yield much more complicated and higher order differential systems when compared with viscous materials. Such complexities in differential systems are due to additional rheological parameters appearing in the constitutive relationships. Even a simpler constitutive equation like for Walter-B gives rise to nonlinear boundary initial value problems which are far from trivial. These boundary value problems have great interest of researchers from different quarters. For example Chang *et al.* (2011) numerically analyzed the free convective heat transfer in viscoelastic flow of Walter-B fluid. Nadeem *et al.* (2015) examined oblique flow of Walter-B fluid in presence of magnetohydrodynamics and nanoparticles. Nandeppanavar *et al.* (2010) explored stretched flow of Walter-B liquid in presence of non-uniform heat source/sink. Hakeem *et al.* (2014) extended such analysis in presence of thermal radiation. Stagnation point flow and Blasius flow for Walter-B liquid were also addressed by Madani *et al.* (2012). Hayat *et al.* (2014a, 2015c) examined heat transfer in flow of Walter-B fluid over a surface with Newtonian heating and convective condition. Talla (2013) studied the flow of Walter-B fluid bounded by an exponentially stretching sheet. Peristalsis of Walter-B fluid in a vertical channel was studied by Ramesh and Devakar (2015).

Falkner-Skan flow is quite popular in fluid mechanics. It is a flow past a wedge placed symmetrically with respect to the flow direction. These types of flows occur frequently to increase oil recovery and in packed bed reactor geothermal industries. Interest of recent researchers in boundary layer flow over a continuos moving surface with prescribed surface heat flux has increased so much. These type of flows have many applications in industrial and metallurgical processes such as glass fiber, wire drawing, paper production and metallic plate cooling in cooling bath, etc. Falkner and Skan (1931) presented some approximate solutions for the boundary layer equation. Yacob *et al.* (2011) studied the Falkner-Skan problem for a static and moving wedge with prescribed surface heat flux in a nanofluid. Falkner-Skan flow of the Maxwell fluid with mixed convection was analyzed by Hayat *et al.* (2012). Khan and Pop (2013) examined the nanofluid flow past a moving wedge. Abbasbandy *et al.* (2014b) discussed numerical and analytical solutions for MHD Falkner-Skan flow of the Maxwell fluid. Hendi and Hussain (2012) found the solution for MHD Falkner-Skan flow over a permeable sheet. Fang *et al.* (2012) studied the momentum and heat transfer in Falkner-Skan flow with algebraic decay. Su and Zheng (2011) presented the approximate solution of MHD Falkner-Skan flow over a permeable wall. Abbasbandy *et al.* (2014a) worked for Falkner-Skan flow of an Oldroyd-B fluid in presence of the applied magnetic field.

The radiation effect in boundary layer flow has much importance due to its applications in physics, engineering and industrial fields such as glass production, furnace design, polymer processing, gas cooled nuclear reactors and also in space technology like aerodynamics of rockets, missiles, propulsion system, power plants for inter planetary fights and space crafts operating at high temperatures. Heat transfer through radiation takes place in form of electromagnetic waves. Radiation emitted by a body is a consequence of thermal agitation of its composing molecules. Hayat *et al.* (2013c) worked on mixed convection radiative stagnation point flow in presence of convective boundary conditions. Hayat *et al.* (2013b) also discussed the effect of thermal radiation in MHD flow of thixotropic fluid. Pal (2013) analyzed the effects of thermal radiation, Hall current and MHD in flow over an unsteady stretching surface. Bhattacharyya *et al.* (2012) analyzed the flow of micropolar fluid over a porous shrinking sheet with thermal radiation. Hayat *et al.* (2013a)studied the three-dimensional MHD flow of Eyring-Powell fluid with radiative effects. Rashidi *et al.* (2014) discussed the influence of thermal radiation in MHD mixed convective flow of a viscoelastic fluid due to a porous wedge. Bhattacharyya (2013) presented the MHD Casson fluid subject to thermal radiation. Sheikholeslami *et al.* (2015) adopted a two phase model for MHD flow of a nanofluid with thermal radiation.

The aim of present study is to venture further in the region of Falkner-Skan flow of a non--Newtonian fluid. Thus flow formulation here is based upon constitutive relationship of Walters-B fluid. Analysis of heat transfer is carried out in presence of heat flux and thermal radiation. Transformation procedure has been used for the reduction of partial differential systems to ordinary differential systems. The homotopy analysis technique has been implemented for the development of convergent series solutions. Influences of pertinent parameters on the velocity, temperature and Nusselt number are pointed out.

2. Problem formulation

We consider the steady two-dimensional Falkner-Skan flow of an incompressible Walter-B fluid. Heat transfer analysis is carried out in the presence of prescribed surface heat flux and thermal radiation. The fluid flow is induced via stretching a wedge moving with the velocity $U_w = cx^n$ and the fluid flow being confined to $y \geqslant 0$. Let T_∞ be ambient temperature. The relevant boundary layer equations are (Hakeem *et al.*, 2014)

$$\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_e\frac{dU_e}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-k_0\Big(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial x}\Big)\\
&u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}
\end{aligned} \tag{2.1}$$

The corresponding boundary conditions are (Yacob *et al.*, 2011)

$$\begin{aligned}
&u=U_w=cx^n \qquad v=0 \qquad \frac{\partial T}{\partial y}=-\frac{q_w}{k} \qquad \text{at} \quad y=0\\
&u\to U_e=ax^n \qquad T\to T_\infty \qquad \text{as} \quad y\to\infty
\end{aligned} \tag{2.2}$$

where (u,v) are the velocities along (x,y) directions respectively, T is temperature, ν is kinematic viscosity, k_0 is elastic parameter, k is thermal conductivity, ρ is density, c_p is specific heat, c and a are the stretching rates and q_w the wall heat flux. Radiative heat flux by using Rosseland approximation is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.3}$$

where σ^* is the Stefan-Boltzmann constant and k^* the mean absorption coefficient. Further, we assume that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series. Hence expanding T^4 about T_∞ and neglecting higher order terms, we get

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{2.4}$$

Using Eqs. (2.3) and (2.4) in $(2.1)_3$, we obtain

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\Big(k+\frac{16\sigma^* T_\infty^3}{3k^*}\Big)\frac{\partial^2 T}{\partial y^2} \tag{2.5}$$

Suitable transformations for the present flow are (Yacob *et al.*, 2011)

$$\begin{aligned}
&\eta=\sqrt{\frac{n+1}{2}}\sqrt{\frac{U_e}{\nu x}}y \qquad v=-\sqrt{\frac{n+1}{2}}\sqrt{\frac{\nu U_e}{x}}\Big(f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\Big)\\
&u=axf'(\eta) \qquad \theta=\frac{k(T-T_\infty)}{q_w}\sqrt{\frac{(n+1)U_e}{2\nu x}}
\end{aligned} \tag{2.6}$$

where x is the distance from the leading edge and n the Falkner-Skan power-law parameter. Using Eq. (2.3), the continuity equation is satisfied automatically and Eqs. $(2.1)_2$-(2.2) take the form

$$\begin{aligned}
&f'''+ff''+\frac{2n}{n+1}(1-f'^2)-k_1\Big((3n-1)f'f'''-\frac{n+1}{2}ff''''-\frac{3n-1}{2}f''^2\Big)=0\\
&\frac{1}{\mathrm{Pr}}\Big(1+\frac{4}{3}R\Big)\theta''+f\theta'+\frac{n-1}{n+1}f'\theta=0
\end{aligned} \tag{2.7}$$

and

$$\begin{aligned}
&f(0)=0 \qquad f'(\infty)\to 1 \qquad f'(0)=\alpha\\
&\theta'(0)=-1 \qquad \theta(\infty)\to 0
\end{aligned} \tag{2.8}$$

where k_1 is the viscoelastic parameter, Pr is the Prandtl number, α is the ratio of stretching rates and R is the radiation parameter. The dimensionless parameters are defined as follows

$$k_1=\frac{k_0ax^{n-1}}{\rho\nu} \qquad \mathrm{Pr}=\frac{\rho c_p\nu}{k} \qquad \alpha=\frac{c}{a} \qquad R=\frac{4\sigma^* T_\infty^3}{kk^*} \tag{2.9}$$

The local Nusselt number in the dimensional form is

$$\mathrm{Nu}_x = \frac{x q_w}{k\Big[\frac{q_w}{k}\Big(\frac{2\nu x}{n+1}\Big)U_e(x)\Big]} \tag{2.10}$$

with

$$q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0} + (q_r)_w \tag{2.11}$$

in which q_r is prescribed as follows

$$(q_r)_w = \frac{-16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}\bigg|_{y=0} \tag{2.12}$$

The dimensionless form of the Nusselt number is

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = -\sqrt{\frac{n+1}{2}}\Big(1+\frac{4R}{3}\Big)\theta'(0) \tag{2.13}$$

3. Homotopic solutions

3.1. Zeroth-order deformation equations

Initial approximations and auxiliary linear operators are taken as follows

$$\begin{aligned} &f_0(\eta) = \eta - (1-\alpha)[1-\exp(-\eta)] \qquad \theta_0(\eta) = \exp(-\eta) \\ &\mathtt{L}_f = f''' - f' \qquad \mathtt{L}_\theta = \theta'' - \theta \end{aligned} \tag{3.1}$$

with

$$\mathtt{L}_f[c_1 + c_2\mathrm{e}^{\eta} + c_3\mathrm{e}^{-\eta}] = 0 \qquad \mathtt{L}_\theta[c_4\mathrm{e}^{\eta} + c_5\mathrm{e}^{-\eta}] = 0 \tag{3.2}$$

where c_i $(i = 1\text{-}5)$ are constants.

Denoting $q \in [0,1]$ as the embedding parameter and $\hbar_f$ and $\hbar_\theta$ as the non-zero auxiliary parameters, then the zeroth order deformation problems are

$$\begin{aligned} &(1-q)\mathtt{L}_f[F(\eta,q) - f_0(\eta)] = q\hbar_f N_f[F(\eta,q)] \\ &(1-q)\mathtt{L}_\theta[\vartheta(\eta,q) - \theta_0(\eta)] = q\hbar_\theta N_\theta[\vartheta(\eta,q), F(\eta,q)] \\ &F(0,q) = 0 \qquad F'(0,q) = \alpha \qquad F'(\infty,q) = 1 \\ &\vartheta'(0,q) = -1 \qquad \vartheta(\infty,q) = 0 \end{aligned} \tag{3.3}$$

where the nonlinear differential operators $\mathtt{N}_f$ and $\mathtt{N}_\theta$ are

$$\begin{aligned} \mathtt{N}_f[F(\eta,q)] &= \frac{\partial^3 F(\eta,q)}{\partial\eta^3} + \frac{2n}{n+1}\Big[1 - \Big(\frac{\partial F(\eta,q)}{\partial\eta}\Big)^2\Big] + F(\eta,q)\frac{\partial^2 F(\eta,q)}{\partial\eta^2} \\ &\quad - k_1\Big[(3n-1)\frac{\partial F(\eta,q)}{\partial\eta}\frac{\partial^3 F(\eta,q)}{\partial\eta^3} - \frac{n+1}{2}F(\eta,q)\frac{\partial^4 F(\eta,q)}{\partial\eta^4} - \frac{3n-1}{2}\Big(\frac{\partial^2 F(\eta,q)}{\partial\eta^2}\Big)^2\Big] \\ \mathtt{N}_\theta[\vartheta(\eta,q), F(\eta,q)] &= \frac{1}{\mathrm{Pr}}\Big(1+\frac{4}{3}R\Big)\frac{\partial^2\vartheta(\eta,q)}{\partial\eta^2} + \frac{\partial\vartheta(\eta,q)}{\partial\eta}F(\eta,q) \\ &\quad + \frac{n-1}{n+1}\frac{\partial F(\eta,q)}{\partial\eta}\vartheta(\eta,q) \end{aligned} \tag{3.4}$$

3.2. m-th order deformation equations

The m-th order deformation problems are

$$\mathtt{L}_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f \mathtt{R}_{f,m}(\eta) \qquad \mathtt{L}_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_\theta \mathtt{R}_{\theta,m}(\eta) \tag{3.5}$$

and

$$f_m(0) = \frac{\partial f_m(0)}{\partial\eta} = \frac{\partial f_m(\infty)}{\partial\eta} = \theta'(0) = \theta(\infty) = 0 \tag{3.6}$$

where $\mathtt{R}_{f,m}(\eta)$ and $\mathtt{R}_{\theta,m}(\eta)$ have the following forms

$$\begin{aligned}
\mathtt{R}_{f,m}(\eta) &= f'''_{m-1} + \frac{2n}{n+1}\Big(1 - \sum_{k=0}^{m-1} f'_{m-1-k}f'_k\Big) + \sum_{k=0}^{m-1} f_{m-1-k}f''_k \\
&\quad - k_1\Big[(3n-1)f'_{m-1-k}f'''_k - \frac{n+1}{2}\sum_{k=0}^{m-1} f_{m-1-k}f''''_k - \frac{3n-1}{2}\sum_{k=0}^{m-1} f''_{m-1-k}f''_k\Big] \\
\mathtt{R}_{\theta,m}(\eta) &= \frac{1}{\mathrm{Pr}}\Big(1 + \frac{4}{3}R\Big)\theta''_{m-1} + \sum_{k=0}^{m-1} f_{m-1-k}\theta'_k + \frac{n-1}{n+1}\sum_{k=0}^{m-1} f'_{m-1-k}\theta_k
\end{aligned} \tag{3.7}$$

and

$$\chi_m = \begin{cases} 0 & \text{for} \quad m \leqslant 1 \\ 1 & \text{for} \quad m > 1 \end{cases} \tag{3.8}$$

The general solutions (f_m, θ_m) comprising the special solutions (f_m^*, θ_m^*) are

$$\begin{aligned}
f_m(\eta) &= f_m^*(\eta) + c_1 + c_2\mathrm{e}^{\eta} + c_3\mathrm{e}^{-\eta} \\
\theta_m(\eta) &= \theta_m^*(\eta) + c_4\mathrm{e}^{\eta} + c_5\mathrm{e}^{-\eta}
\end{aligned} \tag{3.9}$$

where the constants c_i ($i = 1$-5) through boundary conditions (3.6) are

$$\begin{aligned}
c_1 &= -c_3 - f_m^*(0) \qquad & c_3 &= \frac{\partial f_m^*(0)}{\partial\eta} \\
c_5 &= \frac{\partial\theta_m^*(0)}{\partial\eta} & c_2 &= c_4 = 0
\end{aligned} \tag{3.10}$$

4. Convergence analysis

The homotopy analysis method has great advantage to adjust the convergence region by selecting the appropriate values of $\hbar_f$ and $\hbar_\theta$. For this, we plot the $\hbar$-curves for the convergence of velocity and temperature profiles (see Fig. 1). Admissible values of auxiliary parameters are $-0.9 \leqslant \hbar_f \leqslant 0$ and $-0.6 \leqslant \hbar_\theta \leqslant -0.2$. The solution converges in the whole region of η $(0 \leqslant \eta \leqslant \infty)$ when $k_1 = 0.2$, $n = 0.1$, $R = 1.6$, $\mathrm{Pr} = 1.5$ and $\alpha = 0.9$.

Table 1 shows the convergence of functions $f''(0)$ and $\theta''(0)$ at a different order of approximations. Tabulated values show that the 25-th order of approximations is enough for the convergence of $f''(0)$, and the 22-th order of approximation is appropriate for the convergence of $\theta''(0)$.

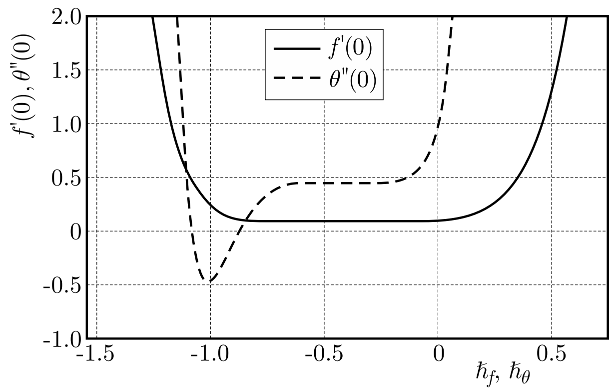

Fig. 1. $\hbar$-curves for $f''(0)$ and $\theta''(0)$ when $k_1 = 0.2$, $n = 0.1$, $R = 1.6$, $\Pr = 1.5$ and $\alpha = 0.9$.

Table 1. Convergence of HAM (homotopy analysis method) solutions when $k_1 = 0.2$, $n = 0.1$, $R = 1.6$, $\Pr = 1.5$,$\alpha = 0.9$, $\hbar_f = -0.2 = \hbar_\theta$

Order of approximation	$f''(0)$	$\theta''(0)$
1	0.09734	0.8154
5	0.09262	0.8203
10	0.09262	0.4547
15	0.09171	0.4454
20	0.09168	0.4447
22	0.09169	0.4446
25	0.09170	0.4446
30	0.09170	0.4446
35	0.09170	0.4446
40	0.09170	0.4446
45	0.09170	0.4446

5. Discussion

In this Section, we discussed the influences of different physical parameters on the fluid velocity, temperature and heat transfer rate.

5.1. Dimensionless velocity profile

Figures 2a-2c show the effect of viscoelastic parameter k_1, power law index n and stretching rates ratio α on the velocity profile. Figure 2a depicts the influence of the viscoelastic parameter on $f'(\eta)$. As k_1 increases, the fluid velocity decreases which corresponds to a thinner momentum boundary layer thickness. The viscoelasticity produces tensile stress which contracts the boundary layer and, consequently, the velocity reduces. Figure 2b represents the impact of α on the velocity profile. Here, the velocity enhances by increasing α. In fact higher values of α correspond to the stronger free stream velocity which enhances the fluid velocity. The effect of Falkner-Skan power law index n is graphed in Fig. 2c. It is observed that velocity is an increasing function of n.

5.2. Dimensionless temperature profile

Figures 3 and 4 show the impact of the Prandtl number Pr, radiation parameter R, viscoelastic parameter k_1, power law index n and ratio of stretching rates α on the temperature profile.

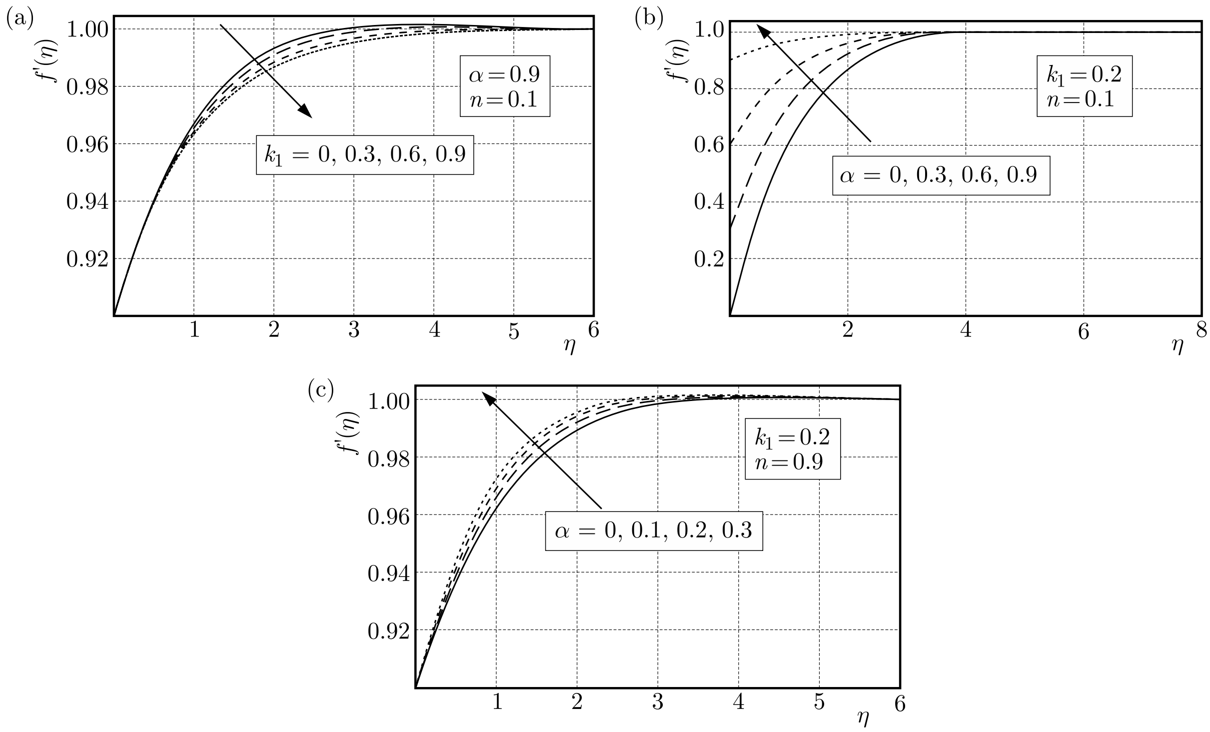

Fig. 2. Impact of: (a) k_1, (b) α and (c) n on $f'(\eta)$

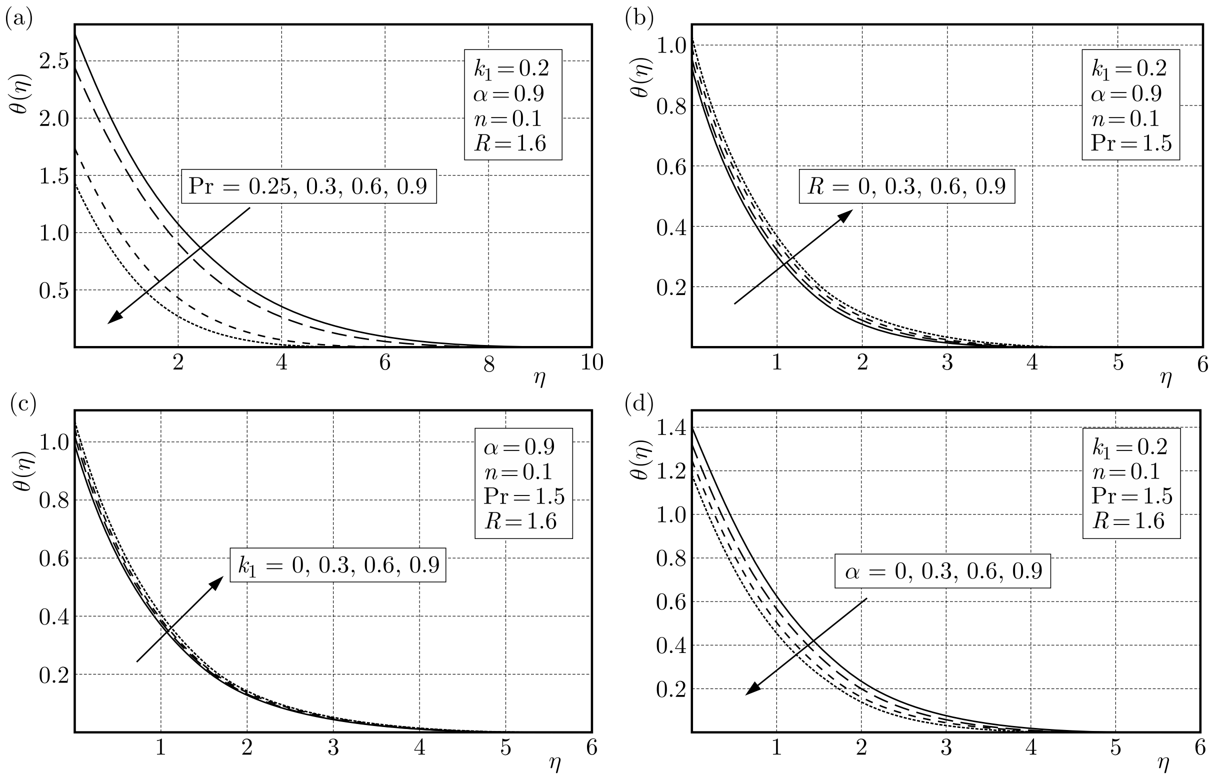

Fig. 3. Impact of (a) Pr, (b) R, (c) k_1, (d) α

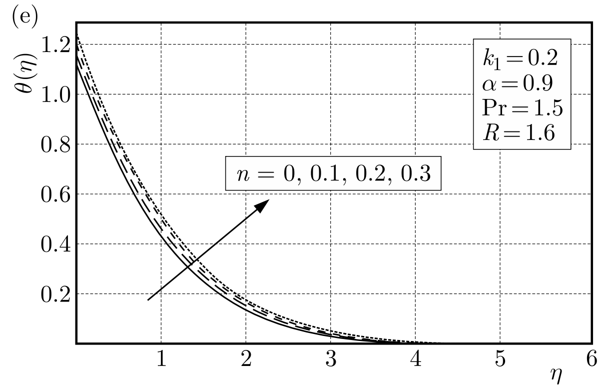

Fig. 4. Impact of n on $\theta(\eta)$

Figure 3a shows the effect of Pr on the temperature profile. For increasing values of the Prandtl number, the temperature decreases. Higher values of Pr correspond to low thermal diffusivity, and the fluid temperature decreases. Figure 3b depicts the behavior of fluid temperature for the radiation parameter R. This figure shows that the temperature profile enhances when radiation effects strengthen. An increase in the radiation parameter corresponds to a decrease in the mean absorption coefficient. Hence the rate of radiative heat transfer to the fluid increases. Figure 3c describes the behavior of temperature for viscoelastic parameter. Fluid temperature enhances for increasing k_1. Figure 3d presents the effect of stretching ratio rates α on the temperature profile. The temperature profile shows decreasing behavior for increasing values of α. The velocity increases when the ratio of stretching rates enhances. There is less resistance for fluid particles motion and, consequently, the temperature reduces. Figure 4 shows the effect of increasing values of n on fluid temperature. The temperature profile and n have a direct relation with each other.

5.3. Nusselt number

In this Section, we show the effects of different physical parameters on the Nusselt number. Figures 5a-5d depict the influence of the viscoelastic parameter k_1, Falkner-Skan power law index n, radiation parameter R and Prandtl number Pr. These figures show that by increasing the viscoelastic and power law index parameters, the rate of heat transfer increases whereas the Nusselt number shows decreasing behavior for increasing values of the radiation parameter and the Prandtl number.

6. Conclusions

The Falkner-Skan wedge flow of Walter-B fluid is studied in presence of thermal radiation and prescribed surface heat flux. Key points of the presented analysis are as follows:

- Fluid velocity is a decreasing function of the viscoelastic parameter and increasing function of the ratio of stretching rates.
- The Prandtl number and radiation parameter have opposite impact on the temperature profile.
- For increasing values of the viscoelastic parameter, the temperature enhances.
- The Nusselt number has opposite impact on the power law index and the Prandtl number.

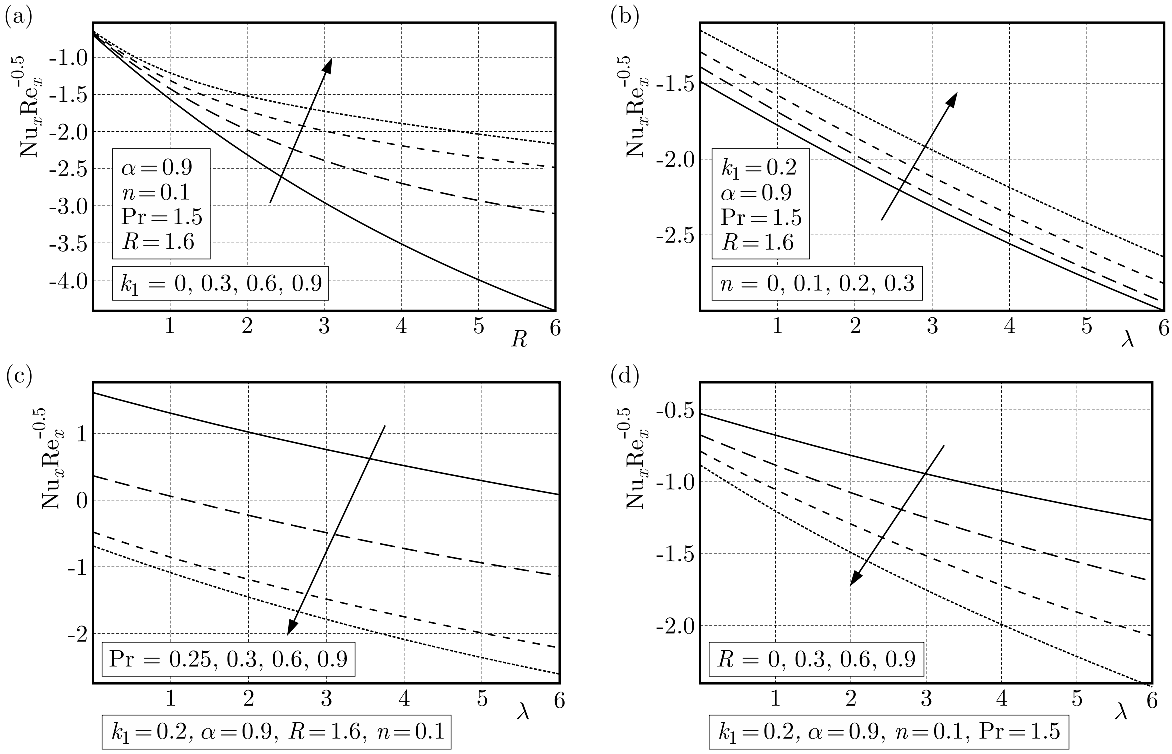

Fig. 5. Impact of (a) k_1, (b) n, (c) R and (d) Pr on $\mathrm{Nu}_x\mathrm{Re}_x^{-0.5}$ (see Eq. (2.13))

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Manuscript received March 30, 2016; accepted for print May 10, 2016